# HYDRAULIC MOTORS:

## TURBINES AND PRESSURE ENGINES

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## TURBINES AND PRESSURE ENGINES

### FOR THE USE OF

*ENGINEERS, MANUFACTURERS, AND
STUDENTS*

BY

G. R. BODMER, A.M. INST. C.E.

*SECOND EDITION, REVISED AND ENLARGED
WITH 204 ILLUSTRATIONS*

# PREFACE TO FIRST EDITION

THE author's attention having been directed to the absence of any modern English work of a sufficiently comprehensive character on the subject of turbines, he was induced to write the present treatise in the hope of to some extent repairing the deficiency. His object has been to give in as elementary a manner as is consistent with accuracy and thoroughness an explanation of the principles underlying the action of turbines and water-pressure engines, and of the application of these principles to the design of such motors, together with descriptions of the most distinctive types, accounts and results of experiments, numerical examples, comparisons of theoretical with practical results, and a brief review of the various methods of water measurement.

Historical matter has been purposely avoided, as well as descriptions of obsolete forms of motors.

The theory of reaction turbines adopted by the author is essentially that of Redtenbacher, modified somewhat in detail on the lines followed by Professor Zeuner in his lectures at continental technical universities.

In applying the theory to the design of turbines, the

author has taken, he believes, a course somewhat different from that generally in vogue, and, in his opinion, better suited for the comparison of different types of turbines, while at the same time it allows the distinctive character of the motor to be determined with accuracy from the very beginning. The leading feature of this method consists in the assumption of the ratio of the guide outflow-area to the bucket outflow-area as a starting-point for the construction of any turbine.

The investigations of the performances and peculiarities of American wheels are, as far as the author is concerned, original, although only the accepted theory has been employed.

The tables of co-efficients of flow, theoretical efficiencies, comparison of theoretical with calculated results, and dimensions of turbines actually made, have been especially compiled and calculated for this work by the author himself.

In writing a technical work on any subject, it is always a matter of some difficulty to know exactly where to draw the line between what should and what should not be included in it, and in any case it will certainly not please all critics; for one there will be too much mathematics, for another too little history, a third will find fault because nothing is said about the actual manufacture in the shop, while a fourth wants to know why all mention of his own particular invention has been omitted !

There is one criticism, however, which in the present case the author thinks it well to meet in advance, as he

feels tolerably certain that it will otherwise be made. This is, that mathematics have been too freely employed in the present work. In reply to this, it may be pointed out that in the majority of cases it is shown how, in applying the principles demonstrated to the design of motors, graphical may be substituted for analytical methods. At the same time, nothing beyond elementary analysis is involved in the mathematics employed, and it is essential to completeness that the algebraical expressions for the laws and rules relating to the subject should be given. To those familiar with the use of formulas, they afford the readiest means of stating facts which in the simplest cases can only be verbally defined in very long and elaborate phraseology. Circumstances, too, often arise in which dimensions have to be determined and estimates made for preliminary purposes, under conditions in which a drawing-board and instruments are not available, and then analytical methods come in. The author has himself on more than one occasion made preliminary calculations while travelling on the railway.

In writing the present treatise the author has consulted to a greater or less extent the works or published papers of the following authors: Francis, Lehmann, Meissner, Redtenbacher, Rittinger, Weisbach, Professors Fliegner, Schröter, Thurston, and Unwin, and Mr. Clemens Herschel, and, especially for hydraulic pressure engines, Knoke's work, 'Die Kraftmaschinen des Kleingewerbes.' He is further indebted for much information and for illustrations to the 'Minutes of Proceedings of the

Institution of Civil Engineers,' 'Minutes of Proceedings of the Institution of Mechanical Engineers,' 'Transactions of the American Society of Civil Engineers,' 'Transactions of the American Society of Mechanical Engineers,' 'Engineering,' 'The Engineer,' 'Industries,' 'Civilingenieur,' and 'Zeitschrift des Vereins Deutscher Ingenieure.'

The following gentlemen and firms the author has to thank for information personally supplied: Professor Fliegner of Zürich, Professor Schröter of Munich, Professor Thurston, Blanchod and Co. of Vevey, Bryan Donkin and Co., Mr. W. Günther of Oldham, Mr. John Hastie, Mr. Clemens Herschel of Holyoke, Messrs. Howes and Ewell, the Humphrey Machine Co. of Keene, U.S., Rieter and Co. of Winterthur, Mr. Arthur Rigg, Mr. Theodore Risdon, Mr. Hamilton Smith, Jun., and Mr. Alphonse Steiger representative of Messrs. Escher, Wyss and Co. of Zürich.

In the matter of illustrations of turbines and hydraulic pressure engines Meissner's and Knoke's works have respectively been especially serviceable.

In selecting certain types of motors for description, the author does not thereby imply that such types are superior to others, but as a rule has chosen those about which he was able to obtain the most authentic experimental data and details. Of many firms applied to, only one, Messrs. Rieter and Co., gave the information desired, which has been tabulated in Table B.

The method of measuring the vane angles adopted in this work is different from that generally in vogue, in so

far that the angle formed between the vane and a perpendicular to the direction of rotation is given instead of the complementary angle. This will be found more convenient for constructive purposes in radial turbines; the effect on the formulas is that the sine takes the place of the cosine, and *vice versâ*. Where, according to the general practice, an angle is given as say 20°, according to the author's method—which he has borrowed from Professor Zeuner—it would be given as $90° - 20° = 70°$.

The author makes no claim to an exhaustive treatment of the subject, but believes that the matter contained in his work will be found useful to those for whom it is intended.

# PREFACE TO SECOND EDITION

SINCE the first edition of this work appeared, little of
a novel character in the form of hydraulic motors has been
introduced to the technical world. In preparing the
second edition, therefore, the task of the author has been
mainly to carefully revise the original text, where neces-
sary, correct a few slight clerical errors and errors of
calculation, and add descriptions of such new installations
as appeared especially deserving of notice. Among the
latter is the Niagara Falls installation.

In several instances descriptive and theoretical matter
has been amplified and supplemented, and numerous
diagrams and illustrations have been added.

The new, revised, and supplementary matter will in the
main be found under the following headings:—Construc-
tion of vane curves; Determination of outflow areas;
Girard turbine with draft-tube; "Vortex" turbine;
"Pelton" wheel; Turbines at Schaffhausen; Niagara
installation; "Purdon-Walters" steam-motor; "Venturi"
water-meter; and curves of kinetic energy, pressure differ-
ence, and hydraulic losses for various diameter-ratios of
radial turbines.

The second part of the work on "Hydraulic Pressure Engines" has not been altered nor enlarged, and is intended merely as a sketch of this branch of hydraulic engineering.

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# HYDRAULIC MOTORS.

## CHAPTER I.

### INTRODUCTION.

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available, and in what forms.

*General principles applicable to hydraulic motors :—*
Flow of water in closed channels.—Hydro-dynamic equation.—
Continuity of flow.—Motion, relative and absolute.—Illustrations.

*Action of water on curved vanes :—*
Deflection of water in turbines.—Force due to deflection.—Avoid-
ance of impact or shock.—Mathematical conditions.—Graphical
construction.—Work done.—Centrifugal action.—Explanation of
increased relative velocity.—Effect of shock.

THE use of water for performing mechanical work dates
back to an extremely early age, some primitive form of
water-wheel having been probably the first hydraulic
motor. In Sicily, what may be considered as a primitive
form of impulse wheel with flat vanes, introduced by the
Saracens, is in use at the present day. The historical
aspect of the subject of water power does not, however,
come within the scope of the present work, which is

B

limited to the principles and practice of the construction of water motors, and the results obtained with the latter.

In many parts of Europe, and especially in Switzerland and the mountainous portions of South Germany, the available water power of the numerous streams and small rivers is taken advantage of to a large extent for industrial purposes, while in the United States a very considerable proportion of the total power expended in manufactories, mills, and similar establishments is derived from hydraulic sources.

In the mountainous districts of Europe, as a rule the falls are high and the quantities of water small, while, on the other hand, in America the falls are moderate or low, and the quantities of water very large, so that the two regions may be taken as illustrating extreme phases of the occurrence of water power. In the United Kingdom there is no doubt that a vast quantity of water power that might be advantageously utilized is entirely neglected, but it may be admitted that in many instances the reasons for this are sufficiently obvious: cheap coal and choice of locality, and sometimes the necessity for expensive works in order to make the water power available. When coal becomes dearer, as it undoubtedly will, more attention must be paid to the hydraulic resources of the country. Of course the absence of large rivers or high falls makes it impossible that a very large proportion of the power required for our numerous manufactories should be drawn from hydraulic sources, but making every allowance on this score, there is yet an extensive field open to the enterprising hydraulic engineer.

Water power is—perhaps after wind power—the most natural, and at the same time truly economic source of energy    Every ton of coal burnt is, as far as our knowledge

goes, irretrievably lost; the process involved in its uti-
lization is *irreversible*. On the other hand, in taking
advantage of the energy now uselessly expended on
friction, erosion, and other resistances, by our rivers and
streams, use is made of a complete cycle already pro-
vided by Nature; the water consumed in producing
power is raised again by the sun, falls as rain and re-
plenishes the river or stream from which the supplies are
drawn.

In most cases where so-called water power is employed
in the performance of mechanical work the real agent is
gravity; the fluid itself being the medium through which
the action of gravity is transmitted to the prime motor.

It is quite possible that stones, earth, sand, &c., should
be utilized in a similar manner, in each case the motive
power being that of gravity.

In order that water may be available for the purpose
of doing work, it must be in such a position that it can
fall from a higher to a lower level, or must be under
pressure produced by some external force, such as that
of a weight or spring acting on the surface of the fluid
through a piston or plunger.

Under the former conditions its utmost capacity for
doing work—potential energy, or energy of position—
is the product of the height through which it can fall
into the weight of water falling, so that if $h$ denote the
available height of the fall, and $G$ the weight of water
falling per second, $h \times G$ equals the energy or power
available per second.

If the fluid is allowed to fall without resistance under
the action of gravity, either free or confined in pipes or
vessels, the power available is expended in imparting
velocity to the water, and the potential energy or energy
of position is converted into kinetic energy or energy of

motion, and in this form is available for performing
work.

It is, however, not necessary that the potential energy [1]
of the water should be first transformed into kinetic energy
in order that it may be employed for motive power ; the
weight of the fluid can also be allowed to act directly on
the prime motor in a manner similar to that in which the
weight of a body attached to one end of a rope, passed
over a pulley for instance, may be made to raise another
body suspended at the opposite end.

A third way of utilizing water power for doing work
is by means of its *pressure*, but the difference between
this method and the preceding is more apparent than
real, and consists chiefly in the form of the motor.
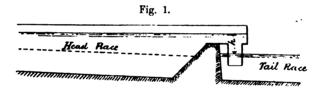
The so-called *pressure* of water is the result of weight or
its equivalent, so that in this, as in the preceding case, the
mechanical work is really done directly by the weight of
the fluid, or in some cases by the pressure of steam or other
elastic fluid acting through an accumulator.   For practical
purposes it may nevertheless be said that there are three
ways in which hydraulic power can be applied to the per-
formance of work : (1) as kinetic energy, or through the
velocity of the fluid ; (2) by weight, and (3) by pressure.

Each of these three methods requires a different type of
motor for its application, denoted respectively as (1) Tur-
bine, (2) Water-wheel, and (3) Water-pressure engine.

The most usual source of hydraulic power in Nature is
a river or stream, but to make this available for practical
purposes some form of works, such as dams, canals, and
aqueducts, is almost invariably necessary.   A river-bed
has always a certain fall or gradient ; for instance, one

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[1] In some quarters the term "potential energy" is objected to, but
the author considers it less cumbersome and more generally expres-
sive than "energy of position."

foot in every hundred feet of length—a very extreme case, as a foot to the *mile* would be more nearly what occurs in England ; to be able to take advantage of this fall for doing work, the portion of it utilized must be applied in one or several nearly perpendicular steps. In order, for instance, to obtain a fall of six feet from a river having a gradient of one foot in every hundred, one plan would be to construct a dam or weir across it of such a height that the level of the surface of the water on the up-stream side of the dam is elevated 6 ft. over that of the water immediately below on the down-stream side ; the surface of the water above the dam will then be raised for a length of 600 ft. *at least,* and instead of having the gradient due to the inclination of the river-

Fig. 1.



bed, will be *approximately* horizontal. This is plainly shown in Fig. 1, where the dotted line represents the original course of the river surface, the full line that after the construction of the dam. As a matter of fact, the effect of a dam on the surface of the river above it extends beyond the point of intersection of the hori-zontal full line with the dotted line indicating the original surface, in some cases nearly twice this dis-tance, and the surface is nowhere truly horizontal.

The head which was previously expended almost entirely in overcoming the resistance of the river-bed over a distance of 600 ft. while the water gradually de-scended 6 ft. down a gentle incline, will now be available

for doing a corresponding amount of useful work by a *sudden drop* through the same height as before. The energy originally wasted in useless friction is accumulated in the form of "head" immediately behind the dam. The water flowing over the dam is taken into the chamber or reservoir containing the motor through which it passes, generally vertically to the tail-race, or, as with an overshot wheel, is conducted by a pen-trough or channel, issuing from the top of the dam, to the point from which it flows into the buckets. In other cases, where it is not admissible or possible to construct a dam, or where only a portion of the water of the river or stream is required, the necessary quantity is drawn off by a separate channel, at a sufficient distance above the point where the power is required to obtain the desired fall.

Occasionally water power is available in the form of a natural waterfall, and then it is simply necessary to guide the fluid in a tube or channel by which it is conducted to the motor. In such cases the greater part of the energy utilized was previously lost by impact in producing eddies, heat, and overcoming the resistance of the air.

Sometimes the velocity only of the flowing stream is employed for working a motor, the "head" in that case being already converted into kinetic energy.

## *General Principles applicable to Hydraulic Motors.*

Before proceeding to consider in detail the theory of each kind of hydraulic motor, it will be necessary to give an outline of the theoretical principles common to all, and investigate, from a general point of view, some portions of the phenomena occurring, previous to applying those principles to the solution of particular problems.

In order to understand the action of hydraulic motors

it is chiefly necessary to have a knowledge of the be-
haviour of water in pipes and channels, its outflow
through orifices, and its action when flowing over curved
surfaces. It is assumed that the reader is acquainted
with the fundamental principles of hydraulics and me-
chanics, so that it will only be necessary to deal briefly
with those portions of the former most closely connected
with the theory of hydraulic motors.

## Flow of Water in Pipes and Closed Channels.

The flow of water under pressure through pipes or
vessels of varying section, always under the assumption
that every part is at all times quite full of the fluid, is
subject to the law of Borda and Carnot. This law is
nothing else than an application of *the principle of the
conservation of energy* to water. Expressed in words it
may be stated generally as follows:

The sum of the potential and kinetic energy of the
fluid flowing through any section of the pipe or vessel is
equal to that sum for any other section plus or minus
any access or loss of energy from external sources.

The potential energy, or energy of position, of, say, a
pound of water at any pressure $p$ is represented by the
product of this pressure and the specific volume or
volume occupied by one pound of water; for any number
of pounds flowing through any section of a pipe, the
potential energy is simply the pressure multiplied by the
total volume. Suppose, for instance, that through some
section of a pipe 62 lbs. of water per second flow at the
pressure of 1000 lbs. per square foot; the total volume of
62 lbs. of water is about one cubic foot, and the potential
energy 1000 foot pounds. It is convenient to express
the pressure of a fluid in the equivalent *head* which
would produce this pressure; as the specific volume is

the inverse value of the specific weight, and since the potential energy of one pound at the pressure $p$ is $p \times$ *specific volume*, it can also be written $\dfrac{p}{\text{specific weight}}$, and this is nothing else than the equivalent head expressed in the unit selected; the equivalent head in feet for a pressure of 1000 lbs. per square foot would be roughly $\dfrac{1000}{62} = 16\cdot1$ ft., and the same figure of course gives the potential energy in foot pounds of one pound of water under the pressure in question.
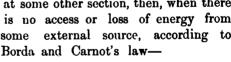
The kinetic energy of one pound of water due to its velocity $c$, is $\dfrac{c^2}{2g}$, or since $g = 32\cdot2$, $\dfrac{c^2}{64\cdot4}$.
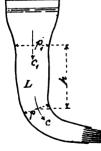
The sum of the potential and kinetic energy of one pound of water flowing with the velocity $c$ at the pressure $p$ through any section is $S = \dfrac{p}{s} + \dfrac{c^2}{2g}$ where $s$ is the weight of one cubic foot of water (when the foot is the unit employed).

If $p_1$ and $c_1$ denote respectively the pressure and velocity at some other section, then, when there is no access or loss of energy from some external source, according to Borda and Carnot's law—



Fig. 2.

$$\frac{p}{s} + \frac{c^2}{2g} = \frac{p_1}{s} + \frac{c_1^2}{2g}.$$

If the point where the pressure $p$ occurs is lower than that corresponding to $p_1$, so that there is an increase of head $h$, then (*vide* Fig. 2):

$$\frac{p}{s} + \frac{c^2}{2g} = \frac{p_1}{s} + \frac{c_1^2}{2g} + h.$$

If, besides this, losses of energy, $L$, from friction for instance, are incurred between the same points, then

$$\frac{p}{s} + \frac{c^2}{2g} = \frac{p_1}{s} + \frac{c_1^2}{2g} + h - L.$$

This is termed the hydro-dynamic equation, and will be subsequently often employed ; it will be convenient in the sequel to denote by $p$, $p_1$, &c., the pressure expressed in equivalent *head* of water, instead of as hitherto the pressure in pounds per unit of surface, so that $p$ takes the place of $\frac{p}{s}$ ; hence the above equation becomes

$$p + \frac{c^2}{2g} = p_1 + \frac{c_1^2}{2g} + h - L.$$

The condition that every part of the pipes or vessels containing the water to which the above equation applies must be *full*, leads to the equation of continuity of flow, which expresses the fact that an equal quantity of water per unit of time flows through every section of the pipe system, the section being measured at right angles to the direction of flow. If $A$ and $A_1$ be the sectional areas at any two points, and $c$ and $c_1$ the corresponding velocities of flow, $Q$ the quantity of water (in cubic feet) per second, then $Q = A c = A_1 c_1$. Owing to this condition, when the dimensions of the pipes or vessels through which the water passes and the velocity at any section are known, the velocity at any other section can be calculated—a change of velocity in any one part affects in the same proportion the motion in every other part.

From the hydro-dynamic equation it is evident that as the velocity increases the pressure decreases, and *vice versâ*. As water flows through a vessel of varying section its velocity must change to satisfy the requirements of continuity of flow and with the velocity the pressure.

There is consequently a continual transformation going on, as the fluid advances, of kinetic into potential and potential into kinetic energy, and provided the changes are gradual, this process is not accompanied by any loss, apart from external causes. Abrupt alterations of velocity of flow due to sudden enlargement or contraction of the inclosing pipes or vessels causes loss, and then the transformation of energy of the one kind into energy of the other cannot take place.

## *Motion, relative and absolute.*

All motion is, strictly speaking, relative, but for the purposes of this work it will be convenient to use the term *absolute motion* or *absolute velocity* in reference to motion or velocity referred to some fixed system on the surface of the earth, and to describe as *relative motion* or *relative velocity* motion or velocity referred to some system in motion relatively to that fixed system.

A passenger sitting in a railway-carriage moving at the rate of forty miles an hour has an absolute velocity referred to the fixed system of the line, of 58·66 ft. per second, but, relative to the moving system of the carriage itself, is at rest. If the same passenger walks across the floor of the carriage at a speed of 3 ft. per second at right angles to the direction in which it is travelling, 3 ft. per second is his relative velocity with reference to the carriage, and his relative direction of motion is at right angles to the longitudinal axis of the carriage ; his absolute velocity is on the other hand compounded of the absolute velocity of 58·66 ft. per second with which the carriage is moving, and the relative velocity of 3 ft. per second, at right angles to the direction of the latter, with which he traverses the floor of the carriage. It is easy to

see that this absolute velocity is the resultant as regards value and direction of the absolute velocity of the carriage and the relative velocity of the passenger with respect to the carriage.

· As another familiar illustration of a similar character : suppose a passenger to jump out of a train in motion, as it is passing a platform, at right angles to the longitudinal axis of the carriage ; although he jump perfectly straight forwards, he will alight on the platform in an oblique direction with respect to the edge of the latter, assuming that edge to be parallel with the wall of the carriage. The absolute direction and velocity with respect to the platform are given by the resultant of the velocity of the train and the relative velocity with which the passenger leaves the carriage (*vide* Fig. 3); this is equally the case whatever the direction relative to the carriage may be.

Fig. 3.
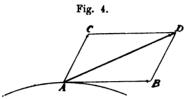


Supposing, again, the carriage were moving at only 3 miles an hour, and that the passenger walks in the opposite direction along the floor of the carriage at the same speed, it is clear that *with respect to the line* his body as a whole remains at rest, and therefore his *absolute* velocity is *nil*, although his velocity *relative* to the carriage is 3 miles an hour.

Motion relatively oblique to that of the carriage can always be resolved into two components, one parallel with and the other at right angles to the direction in which the carriage moves, and from this it is easy to see that the absolute velocity of any body moving with a certain velocity relative to another, which again itself has a given absolute velocity, is the resultant both as regards direction

and value of the relative velocity of the one body and the
absolute velocity of the other.

In the sequel this principle will be applied to particles
of water moving over rotating surfaces.    Let $A$, Fig. 4, be
a particle of water at any
point on a rotating sur-
face which has moment-
arily the absolute velocity
$A$ $B$ tangential to the
circular path of that
point, while the particle



Fig. 4.

at the same moment has the relative velocity and direction
$A$ $C$ with respect to the rotating surface; then $A$ $D$, the
resultant of $A$ $B$ and $A$ $C$, is the absolute velocity and
direction taken by $A$.

## The Action of Water on curved Vanes.

The construction of turbines is based on the fact that
when a mass of water (or other material) moving in a
certain direction with given velocity is compelled to
change this direction, force is required to effect the
change.   The intensity of the force necessary is obviously
dependent on the extent to which the mass is deflected
from its original course.   In a turbine a jet of water is
deflected by being brought into contact with a curved
vane, which, preventing further progress in the initial
direction, compels the jet to follow its surface; owing to
the resistance offered by the water to this compulsion, a
reactionary force is exerted on the vane which is em-
ployed in driving the turbine-wheel.   When the course
of a particle of water is altered, its motion in certain
directions is retarded or accelerated, and the force exerted
in any of these is measured by the amount of the retarda-

or acceleration. Let $A$ $B_1$ $B_2$ $B_3$, &c., Fig. 5, represent ons of a curve, each so short that it may be considered a straight line, and let $A$ $C$ be the initial
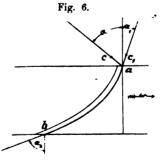
Fig. 5.

direction of a particle of water compelled to follow this curve; in passing from the element $A$ $B_1$ to that of $B_1$ $B_2$ the particle is deflected through the angle $a$, and on reaching $B_2$ has traversed the distance $B_2$ $C$ relatively to its original course at right angles to the latter; its velocity in this direction, at the point $B_1$, is proportional to its velocity $u$ in the direction $B_1$ $B_2$, and is $u$ $sin$ $a$; when traversing $A$ $B_1$ the city parallel with $C$ $B_2$ was *nil*, hence there has been a :ive acceleration of velocity at right angles to the inal course, and if $t$ be the time required for ersing $B_1$ $B_2$, this acceleration will be expressed by

$\frac{n\ a}{t}$. By applying this method of reasoning to

emely small portions of a curved vane over which a ım of water is flowing, and extending it to the whole

Fig. 6.
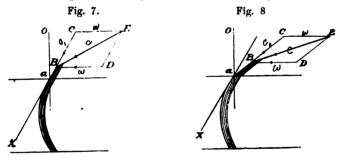
vane, an expression can be found for the total force exerted on the latter by a given quantity of water per second flowing over it.

Let $a$ $b$, Fig. 6, represent a very narrow curved channel through which a certain mass of water $m$ flows per second.

ı channel moves with the velocity $w$, in the direction

indicated by the arrow.   The water enters at $a$ with the
relative velocity $c_1$, at the angle $a_1$ formed by the tangent
to the curve of the channel at $a$, with a perpendicular
to the direction of motion of the latter; it leaves the
channel at the angle $a_2$ and with the relative velocity $c_2$.

The *absolute* velocity $c$—with respect to some fixed
point—with which the water is moving at $a$, and the
corresponding angle of direction, bear definite relations
to the quantities $c_1$ $w$ and $a_1$; resulting from the condition
that the water must enter the channel in a direction
coinciding with that of the tangent to the curve of the
vane at the point of entry $a$, that is, *without impact* or *shock*.

Fig. 7.                              Fig. 8



The moment before entering the channel the water
has a certain velocity and direction *relative* to the latter,
which, as has been explained in dealing with absolute
and relative motion, is determined by the absolute
velocity of the water in combination with the absolute
velocity of the vane.   When the water comes in contact
with the vane, no immediate change must be caused by
the latter in the relative direction of flow if impact is to
be avoided, hence the first portion of the vane at $a$ must
be parallel with the relative course of the water at the
moment of entering the channel.   Fig. 7 illustrates
the entry of the fluid *without*, and Fig. 8 *with* impact;

in both cases $E\ B$ represents the value and direction of the absolute velocity of flow of the water immediately before coming into contact with the vane, $E\ C$ the absolute velocity and direction of the motion of the vane; $E\ B$ must be the resultant of $E\ C$ and the required *relative* velocity of flow; the latter is found simply by completing the triangle $E\ C\ B$, in which the side $C\ B$ gives its value and direction. The complete parallelogram of velocities is $B\ C\ E\ D$.

In Fig. 7, $a\ X$, the tangent to the vane-curve at $a$, is a prolongation of $C\ B$, while in Fig. 8 the direction of $C\ B$ does not coincide with $a\ X$, but the course of the fluid on arriving at $a$ is suddenly changed.

From the preceding explanation, the graphical construction necessary to determine in any given case the direction of the first portion of (or tangent to) the vane at $a$, will be clear, bearing in mind that $B\ E = c$, $B\ C = c_1$, and $C\ E = w$.

Mathematically expressed, the conditions for entry of the water into the channel without impact are the following :—

$(A)...c\ sin\ a = w - c_1\ sin\ a_1,$

$(B)...c\ cos\ a = c_1\ cos\ a_1,$

$(C)...c\ :\ w = cos\ a_1\ :\ sin\ (a + a_1),$

$(D)...c_1\ :\ w = cos\ a\ :\ sin\ (a + a_1).$

The derivation of these will be evident from elementary trigonometrical principles with the assistance of the diagram Fig. 9.

According as the angle $a_1$ is measured on the same side of the line $O\ Y$ as the angle $a_2$ (*vide* Fig. 10), or on the opposite side, it is either positive or negative; in the latter case it must not be forgotten that the sine is also negative, consequently formula $(A)$ then becomes

$$c\ sin\ a = w + c_1\ sin\ a_1.$$

The absolute or *residual* velocity $u$ with which the
water leaves the vane,
Fig. 10, is the resultant
of $c_2$ and $w$.
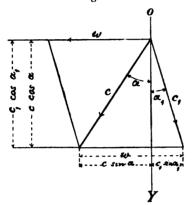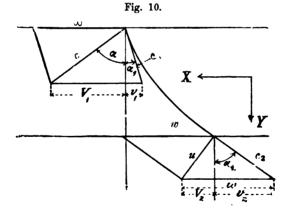
Fig. 9.

All the forces ex-
erted on the vane or
channel by the water
may be resolved into
two resultant forces,
one acting parallel with
the direction of $w$, the
other at right angles to
it; the first of these
may be denoted by $X$,
the second by $Y$.

The momentum parallel with $X$ of the water entering
with the velocity $c$, is proportional to the component

Fig. 10.

$V_1$ of the latter, and $V_1 = c \sin a$; on leaving the vane
with the residual velocity $u$, the momentum in the same

direction is proportional to the component $V_2$ of $u$, where $V_2 = w - c_2 \sin a_2$. Hence the loss of momentum is proportional to $V_1 - V_2$, the actual value being $m (V_1 - V_2)$, and this loss represents the force $X$ exerted on the vane; it is obvious from the diagrams that the difference in the components $v_2$ and $v_1$ of $c_2$ and $c_1$ parallel with $X$, representing the increase of momentum *relative* to the moving vane, is identical with $V_1 - V_2$ in absolute value.

Hence $v_2 - v_1 = V_1 - V_2$, or, expressed in terms of the velocities and angles :—

$$c \sin a - (w - c_2 \sin a_2),$$
$$= c_2 \sin a_2 - c_1 \sin a_1,$$
$$X = m \left( c \sin a - (w - c_2 \sin a_2) \right\} \qquad (1$$
$$= m (c_2 \sin a_2 - c_1 \sin a_1).$$

The work done by $X$ at the velocity $w$, per second, is—

$$X w = m w (c \sin a - (w - c_2 \sin a_2),\}$$
$$= m w (c_2 \sin a_2 - c_1 \sin a_1), \qquad \text{or}$$
$$W_m = X w = m w (V_1 - V_2) = m w (v_2 - v_1).$$

The value of $V_1 - V_2$, it is evident, can very easily be determined graphically without the aid of trigonometrical calculations, by simply measuring $V_1$ and $V_2$ (or $v_2$ and $v_1$), from the diagram, Fig. 10, already constructed to find $c$ and $u$.

The equivalence of $V_1 - V_2$ and $v_2 - v_1$ may be thus stated : The *absolute loss* of momentum is equal to the *relative gain* in momentum.
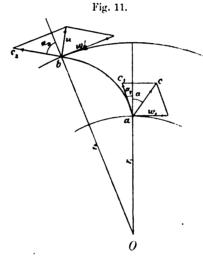
The force $Y$ is found in a similar manner to $X$, and is simply proportional to the difference of the components of $c_2$ and $c_1$ in a direction parallel to $Y$; hence

$$Y = m (c_2 \cos a_2 - c_1 \cos a_1) \qquad (2$$

as the vane is not assumed to have any motion parallel with $Y$, no work is performed by the latter force; in practice it tends by a downward pressure on the bearings

of a turbine to increase the frictional resistance of the shaft.

If, instead of moving in a straight line, the vane or channel rotates about an axis at right angles to the parallel planes in which the filaments of water flow, the expressions for the forces exerted by the water on its passage over the vane are somewhat modified.

Fig. 11.



Let $a$ $b$, Fig. 11, represent a vane or channel rotating round the axis $O$ (perpendicular to the plane of the diagram); the distance or radius $O$ $a = r_1$, and $O$ $b = r_2$,—the water being assumed to enter at $a$ and leave at $b$; the circumferential velocities at $a$ and $b$ are respectively $w_1$ and $w_2$. The angles $a$, $a_1$, $a_2$, are those formed by the direction of the velocities $c$, $c_1$ and $c_2$ respectively with the radii $r_1$ and $r_2$.
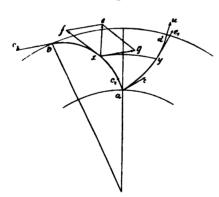
The work performed by the forces acting tangentially at all points of the vane is in this case expressed by

$$3) \quad W_m = m \left[ w_2 \, c_2 \, sin \, a_2 - w_1 \, c_1 \, sin \, a_1 - (w_2^2 - w_1^2) \right]$$

The additional quantities, as compared with the corresponding formula for rectilinear motion, are due to the varying velocities of the different points of the vane. Some writers have treated the influence of the varying velocity of rotation at different radii as due to the action of centrifugal force resulting from the rotation of the

vane. This view is misleading, as it gives the impression that there is a fundamental distinction between the cases of rectilinear and rotary motion, and that the latter requires the application of principles different from those which suffice for the former. In fact, precisely the same reasoning applies to both cases. The water in passing over a moving vane takes a certain absolute path, the direction of which at any point $x$ is that of the resultant of the velocity of flow relative to the vane and the absolute velocity of the latter. This (absolute) path may be easily found when the relative velocity of the water at all points of the vane and the velocity of the latter are known.[1]

Fig. 12.



Let $a\ d$, Fig. 12, represent the absolute path corresponding to the vane-curve $a\ b$.

A particle of fluid at the point $x$ of the vane-curve rotating with the velocity $x\ g$, and with the relative velocity of flow $x\ f$, has the absolute velocity $x\ e$. By the time a particle entering at $a$ has reached $x$, this point will have advanced to $y$, and therefore $y$ lies in the absolute path of that particle, which has the absolute velocity $y\ e_1 = x\ c$, the direction of $y\ e_1$ being tangential to the curve of the path, and obtained by rotating the vane-curve $a\ b$, and with it the constructive parallelogram $x\ f\ e\ g$, until the point

[1] *Vide* Chapter V.

$x$ coincides with $y$, when the new position of $x\,e$ gives the direction $y\,e_1$.

The force exerted by the water on the vane depends solely on the absolute velocity of the former and *the form* of the path; whether this be the result of a rotary or rectilinear motion is immaterial. The effect of the varying speed of the vane at different points is to modify the curve of the absolute path and the absolute velocity; or if the absolute path be given, then the *relative* course (or vane-curve) is affected from the same cause. A particle of water in passing from $a$ to $d$ over—let it be supposed—a frictionless revolving plane surface, will come into contact as it proceeds with points passing under it with increasing speed, and this of course increases the resultant relative velocity. To consider the so-called effect of the centrifugal action separately, destroys unnecessarily the unity of treatment.

The action of water flowing along a curved vane or channel depends, with given velocities of flow, entirely on the extent to which it is deflected from its original course, and the fact that the motion of all points of the vane is not uniform does not in principle introduce any new element into the problem.
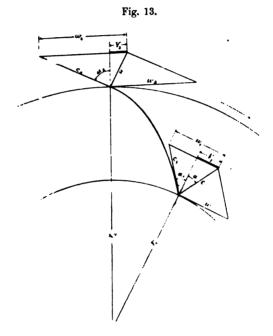
Returning now to the expression previously found for the work done by water flowing over a rotating vane

$$W_m = m\,[w_2\,c_2\,sin\,a_2 - w_1\,c_1\,sin\,a_1 - (w_2^2 - w_1^2)]$$

it may be stated in the form

$$W_m = m\,[\,w_2\,(c_2\,sin\,a_2 - w_2) - w_1\,(c_1\,sin\,a_1 - w_1)]$$

in which the quantities $(c_2\,sin\,a_2 - w_2)$ and $(c_1\,sin\,a_1 - w_1)$ correspond respectively to $V_2$ and $V_1$ as previously defined for the case where the vane has a uniform motion; they can be easily determined graphically in an exactly similar manner, as shown in Fig. 13; $w_2$ and $w_1$ however take the place of the uniform velocity $w$ in the expression for $W_m$.

Hence $W_m = m \left[ w_1 \, V_1 - w_2 \, V_2 \right]$, $\qquad$ (3a

where $V_1 = \underline{\hphantom{xxxxxxxxxx}} = c \sin a$,

and $V_2 = w_2 - c_2 \sin a_2 = u \sin \beta$,

$\beta$ being the angle formed with the radius $r_2$ by the direction of $u$.

Fig. 13.



It will be seen that with given velocities the action of the water upon a vane *depends only on the angles of entrance and exit, and not upon the intermediate form of the curve,* assuming the latter to be continuous and the curvature not too abrupt.

By making $w_1 = w_2 = w$ in the formula for $W$, the latter becomes

$$W_m = m\,w\,(V_1 - V_2)$$

the expression for work done with a uniform velocity $w$ of the vane, as previously demonstrated.
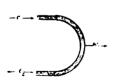
So far it has been assumed that the water comes into contact with the vane without impact, that is, it *glides* along the latter, and the algebraical condition for this [*vide* (A) page 15] is

$$w_1 - c_1 \sin a_1 = c \sin a.$$

$c \sin a$ or $V_1$ is the component of the absolute velocity of flow in the direction of the motion of the vane, and for a given velocity is independent of the question whether impact takes place or not, the force required to deflect the stream remaining the same; hence the work $W_m$ can always be expressed by $W_m = m\,[w_2\,(c_2 \sin a_2 - w_2) + w_1\,c \sin a]$, whether the water enter without shock or not; *only* in the *former* case, however, does $c \sin a$ bear the relationship above given to the speed of rotation $w_1$ and the relative velocity $c_1$.

It might be supposed at first sight that as the expression for the work done remains unchanged, it is immaterial whether impact occurs or not, as the work done would in either case be the same; a little consideration will show that this is not so. Impact or sudden retardation of flow causes a loss of energy from which results a diminution of the absolute and relative velocities of flow. Hence under given conditions, although the formula remain the same, different values have to be assigned to the constituents when impact takes place than when the water enters parallel with the guide-vane.

Fig. 14.



A special case will illustrate the use of the preceding formulas.

Assume that the water enters the vane in a direction parallel with the motion of the latter, and leaves in exactly the opposite sense, so that it is deflected through 360°, the velocity of the vane $w$ being the same at all points. Under these conditions $a = 90°$ and $a^2 = 90°$, consequently $\sin a = \sin a_2 = 1$, and the force $X$ exerted on the vane is by (1)—

$$X = m \left(c \sin a + c_2 \sin a_2 - w\right) = m \left(c + c_2 - w\right);$$

if the vane be held fast, so that $w = o$, and assuming $c = c_2$,

$$X = m \, 2 \, c.$$

The relative velocity $c_1 = c - w$, or $c = w + c_1$, when the vane is in motion; assuming $c_1 = c_2$,

$$X = m \left(w + c_1 + c_1 - w\right) = m \, 2 \, c_1,$$

and the work done $W = m \, 2 \, c_1 \, w$.

# CHAPTER II.

## ON TURBINES IN GENERAL.

Definition of turbine.—Classification.—Summary of classification and subdivision.—Description of typical turbines.—Reaction turbines.—Impulse turbines.

## *Definitions.*

A TURBINE is a motor for utilizing the energy of water by causing it to flow through curved buckets or channels on which it exerts a reactionary pressure constituting the motive force.

As distinguished from a water-wheel, in the older and narrower sense of the word, a turbine may be defined as a water-wheel in which a motion of the water *relatively to the buckets* is *essential* to its action.

A turbine consists essentially of a ring, or pair of rings, to which are attached curved vanes arranged uniformly round the circumference, revolving on a shaft or spindle to which the ring or pair of rings is connected by a boss and arms, or other suitable means.

## *Classification.*

Turbines may be divided as regards their construction into three classes: radial, axial, and combined or mixed flow.

In *radial* turbines the water in passing through the

wheel flows in a direction at right angles to the axis of rotation, or *approximately radially.*

In *axial* turbines, otherwise called *parallel-flow* turbines, the water flows through in a direction generally *parallel* with the axis of rotation.

In *combined* or *mixed-flow* turbines, both the previously-named systems are combined.

All turbines, as regards the behaviour of the water in them, belong to one of two kinds—that which works with all parts entirely full or *drowned,* or that in which free deviation and air in the buckets is required; these have been named respectively Reaction and Impulse Turbines, and although the terms are not free from objection, it will be convenient to adhere to them as being generally adopted by writers on the subject.

In *reaction* turbines it is essential that there should be continuity of flow of the water in every part of the motor and apparatus connected with it. Reaction turbines may be used in conjunction with a suction-tube, the nature of which will be subsequently explained.

In *impulse* turbines, the construction is such that the buckets are only partially occupied by the water passing through them, and the atmosphere has free access to the remaining space, so that the inflow always takes place under atmospheric pressure.
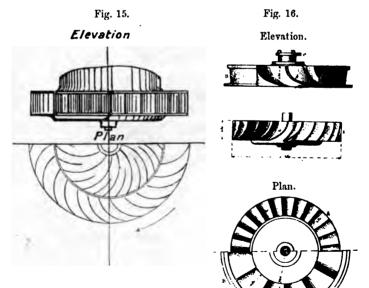
In some cases the water only enters into a part of the whole number of buckets at a time; the turbine is then said to have *partial admission.*

Generally both reaction and impulse turbines are provided with guide-vanes from between which the water enters the buckets of the wheel; as the name implies, they cause the water to enter in the desired direction.

The classification and subdivision of types of turbine may be summarized as follows:—

| Reaction Turbines. | | | Impulse Turbines. | | |
|---|---|---|---|---|---|
| Radial | { | Outward flow. | Radial | { | Outward flow. |
| | | Inward    „ | | | Inward    „ |
| Axial | { | Downward „ | Axial | { | Downward „ |
| | | Upward    „ | | | Upward    „ |
| Combined | { | Inward    „    up | Combined | { | Inward    „    up |
| | | or | | | or |
| | | Outward    „    down. | | | Outward    „    down. |

With or without suction-tubes.
With   vertical   or   horizontal
axes.

Without suction-tubes.
Without total or partial admis-
sion ; vertical or horizontal axes.

Figs. 15, 16, and 17 show in principle respectively the
construction of a radial, axial, and combined turbine.

Fig. 15.                          Fig. 16.

*Elevation*                     Elevation.



*Plan*

Plan.

In Fig. 15, the guide-vanes and wheel of a radial outward-flow turbine are shown. In Fig. 16 both the guide-apparatus and wheel of a Jonval or axial turbine are represented; *B* is the outer casing of the guides, of which *b* is one of the vanes and *c* the stuffing-box through

Fig. 17.[1]



which the shaft passes. *A* is the wheel, and *a* one of the bucket-vanes.

Fig. 17 illustrates the construction of the wheel of a combined or mixed-flow turbine, and Fig. 17A shows a horizontal section through guides and wheel, *B B* being

[1] Figs. 17 and 17A are illustrations of a " Risdon " turbine, for which the author is indebted to the makers.

the guides, and *A A* the wheel-vanes. The guides are similar to those for a radial inward-flow wheel.

Fig. 17A.



*Description of Typical Turbines.*
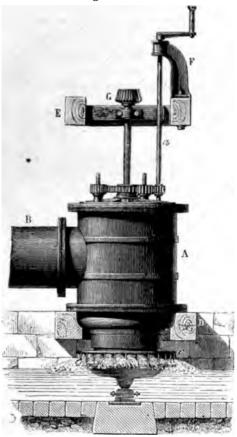
*Reaction Turbines.*

*Radial Outward Flow.* As representative of the *radial outward-flow* class may be taken the *Fourneyron turbine*— so called, as in other instances, from its inventor; it was practically the first of the turbines in the modern sense of the term, although there existed previously primitive forms of motors which, strictly speaking, were turbines as regarded the mode of action of the water on them.[1]

[1] For the illustrations Figs. 16 and 18, as well as some others, the author is indebted to Armengaud's work, " Les Moteurs Hydrauliques."

In Fig. 18 is shown a Fourneyron turbine of an early type discharging above water. *B* is a tube through

Fig. 18.



Fourneyron Turbine.

which, under the pressure due to the head, the water enters the casing *A*; from there it passes through the
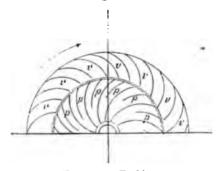
guide-passages *p  p*, &c., shown in the sectional plan
Fig. 19, into the wheel *c* of which *v v*, &c. are the vanes.

The regulation is effected by means of a circular sluice-
gate, by which the depth of the passages through which
the water enters the wheel can be reduced ; motion is
transmitted to the gate, through the spindle *a* and an
arrangement of a wheel and pinions, from the hand-crank
*F.*   Through the bosses of the pinions, which act as nuts,
pass the screwed rods by means of which the gate is raised

Fig. 19.



Fourneyron Turbine.

and lowered.   The vertical shaft on which the turbine
revolves carries at its upper end a bevel pinion, from which
the power is transmitted to the machinery to be driven.
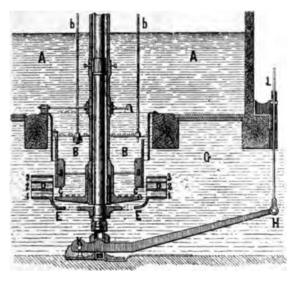
The water enters the wheel-buckets *v  v*, &c., in the
direction imparted to it by the guide-passages *p  p*, &c.,
but after entering is deflected from its original course, and
in consequence exerts a certain pressure on the *concave*
side of the vanes *v* by which the wheel is driven round.

The guide-vanes and apparatus are stationary, and
arranged concentrically with the wheel.

Fig. 20 illustrates another arrangement of a Fourneyron
turbine for a low fall.

The wheel *D* is divided by horizontal partitions into three parts, 1, 2, and 3, which are in fact separate turbines connected together, one above the other. The water enters from a wooden reservoir *A* into the guide-wheel *C*, which is furnished with vanes similar to those shown in Fig. 19, and extending from *B* to *C*; thence it flows into the wheel *D*. By means of the sluice-gate *a*, in which

Fig. 20.



Triple Fourneyron Turbine.

there are slots to admit the guide-vanes, the water can be shut off in succession from the parts 3, 2, and 1 of the wheel. This turbine is shown working drowned; the mechanical construction is essentially the same whether the turbine works in that way, or discharges above water.

When however the discharge takes place above water, there is always a risk of the wheel working as an impulse

turbine owing to the buckets not being filled with water at starting.

There are, of course, many modifications of the Fourneyron turbine, but they are all essentially of the same construction, and the same theory applies to all. This type
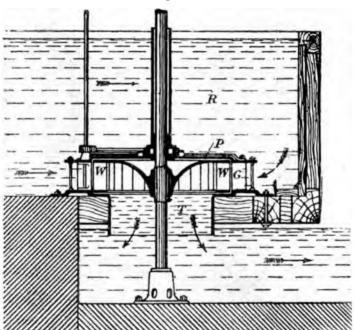
Fig. 21.



Inward-Flow Turbine.
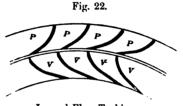
is chiefly adopted in France, where it originated; in other parts of Europe the axial construction is more in favour, while the Americans prefer inward and mixed-flow turbines.

Fourneyron's wheel is *not* used with a suction-tube.

*Radial Inward Flow.* Of this class the Vortex Turbine

of Prof. James Thomson and the radial inward-flow wheel of Francis may be taken as types. Fig. 21 shows the general arrangement of one of the latter working with a suction-tube. The water from the reservoir *R* enters the guide-wheel *G* which surrounds the wheel *W*, flowing towards the centre; after passing through the buckets, it is gradually deflected downwards by the curved surface of the top part of the wheel, so that it enters the suction-tube *T* in a vertical direction, and passes out into the tail-race. The arrangement of the guide and wheel-vanes *P* and *V* is illustrated in Fig. 22. As compared with the Fourneyron turbine, guide apparatus and wheel have changed places, the former being placed concentrically outside the latter.

Fig. 22.



Inward-Flow Turbine.

It is easy to see how the mixed-flow has been developed out of the inward-flow turbine by continuing the vanes into that part of the wheel where the water assumes a vertical direction, so that the radial flow is changed into an axial flow *while* the water is in the buckets instead of *after* it has left them.
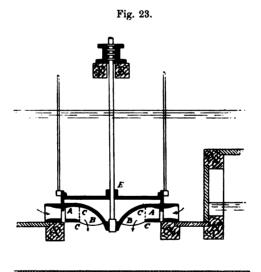
In the turbine under consideration the plate *P* sustains the pressure of the column of fluid above the wheel; it is connected at various points with the guide-wheel *G*. The vertical shaft is supported in an ordinary footstep on the floor of the tail-race, and passes through a stuffing-box in *P*.

The outflow from the suction-tube always takes place under water, this being essential to its efficient action with whatever type of turbine it may be employed.

As generally constructed, the suction-tubes of turbines

D

are of uniform diameter throughout the greater part of their length. Their action, however, is improved, as will be subsequently explained, by a gradual enlargement towards the lower end ; this, however, somewhat increases the cost.
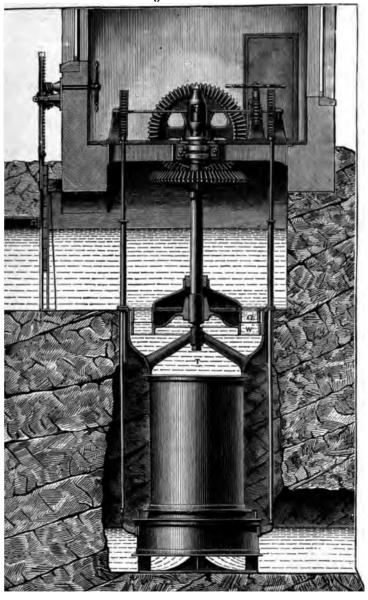
The chief purpose of a suction-tube is to enable the wheel to be placed at a considerable height above the tail-water level, an object which in many cases is for practical reasons very desirable, as it renders the wheel more easily accessible, and a shorter shaft is required than would otherwise be necessary.

Fig. 23.



Mixed-Flow Turbine.

*Mixed Flow.* Fig. 23 shows an inward mixed-flow turbine without suction-tube, in which the regulation

Fig. 24.



*Jonval Turbine* with Suction-Tube.

is effected by means of a circular sluice-gate passing between the guide apparatus and wheel, in the clearance space.

The wheel-vanes extend from *A* to *B*, instead of terminating at *C*, as represented by the dotted lines.

The shaft is suspended from a collar bearing, and passes through a second bearing at *E*.

*Parallel-Flow or Axial Turbine.*   Fig. 24 illustrates an axial or parallel-flow turbine of the usual construction, which may be taken as representative of this class.

It is of the type generally known as a Jonval turbine, from the name of the engineer who first practically introduced it, and is more extensively adopted in Europe than any other form of reaction wheel.

It consists essentially of three parts: the guide apparatus *G*, the wheel *W*, and the suction and off-flow tube *T*.

The guide apparatus consists of two concentric casings, in the annular space between which are secured the guide-vanes, and a dished or conical cover or plate over the space within the inner casing, having in the centre a water-tight bearing or stuffing-box, through which the shaft passes. This cover is so constructed as to bear the pressure of the column of water above it, and forms, so to speak, a part of the bottom of the reservoir from which the turbine draws its supply.
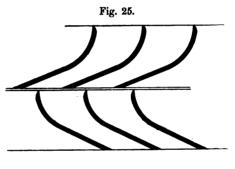
The shape of the casings and cover should be such as to offer as little resistance to the flow of the water as possible.

The wheel or runner consists of two concentric rings connected by curved vanes which form the buckets, and secured by arms to a boss keyed to the vertical shaft. In some of the older turbines, a single ring only was used for the wheel, with the vanes projecting from the circumfer-

ence and unconnected with each other at their outer
ends.

The vanes are either cast in one with the rest of the
wheel, or made of separate sheets of wrought-iron or steel
let into the cast-iron rings. They are shown in section in
Fig. 25. The water enters vertically between the guide-
vanes, is given by them the desired direction in which it
enters the wheel,
and is then dur-
ing its passage
through the latter
deflected by the
wheel-vanes, and
issues from the
passages between
them, into the
suction - tube or
tail-race.

Fig. 25.



The section shown is on the development of a cylindrical
surface of the mean diameter of the wheel.

The suction-tube—or draft-tube—is a large cast-iron
pipe, which corresponds in diameter at its upper end with
the outer ring of the wheel, where it surrounds the latter
with sufficient clearance to insure free motion.

At the lower end is a cylindrical sluice of cast- or
wrought-iron, under which the water flows off into the
tail-race; the out-flow orifice of the suction-tube must
always be submerged to secure its proper action.
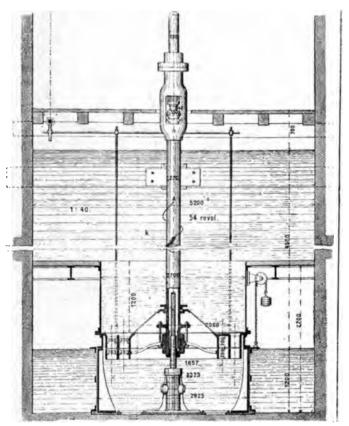
The sluice can be raised or lowered from the floor
above the turbine by rods actuated by a suitable
mechanism.

Fig. 26 represents a Jonval turbine with double buckets,
and without suction-tube.

The wheel consists practically of two concentric wheels

which can be shut off from the action of the water by a
series of valves or covers actuated by rods each worked

Fig. 26.



Doubel Jonval Turbine.

from a rack and pinion. The rest of the construction will
be quite clear after the preceding descriptions.

## Impulse Turbines.

*Radial Outward Flow.* In some cases, as has been already mentioned, Fourneyron's turbines discharged above the tail-water level, and under those circumstances worked as impulse turbines, in which the water leaves the guide passages against atmospheric pressure : generally speaking, however, a Fourneyron wheel is intended to work drowned, and does not come within the system of impulse turbines.

Zuppinger's so-called tangent wheel was one of the first types of radial outward- as well as inward-flow impulse turbines, intended to be used for high falls, for which the dimensions of reaction turbines, obliged to work full of water, would become impracticably small. An illustration of this class of motor is given in Fig. 27, with inward flow.
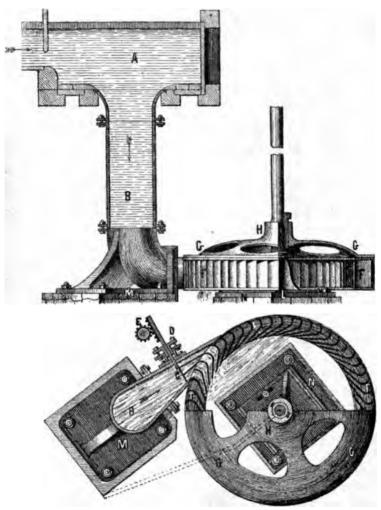
The water is conducted from the tank or reservoir *A* through the pipe *B* into the nozzle or mouthpiece *C*, which is furnished at the outlet with guide-vanes if sufficiently wide.

From the nozzle the water enters the wheel *F* on the outer circumference, and flows through the buckets in a radial direction, leaving at the inner circumference.

It will be seen that the quantity of water used is only sufficient to act upon a few vanes at a time; hence the tangent wheel is a turbine with only *partial admission*. The regulation is effected by means of the slide *D* actuated by a rack and pinion *E*. The tangent wheel has generally been made for inward flow, but many with outward flow have been constructed.
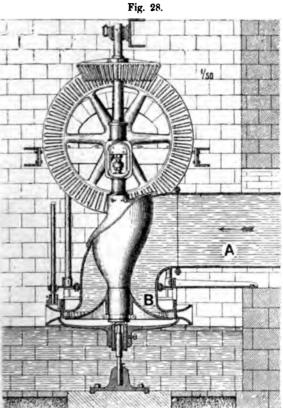
The French engineers Callon and Girard commenced in 1856 to design impulse turbines for all possible conditions,

Fig. 27.
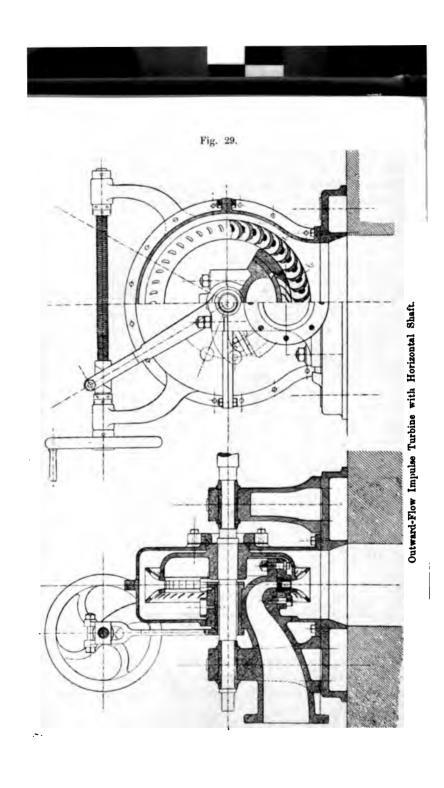
Tangent Wheel.

high and low falls, large and small quantities of water.
These turbines were made both axial and radial, with
horizontal, vertical, and inclined axes. The impulse turbine

Fig. 28.



Outward-Flow Impulse Turbine.

has in consequence become associated with the name of
Girard, and every variety of impulse turbine now goes by
the name of a "Girard" turbine. One of the improve-

Fig. 29.



Outward-Flow Impulse Turbine with Horizontal Shaft.
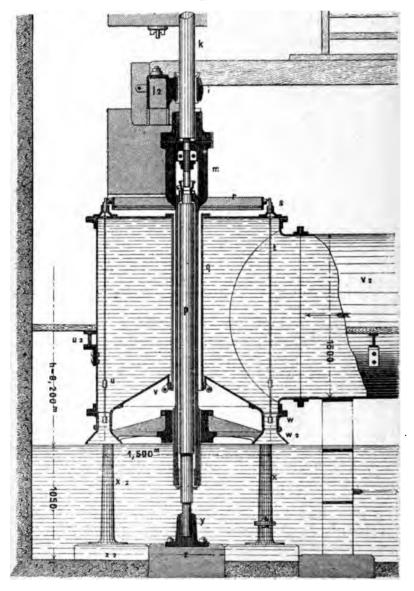
ments introduced by Girard consisted in the ventilation of the buckets for the purpose of insuring atmospheric pressure in the latter, and preventing the water from filling them and working as in a reaction wheel.

*Radial Outward-Flow Girard Turbine.* Fig. 28 shows a turbine of this class, in which the water is conducted to the guide apparatus *B* through the tube *A*, entering the wheel at the inner circumference. It will be seen that the cross-section of the wheel-casing is bell-mouthed, the buckets being wider at the outlet than at the inlet; the object of this construction will be subsequently explained —nearly all turbines of the Girard type are made in this way. The bearings for the shaft are so arranged as to be both above water. This is effected by using a hollow shaft having in its upper part a kind of open frame or lantern in which the pivot is fixed which carries the weight of the motor; this pivot bears on a fixed spindle carried up through the hollow shaft and supported in a socket on the bed of the tail-race.

In Fig. 29 is shown a radial outward-flow Girard turbine with a horizontal axis. The admission of water through the pipe and guide passages is only partial; the water leaves the wheel-buckets in a vertical absolute direction. Turbines of this pattern have recently been made in considerable numbers for very high falls.

*Axial-Flow Girard Turbine.* Fig. 30 illustrates an axial impulse turbine. In general arrangement it resembles a Jonval turbine, but has ventilated buckets, and the latter are widened at the lower end, where the outflow takes place. The regulation of this type of wheel is effected by various means, to be subsequently described; that shown in the illustration consists of a series of vertical slides, by which the guide passages are closed. These slides are raised or lowered from the floor above the turbine by
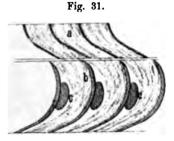
Fig. 30.



**Axial Impulse Turbine.**

vertical rods actuated from a kind of horizontal cam ; each rod works three slides.

The cross-section of the wheel-casing and buckets and form of the vanes is shown in Figs. 31 and 31A, with the apertures for ventilation, $a$ being the guide-vanes, $b$ the wheel-vanes, and $c$ the ventilating apertures.

Fig. 31.
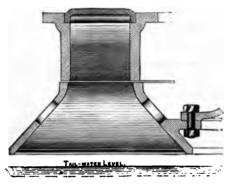


Referring again to the illustration Fig. 30, $w$ is the guide apparatus, $w_2$ the wheel, $x$ $x_2$ cast-iron columns, four in all, supporting the guide-casing; $v$ is a cover, connected with the guide apparatus, which supports the column of water in the chamber above the wheel; $y$ a socket carrying the fixed spindle $p$ passing up through a hollow cast-iron shaft terminating at the top in the lantern $m$, which is connected with a bearing spindle running on the upper end of $p$; $k$ is the vertical wrought-iron driving shaft coupled to the hollow cast-iron shaft; $i$ a bearing for $k$, bolted to a horizontal cast-iron girder $l_2$; $r$ the horizontal cam by which the rods $t$ $u$, attached to the regulating slides of the guide apparatus, are raised or lowered; $v_2$ the conducting tube by which the water is conveyed to the turbine; $u_2$ a girder supporting the floor of the turbine house; $z$ and $z_2$ foundations for the socket $y$ and the columns $x$ and $x_2$. The dimensions shown are metric.

There is of course a vast number of varieties of turbines, but the preceding examples are sufficient to give the reader a fair idea of the leading features of various types, and it does not come within the scope of this work to enter at length into details of construction.

Several kinds of turbines have, moreover, merely an historic interest, their construction having become obsolete. It

Fig. 31A.

Tail-water Level.

should be mentioned that some turbines have been made without guide-vanes, in which case the water is generally assumed to have on entering an absolute direction at right angles to that of the motion of the wheel-vanes.

The relative merits of reaction and impulse turbines, the methods of regulation, construction of the vanes, &c., will be discussed in another part of this work; but before dealing with the theory of turbines, the author has considered it necessary to give the foregoing description of the essential construction of different systems.

# CHAPTER III.

## GENERAL THEORY OF REACTION TURBINES.

Essential parts of reaction turbine with suction-tube.—Comparison with system of pipes.—Available energy.—Nature of resistances to flow of water.—Fundamental relations.—General equations for work and efficiency.—Theory of suction-tube.—Pressures at orifices.— Available energy for wheel.—Formula.—Pressure-differences at orifices.—Limit of height for wheel in suction-tube.—Further explanation of action.

Useful work.—Expression for useful work.—Application to turbines of action of water on curved vanes.—Arrangement of vanes. —Formulas for total work done for radial, axial, and mixed-flow wheels.—Velocity of flow.—Interdependence of velocity of flow in all parts of turbine.—Differences between reaction and impulse turbines.—Method of determining velocity of flow.—Formulas for velocity of flow.—Numerical example.—Dependence of velocity of flow on speed of rotation.—Effect of absence of suction-tube.—Effect of losses on velocity of flow.—Complete formula.—Range of velocity of flow compared with that due to head.—Experimental results showing relation between velocity of flow and speed of rotation.

## *Notation employed.*

$c_o$ = velocity of flow in supply or conducting-pipe.

$c$ = absolute velocity of water leaving guide passages = " velocity of flow."

$c_1$ = relative velocity of water entering wheel-buckets.

$c_2$ = relative velocity of water leaving wheel-buckets.

$c_3$ = velocity of water in suction-tube.

$c_4$ = velocity of water leaving suction-tube.

$w_1$ = velocity of rotation of wheel at inflow.

$w_2$ = velocity of rotation of wheel at outflow.

$a$ = angle of outflow from guides.

$a_1$ = relative angle of inflow into wheel.

$a_2$ = relative angle of outflow into wheel.

$A$ = effective outflow area of guide passages.

$A_1$ = effective inflow area of wheel-buckets.

$A_2$ = effective outflow area of wheel-buckets.

$A_3$ = sectional area of flow for suction-tube.

$A_4$ = effective outflow area for suction-tube.

$r_1$ = radius of wheel at inflow.

$r_2$ = radius of wheel at outflow.

$E_u$ = available energy per unit weight of water.

$\epsilon$ = relative efficiency.

$W_u$ = useful work done per unit weight of water per second.

$W$ = total useful work per second.

$L_u$ = losses per unit weight of water by hydraulic friction, &c.

$\zeta_1$ $\zeta_2$, &c. = experimental co-efficients for frictional losses, &c., corresponding with velocities of flow, $c$, $c_1$, &c.

$p_1$ = pressure of water (in "head") on leaving guide passages.

$p_2$ = pressure of water on leaving wheel-buckets.
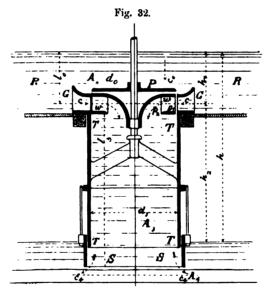
$a$ = atmospheric pressure.

$g$ = acceleration due to gravity = 32·1889 at sea-level, London, taken for practical purposes as 32·2.

$h$ = available head.

For the purpose of investigation it will be convenient to take an inward-flow (Francis) turbine as a type of the reaction turbine. Fig. 32 represents the general construction of one of these, which, with the aid of the illustrations previously given, will be clear without detailed description. The water enters the guide passages from a conducting channel or tube of the length $l_o$ and sectional area $A_o$, in which it has the

velocity $c_o$; it passes through the guide passages into the turbine wheel, and from the latter with the velocity $c_3$ through a suction-tube of the length $l_3$, sectional area $A_3$ and diameter $d_3$, out under a cylindrical sluice. The area of the outflow orifice below the sluice is $A_4$, the corresponding velocity of outflow $c_4$.

Fig. 32.



Inward-Flow Turbine.

The total available head of water is $h$; the depth of the middle of guide passages below the upper water level $h_1$; that of the tail-water level from the former $h_2$; hence $h_1 + h_2 = h$.

The whole arrangement forms, so to speak, *one system of pipes,* to which apply the general principles of hydrodynamics for the relations between pressure and velocity with continuity of flow.

E

In passing through the system the water meets with various resistances, to overcome which it has to do work, and leaves the apparatus with a certain residual velocity which has been denoted by $c_4$.

The available energy for each pound of water passing through the system under the action of the pressure due to the head $h$ is $E_u = h$; this is expended in overcoming resistances and producing velocity.

Now the resistances are of two kinds—that offered by the wheel, in overcoming which useful work is done, and that due to friction, impact, and other obstructions, the energy expended on which is lost.

Hence there results the equation :—

Available energy = Useful work,
+ Work lost on friction, &c.,
+ Energy remaining in water (due to velocity $c_4$).

Denoting these three quantities respectively by $W_u$, $L_u$, and $\dfrac{c_4^2}{2g}$ —where $g$ is the acceleration due to gravity— there follows :—

$$(1) \quad E_u = W_u + L_u + \frac{c_4^2}{2g}$$

and
$$W_a = E_u - \left( L_u + \frac{c_4^2}{2g} \right).$$

Assuming for a moment that there are no losses by friction, impact, &c., and therefore that $L_u = 0$, then

$$(1a) \quad W_u = E_u - \frac{c_4^2}{2g}$$

and the only portion of the available energy not expended in doing useful work is that carried off in the kinetic form by the outflowing water.

It is evident from this that the less the value of $c_4$ the

greater will be $W_u$, the work done by the turbine, and consequently the greater the efficiency of the latter; for the extreme case that $W_u = 0$, i. e. no work is done,

$$E_u = \frac{c_4^2}{2g} = h \text{ or } c_4 = \sqrt{2gh},$$

the whole available energy is converted into kinetic energy, and the velocity $c_4$ is that due to the whole head.

The efficiency of the turbine is the ratio of the useful work done to the available energy or $\dfrac{W_u}{E_u}$, substituting the foregoing values from (1)

$$(2) \text{ Efficiency } \epsilon = \frac{W_u}{W_u + L_u + \dfrac{c_4^2}{2g}} = \frac{W_u}{h}$$

The preceding formulas apply both to reaction and impulse turbines; in the case of the latter, and of reaction turbines without a suction-tube, $c_4$ denotes the absolute velocity with which the water leaves the wheel.

To reaction turbines the principle of continuity and the hydro-dynamic equation apply.

The conditions of continuity are, *vide* p. 9,

$$A c = A_1 c_1 = A_2 c_2 = Q,$$

where $A \ A_1 \ A_2$ represent the sectional areas of the pipes and passages through which the water flows at different points, measured in planes at right angles to the direction of motion; $c \ c_1 \ c_2$ the velocities of flow at these points—similar indices denoting corresponding quantities—and $Q$ the quantity of water passing through the system per second.

The hydro-dynamic equation has the form, *vide* p. 8,

$$p + \frac{c^2}{2g} = p_1 + \frac{c_1^2}{2g} + h_1,$$

where $p$ and $p_1$ denote the pressures expressed as *heads* of water (in feet) at any two points, $c$ and $c_1$ respectively the corresponding velocities, and $h_1$ the difference in level of the two points, the increase or diminution of head.

## *Theory of Suction-Tube.*

It has been already stated that within certain limits the wheel may be placed at any height within the suction-tube; it is now necessary to explain the reasons for this fact.

The water leaving the guide passages and entering the wheel-buckets has a certain absolute velocity $c$ representing a corresponding amount of kinetic energy available for doing work on the turbine. At the mouth of the guide passages, in an imaginary cylindrical surface dividing the guide apparatus from the turbine, there exists a pressure $p_1$; at the orifices of the wheel-buckets where the water leaves the latter a pressure $p_2$, which may be assumed to be less than $p_1$. It is evident that the difference in the pressures $p_1$ and $p_2$ is also available for doing work on the turbine, by increasing the velocity of the water during its passage through the buckets.

Hence the *total available energy* for the *wheel* is that due to the velocity of entrance $c$, *plus* that due to the difference of pressure $p_1 - p_2$ or

(3) Available energy $E_1 = \dfrac{c^2}{2g} + p_1 - p_2$ for the unit of

weight.

If the water leaves the buckets at a lower level than that at which it enters, as in a mixed or axial-flow turbine, then, $h_o$ being the difference of level,

$$E_1 = \frac{c^2}{2g} + p_1 - p_2 + h_o.$$

Now with reference to Fig. 32 it will be clear that

$$h_1 - p_1 + a = \frac{c^2}{2g} + \text{Loss of energy } L_1;\ L_1 \text{ being the}$$

loss of energy incurred by the water up to the moment of its exit from the guide passages, and $a$ the atmospheric pressure on the upper surface of the water. Similarly

$$(3a)\quad \frac{u^2}{2g} + p_2 + h_2 - a = \frac{c_4^2}{2g} + L_2,$$

where $L_2$ is the loss of energy sustained by the water from the moment of leaving the wheel-buckets to its exit from the suction-tube, and $u$ the absolute velocity with which the water leaves the wheel.

From a combination of the two equations the value of the difference of pressure is obtained—

$$(4)\quad p_1 - p_2 = (h_1 + h_2) - \frac{c^2}{2g} - \frac{c_4^2}{2g} - L_1 - L_2 + \frac{u^2}{2g}.$$

Now $h_1 + h_2 = h$, and owing to the condition of continuity all the different velocities may be expressed in terms of any one of them—$c$ for instance—and the ratios of the corresponding sectional areas $A\ A_4$, &c. Further, the losses $L_1\ L_2$ may be stated in the form

$\zeta_1 \dfrac{c^2}{2g}$ and $\zeta_2 \dfrac{c^2}{2g}$, where $\zeta_1$ and $\zeta_2$ are factors made up of the ratios of the sectional areas and certain experimental co-efficients; they may be combined with the equivalents for $\dfrac{c_4^2}{2g}$ and $\dfrac{u^2}{2g}$ and all expressed in the form

$$(5)\quad \zeta_1 \frac{c^2}{2g} = \left(\frac{c_4^2}{2g} - \frac{u^2}{2g} + L_1 + L_2\right).$$

Substituting this in equation (4) there follows—

$$(6)\ p_1 - p_2 = h - (1 + \zeta^1)\frac{c^2}{2g},$$

and this again in equation (3) —

$$(7)\ E_1 = h - \zeta^1 \frac{c^2}{2g};$$

that is to say, the energy available for doing work in the wheel is that due to the *total head h, less* the various losses incurred previous to entering and after leaving the wheel-buckets, and is quite independent of the level at which the wheel is placed in the suction-tube, within the limits already mentioned.

The velocity of flow $c$ from the guide passages is—as will be subsequently shown—determined quite independently of the position of the wheel relative to the tail-water level. Hence it is clear from equation (6) that the pressure difference $p_1 - p_2$ is also unaffected by the height of the wheel in the suction-tube.

Apart from mathematical demonstration, this phenomenon is explained by the statement that, as the wheel is placed higher or lower, the pressures where the water enters and where it leaves the buckets are respectively lowered or raised by the same amount, so that *the difference* remains constant.

It must be borne in mind that each of these pressures $p_1$ and $p_2$ depends not only on the depth of the points where it occurs below the surface of the water, but also on the velocity of flow at those points.

It remains to be shown what is the limit of height at which the wheel may be placed in the suction-tube above the tail-water surface.

By transposing the equation (3$a$) the following expression for $p_2$ is obtained—

$$p_2 = a - h_2 + \frac{c_4^2 - u^2}{2g} + L_2;$$

if no flow were taking place, or the velocity of flow were so slow as to be negligible, $c_4$ and $u$ would both be *nil*, and therefore also the sum of the losses $L_2$; in that case

$$p_2 = a - h_2.$$

The smallest possible value of $p_2$ is of course *nil*, and would occur when

$$a = h_2,$$
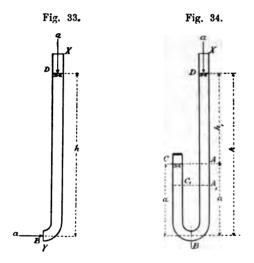
that is to say, if there were no flow the maximum possible height of the wheel above the tail-water would be equal to the atmospheric column; the atmospheric pressure exactly balancing the head $h_2$. If under these conditions flow commences, it is clear that the pressure of the atmosphere is no longer capable of supporting the same height of column as before, since to the action of the head $h_2$ is now added the momentum of the water issuing from the tube.

The following considerations will serve to render the action of the suction-tube in a turbine more clear :—

Let $X$ $Y$, Fig. 33, be a tube in which a column of water of the height $B$ $D$ $= h$ is maintained; at the lower end $B$ there exists atmospheric pressure $a$ (expressed in head of water), opposed to which is the head $h$, *plus* the atmospheric pressure $a$ acting on the upper surface at $D$; the atmospheric pressure at the lower end balances that on the upper surface, so that the fluid flows out under the pressure due to the head $h$. Suppose the tube continued and bent up as shown in Fig. 34, and that at the end $C$, which is closed, there is a vacuum, while instead of the atmospheric pressure at $B$ there is substituted a column of water $B$ $C$ $= a$.

The column $B$ $C$ balances an equal column $B$ $A$ in the other arm of the tube, so that $B$ $A$ $= a$; therefore the excess of pressure at $A$ tending to produce a flow of the water is $A$ $D$ $= h_1$ *plus* the atmospheric pressure at $D$,

but this is nothing else than the total head $h$, since $A\,B$ = atmospheric pressure and $A\,B + A\,D = a + h_1$ = $h$. The *effective* head, therefore, at $A$, 34 feet above $B$, is the same as at the latter point. $A$ corresponds to the extreme limit of height at which a turbine may be placed in the suction-tube. If any other point below

Fig. 33.                     Fig. 34.



$A,\,A_1$ for instance, be taken, the column $B\,A_1$ may be considered as balanced by the equal column $B\,C_1$ in the other arm of the tube; the effective pressure at $A_1$ is consequently that due to the column $A_1\,D$ *plus* atmospheric pressure, or $A_1\,A + A\,D + a$ *less* that equivalent to $C\,C_1$; $C\,C_1$ however equals $A\,A_1$, so that $A_1\,A + A\,D + a - C\,C_1 = A\,D + a = h$ as before.

Hence for any point below $A$ the effective head for producing flow is the same as at $B$. In practice the height $A\,B$ is very considerably less than $a$.
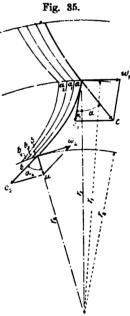
## Useful Work.

Under "*useful work*" is to be understood the work done by the force, resulting from the deflection by the vanes of the streams of water entering the wheel-buckets, turning the wheel at a uniform speed, and overcoming an equivalent resistance. In the "useful work" is included that required to over-come the friction of the bear-ings of the turbine shaft and of the surface of the wheel against the surrounding water; it is therefore the equivalent of the indicated work of a steam-engine.

Fig. 35.

The "effective work" is the work actually given off by the turbine shaft, as measured for instance by a brake; it equals the "useful work" *less* that required to overcome friction.

It has been shown (*vide* p. 22) that the work performed on a curved vane by the action of a jet of water passing along it, when the vane has a rotary motion about an axis at right angles to the planes in which successive parallel filaments of the jet move—or approximately at right angles to a plane parallel with the resultant course of the jet—is expressed by

$$W_m = m \left[ w_2 \left( c_2 \sin a_2 - w_2 \right) + w_1 c \sin a \right],$$

the notation being as already explained and indicated by the diagram Fig. 35.

Strictly speaking this applies only to a very shallow stream or jet moving along the vane; but provided the depth be not too great, the formula may be used without appreciable error for a stream of a relatively considerable depth, which can be considered as made up of a series of very shallow streams $a\ b$, $a_1\ b_1$, &c., Fig. 35, superimposed, and all forming the same angle at the points of entry and exit respectively with the radii—drawn from the axis of rotation—at those points.

In an actual turbine vanes are arranged round the circumference of the wheel at equal distances, each in the same position relative to the corresponding radii, and the formula given above may be applied to express the total work done on the wheel, provided that, instead of the small mass $m$ the total mass $M$ of all the water passing through the turbine per second be substituted; then

$$(8)\quad W = M\left[w_2\left(c_2\ sin\ a_2 - w_2\right) + w_1\ c\ sin\ a\right].$$

If $G$ be the *weight* of water flowing through the turbine per second, then $M = \dfrac{G}{g}$, and if further $Q$ be the corresponding *quantity* in cubic feet, $G = Q\ s$, where $s$ is the weight of a cubic foot. It is clear from the conditions of continuity,

$$(9)\quad Q = A\ c = A_1\ c_1 = A_2\ c_2 = A_x\ c_x,$$

that all the velocities of flow at various points may be expressed in terms of any one of them and of the ratio between the respective sectional areas; it will be convenient to express all these velocities in terms of $c$, the absolute velocity with which the water leaves the guide passages; hence—

$$(10)\quad c_2 = \frac{A}{A_2}\ c\ ;\ c_1 = \frac{A}{A_1}\ c\ ;\ c_x = \frac{A}{A_x}\ c_x.$$

The formula (8) may be written, *vide* p. 21,

$$(8a) \quad W = (w_1 \, V_1 - w_2 \, V_2) \, M;$$

it applies to *radial turbines* with either inward or outward flow.

In axial turbines a very narrow stream of water following the course of a vane on a cylindrical surface at a certain radius from the axis, advances with an equal velocity of rotation at all points, and for the work done by the water the formula applies—

$$W_m = m \, w \, (c_2 \sin a_2 - w + c \sin a).$$

The actual stream of water flowing over the vane of an axial turbine may be considered as made up of a large number of such very narrow streams, the distance of which from the axis, and consequently also their velocity of rotation, varies. It can be proved that the combined work of all these streams acting at different distances from the axis is represented by simply substituting for $w$ in the above formula the *mean* velocity of rotation, that is, the velocity of the middle of each vane at the mean radial distance $r$ from the axis; then

$$(11) \quad W = M \, w \, (c_2 \sin a_2 - w + c \sin a), \text{ or}$$
$$(11a) \quad W = M \, w \, (V_1 - V_2).$$
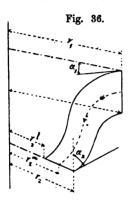
The formula (11) results out of (8) if for both $w_1$ and $w_2$ the same value $w$ be used

$$w_1 = w_2 = w,$$

and hence the expression $W$ for radial turbines includes axial turbines as a special case.

There still remains the mixed-flow turbine to be considered. Generally speaking, in this the water enters radially, flowing inwards, and leaves axially, and the vanes have consequently a double curvature. Assuming, as before, that the total stream flowing over any vane is

made up of a large number of very narrow streams or filaments, each one of these enters *radially* at the radius $r_1$, *vide* Fig. 36, and leaves axially at a radius which varies from $r_2$ for the filament furthest from the axis, to $r^1_2$ for that nearest to the axis. To each of these filaments or very narrow streams may be applied the formula for the work done in radial-flow turbines, so that the work done by the outermost filament will be



Fig. 36.

$$W_1 = m\ (w_1\ V_1 - w_2\ V_2),$$

and that performed by the innermost filament

$$W_2 = m\ (w_1\ V_1 - w_{21}\ V_2),$$

where $w_2$ and $w_{21}$ are the velocities of rotation corresponding to $r_2$ and $r^1_2$ at the inner and outer extremities of the lower edge of the vane; the sum of the work done by these and all the intermediate filaments is expressed by

$$(12)\quad W = M\left( w_1\ c\ sin\ a\ +\ c_2\ sin\ a_2\ \frac{w_2 + w_{21}}{2} - \frac{w_2^2 + w_{21}^2 + w_2\ w_{21}}{3} \right).$$

It will be seen that when $w^2 = w_{21}$ this formula becomes identical with (8); hence it is applicable with the necessary special assumptions to all three classes of turbines, the radial, axial, and mixed-flow.

It is obvious that

$$w_1 = \frac{r_1}{r_2}\ w_2 = \frac{r_1}{r_2^1}\ w_{21},$$

and consequently $W$ can be expressed in terms of one of the velocities $w_1$, $w_2$, or $w_{21}$ only.

Generally speaking, therefore, $W$ may be written in the form
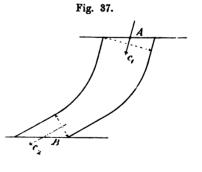
$$(13) \quad W = (B\, w_1\, c - C\, w_1^2)\, M,$$

bearing in mind equations (10); so that the work done on the wheel with given dimensions depends both on the velocity of flow $c$ and the velocity of rotation $w_1$ (or angular velocity $\omega$).

The angle $a$ in mixed-flow turbines is that formed by the direction of flow at any point where the water enters the buckets with a *radial* line to that point; the angle $a_1$ is that formed by the relative direction of outflow at any point with a line through that point *parallel* with the axis, or, more generally expressed, with a line perpendicular to the momentary direction of rotation in the plane defined by the latter and the relative direction of outflow. This more universal definition is necessary, because in many turbines the plane in question is not parallel with the axis of the wheel.

In a reaction turbine the water as it traverses the vanes of the wheel completely fills the space between any two of them, and when— as generally happens— the sectional area of the passage diminishes towards the outflow orifice, the relative velocity is increased from $c_1$ on entering to $c_2$ on leaving, in the inverse ratio of the sectional areas, and as a necessary consequence the pressure at $A$, Fig. 37, is in excess of that at $B$. This does not, however, modify in any way the method of calculating the work due

Fig. 37.

to the deflection of the stream by the vanes, which, whether
the fluid be under pressure or not, is expressed solely in
terms of the velocities of flow.   The effect of an excess of
pressure at $A$ makes itself felt in a corresponding accele-
ration of the relative velocity from $c_1$ to $c_2$, or when the
pressure at $B$ is greater than at $A$, by a retardation of the
flow.   As the proportions between $c_1$ and $c_2$ can be
determined solely from the ratio of the sectional areas of
the passages at $A$ and $B$, the calculation is much simpler
than if the pressures were introduced, and as a matter of
fact the latter can only be expressed through the velocity
of flow.

In applying the hydro-dynamic equation to the flow and
pressure of water in turbines, care must be taken to
compare *relative velocities with relative velocities only*, and,
similarly, *absolute with absolute velocities*, otherwise the
deductions will be wrong.

## *Velocity of Flow.*

It has been seen that the velocity of flow at any one
point in a reaction turbine determines that at every other
point, assuming that the machine is working as it should
do.   In dealing with this part of the subject, it will be
convenient for purposes of comparison and otherwise to
consider the velocity $c$, with which the water leaves the
guide passages, as the standard velocity.

If—as in impulse turbines—the water left the guide
passages at atmospheric pressure and entered a space to
which the atmosphere had free access, the velocity of flow
$c$ would depend—apart from friction and similar influences
—solely on the head of water above the point of outflow,
and would be quite independent of what subsequently
became of the fluid.

The pressure, however, at which the water in a reaction turbine leaves the guide passages is not atmospheric pressure, but, as has been seen, a greater or less pressure $p_1$, and this in its turn is a variable quantity which cannot be independently determined.

Owing to the continuous connection between the fluid in every part of the apparatus both before, during, and after its passage through the wheel, any resistance or facility offered to the flow of the water in any one part reacts on that in every other part. Hence, if after leaving the guide passages the water has to do work in turning the wheel, the velocity of flow $c$ from the former will not be the same as when no work is being done.

The velocity of flow is determined from equation (1) (p. 50).

$$E_u = W_u + L_u + \frac{c_4^2}{2g}$$

Both $W_u$ and $L_u$ are functions of the velocity of flow $c$ and the velocity of rotation $w_1$ or $w_2$, while $E_u$ is proportional to the head only. Hence from equation (1) $c$ can be calculated in terms of $h$ and $w_1$ (or $w_2$).

In making this calculation, the losses of energy by shock as the water enters the wheel must be taken into account, as they very materially affect the result. The detailed calculation is very lengthy, and is never employed in practice; it will therefore be sufficient to state the result, which is as follows :—

$$(14) \quad c = \left( \frac{Bw_2}{1+\zeta} + \sqrt{\frac{2gh}{1+\zeta} + \frac{C+B^2}{(1+\zeta)^2} w_2^2} \right) \frac{A_2}{A}$$

in which

$$B = \frac{r_1}{r_2} \frac{A_2}{A_1} \sin a_1$$

$$C = (1+\zeta) \left[ 1 - \left( \frac{r_1}{r} \right)^2 \right]$$

$\zeta = a$ co-efficient depending on various experimental co-efficients and on the dimensions of the turbine, including guides and supply-pipe.

The value of $\zeta$ can be experimentally determined by holding the wheel fast, and allowing the water to flow through under the pressure of the head. In that case the velocity of rotation $w_2 = 0$, and hence

$$c = \frac{A_2}{A} \sqrt{\frac{2gh}{1 + \zeta}}.$$

$\frac{A_2}{A} \sqrt{\frac{1}{1 + \zeta}}$ is then the ratio of the actual velocity of flow

to the velocity due to the head. Assuming the formula to be correct, $\zeta$ should be constant for all speeds of a given turbine.

It is probable that the above formula for $c$ does not give in every instance values which coincide with practical results in detail, since the assumptions involved regarding the loss by shock are not correct for extreme values of the velocity of rotation. The losses by shock, as given by the formulas employed in the calculation—which will be subsequently considered—are too great for speeds considerably above or below the best speed.

In spite of this, the formula in question is useful for showing the general tendencies of an increase or decrease of speed with turbines of various systems.

In the case of a Jonval turbine, in which $r_1 = r_2$, the

term $C = 0$, since $1 - \left(\frac{r_1}{r_2}\right)^2 = 0$. If it be further as-

sumed, that $a_1 = 0$ — as is very usual — then $\sin a_1 = 0$ and therefore also $B = 0$.

Under these conditions

$$c = \frac{A_2}{A} \sqrt{\frac{2gh}{1 + \zeta}};$$

that is to say, the velocity of flow is *constant* for all speeds of the turbine. This is quite in accordance with experience, as will be subsequently proved.

On the other hand, if in a Jonval turbine the value of $a_1$ is *negative*, then the first term in the formula for $c$, viz. $\dfrac{B\,w_2}{1+\zeta}$ also has a negative value, and consequently the velocity of flow diminishes as the speed increases.

For a radial inward-flow turbine in which $\dfrac{r_1}{r_2} > 1$, $C$ is *negative*, and hence the flow decreases as the speed increases. The greater the ratio, $\dfrac{r_1}{r_2}$, the more rapid is this decrease of the velocity of flow. This peculiarity of the inward-flow turbine has a certain practical importance, as if carried far enough it tends to make the motor self-regulating. Let it be supposed that when such a turbine is running at its best speed, a portion of the load is suddenly removed, the wheel will begin to run faster, and the reactionary force exerted by the water on the vanes will in the first place diminish, owing to the increase in the velocity of rotation.

From equation

$$(8) \quad W = M\left[w_2\left(c_2\sin a_2 - w_2\right) + w_1\,c\sin a\right]$$

it is evident that this diminution commences as soon as $w_2 > c_2\sin a_2$. For normally designed turbines $w_2 = c_2 \sin a_2$ corresponds with the best speed.
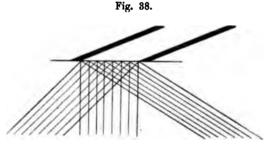
If simultaneously with the increase in $w_2$ (and $w_1$) the velocity of flow is reduced, so that $c$ and $c_2$ diminish, then the decrease in the reactionary force will be more rapid than otherwise, and the more rapid this decrease, the lower is the speed at which effort and load again balance each other.

F

A self-regulating action of this kind is promoted by making the ratio $\dfrac{r_1}{r_2}$ as great as practicable; it will also be intensified by giving the angle $a_1$ a negative value.

In radial outward-flow turbines $\dfrac{r_1}{r_2}$ has a positive value, and it is obvious from the formula that an increase in the speed is accompanied by an increase in the velocity of flow.

Fig. 39 illustrates the relations between speed and velocity of flow for the three systems of turbine. Theoretically, the velocity of flow is affected by the presence or otherwise of a suction-tube with the turbine.
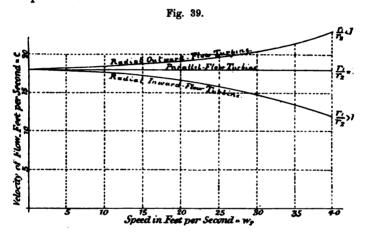
If there is a suction-pipe, the velocity of discharge

Fig. 38.



$c_4$ from the latter bears a fixed ratio to the velocity of flow $c$, which ratio is unaffected by variations of speed.

When, however, no suction-pipe is employed, and the wheel discharges direct into the tail-race, the velocity of discharge $u$ from the wheel varies with the speed. As has previously been shown, it is the resultant of the relative velocity of outflow $c_2$ and the velocity of rotation $w_2$. It is clear that (*vide* Fig. 38) the section of the stream of water leaving each bucket must vary in area according to the direction in which it issues; in other

words, the sectional area of discharge will change with different values of $c_2$ and $w_2$, but as the variation of $c_2$—proportional to $c$—is slight compared with that of $w_2$, this area will depend chiefly on $w_2$. Practically, the effect of the suction-tube on the velocity of flow is of little importance.

Fig. 39.



The following figures, taken from the results obtained by experiment with several Jonval turbines by Mr. P. Rittinger of Vienna in 1862, illustrate the effect of the speed on the velocity of flow as indicated by the quantity of water used per second :—

*Turbine No. 1.*

Number of revolutions = 160, 147, 140, 122, 110, 99, 93, 80.

Quantity of water    =  4·98, 5·25, 5·40, 5·74, 5·92, 6·12, 6·25, 6·43.

*HYDRAULIC MOTORS.*

### *Turbine No. 2.*

Number of revolutions = 242, 201, 177, 150, 135, 127, 102, 84.
Quantity of water = 5·25, 5·15, 5·11, 5·06, 5·04, 5·02, 4·99, 4·97.

### *Turbine No. 3.*

Number of revolutions = 268, 224, 198, 168, 151, 142, 113, 91.
Quantity of water = 6·97, 6·66, 6·55, 6·43, 6·38, 6·35, 6·24, 6·19.

### *Turbine No. 4.*

Number of revolutions = 289, 186, 171, 164, 156, 141, 126, 112.
Quantity of water = 3·06, 3·78, 3·89, 3·94, 4·00, 4·05, 4·17, 4·30.

### *Turbine No. 5.*

Number of revolutions = 289, 184, 176, 167, 159, 150, 143, 115.
Quantity of water = 3·03, 3·60, 3·66, 3·71, 3·77, 3·82, 3·95, 4·08.

### *Turbine No. 6.*
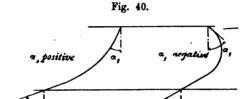
Number of revolutions = 586, 540, 493, 400, 354, 308, 285, 216.
Quantity of water = 2·41, 2·48, 2·55, 2·69, 2·75, 2·82, 2·86, 2·96.

## Explanation of Effect of Speed on Velocity of Flow.

It will be seen from these figures that in the case of turbines 1, 4, 5, and 6, the quantity of water, and with it the velocity of flow, decreases as the speed increases,

Fig. 40.
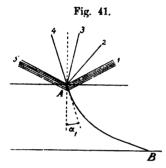


while for turbines 2 and 3 the contrary is the case. The reason for this difference is to be found in the fact that in the former the relative angle of entrance $a_1$ is negative as compared with $a_2$, while in the latter it is positive, *vide* Fig. 40. When $a_1$ is negative, $sin\ a_1$ also becomes negative, and consequently the co-efficient $B = \dfrac{A_2}{A_1} sin\ a$, for a Jonval turbine, has a *minus* sign, and $\dfrac{B}{1 + \zeta}\ w$ must be deducted from the quantity under the root sign; it increases with $w$ more rapidly than the latter, and $c$ consequently decreases simultaneously.

For turbines 2 and 3, $a_1$ is positive, and therefore $\dfrac{B}{1 + \zeta}\ w$ also positive, so that $c$ becomes greater with an increase of speed.

According, therefore, to the proportions of the buckets, the velocity of flow may be either increased or retarded

by the motion of the wheel, while in radial turbines the ratio of the radius at which the water enters to that at which it leaves exerts an additional influence.

Fig. 41.



The effect of the inclination of the vanes where the water enters the buckets on the velocity of flow may be explained as follows:—

Let $A$ $B$, Fig. 41, represent the wheel-vanes of a turbine for which the angle $a_1$ is positive. When the motor is held fast, the direction relative to the vanes of the stream entering the buckets is $A$ 1, and is in that particular case the same as the absolute direction of flow with which the water leaves the guide passages. The fluid impinges against the surface of the vane at an obtuse angle, and owing to the shock a loss of energy takes place, and the velocity of flow is retarded. When the wheel moves the relative direction in which the water impinges on the vanes is changed and becomes less obtuse; at a certain low speed let this direction be represented by $A$ 2, and at higher speeds by $A$ 3 and $A$ 4. As the angle of impact becomes less obtuse, the effect of the shock in retarding the flow is reduced, until finally some speed is attained at which the entrance of the water into the buckets occurs without shock; $A$ 4 represents the relative direction of flow under these conditions.

Up to this point the *shock* has diminished, and consequently the velocity of flow increased. Suppose the speed still further accelerated, so that the relative direction of inflow is $A$ 5. Now it is evident that the direction of impact has changed, and that the shock acts on the

back of the vanes in a sense opposed to the motion of the latter; the upper portion of the back convex vane surface tends to *force* the water through the buckets, acting in fact *as a pump,* while in the lower part the water still exerts a pressure on the concave surface, and drives the wheel. The turbine is therefore working simultaneously as a pump and motor, a certain proportion of the power developed in the latter capacity being absorbed in the former. If the increase of the speed be continued, a point is at length reached where the work done as motor is exactly balanced by that expended as pump, and no useful effect is produced. Beyond this the turbine works simply as a pump, and requires power from some external source to drive it.

With a negative angle $a_1$ the action of the vanes on the water is the reverse of that just described. From a condition of rest up to the speed at which the water enters without shock, there is a diminution of the resistance—from impact—offered to the entry of the water which tends to increase the velocity of flow, but this is generally more than counteracted by the tendency of the velocity of flow, apart from shock, to diminish. When the speed for inflow without shock is exceeded, the impact, as before, is on the back of the vanes; but now, in consequence of their being inclined in the opposite sense, an upward push is exerted on the inflowing water by the convex surfaces of the *negatively* inclined parts, which increases the resistance offered to the flow. As a result of the action just explained, in turbines with a negative value of the angle of inflow $a_1$ the velocity of flow is greatest when the wheel is held fast, and *decreases* as the speed increases. Theory on this point is fully confirmed by experiment, as a careful inspection of the various tabulated data relating to trials with turbines will show.

## Efficiency and Best Speed of Turbines.

By equation (2) the general expression for the efficiency of a turbine is $\epsilon = \dfrac{W_u}{h}$, or $\epsilon = \dfrac{W_u}{W_u + L + \dfrac{c_4{}^2}{2g}}$, or, including under $L$ all losses, whether from friction, impact, or velocity of off-flow, $\epsilon = \dfrac{W_u}{W_u + L}$.

Substituting for $W_u$ its value from equation (8), there follows :—

$$(15) \quad \epsilon = \frac{w_2\,(c_2\,\sin\,a_2 - w_2) + w_1\,c\,\sin\,a}{gh}.$$

It is evident that if $c$ and $c_2$ in the above expression are stated in terms of $w_2$ from (14), the efficiency $E$ will then be given in terms of the latter only as $w_1 = w_2\dfrac{r_2}{r_1}$; it is then possible to find a value of $w_2$ for which the efficiency $E$ is a maximum. The calculation of this value is somewhat complicated, requiring the use of the differential calculus, and it is unnecessary to give it here.

It is clear, however, from the following considerations, that there must be a certain speed of rotation at which the turbine is more efficient than at any other.

The force exerted on a vane by the water flowing over it depends, with a given velocity of the latter, on the extent to which the stream is deflected from its original course; it will therefore be greatest when the vane is at rest, or when the wheel is held fast, because as soon as the latter moves in the opposite direction to that in which the stream is deflected, the deflection must become less. The work done is the product of the

force exerted on the vanes and the velocity with which they move. When the wheel is held fast there is no motion, and consequently no work done, although the force acting on the vanes under these conditions is a *maximum;* on the other hand, when a certain speed of the vanes is attained, this force becomes *nil*, the wheel revolves—supposing there is no friction—without doing work. Between these two extremes the efficiency has risen from zero to some maximum value and fallen again to zero, and the speed at which this maximum efficiency occurs is the *best speed*, at which the wheel ought to be worked in order to obtain the highest results. Assuming the velocity of flow $c$ to be constant for all speeds of the turbine, it can be proved mathematically that the *best speed* is half that at which the wheel would run without a load. The efficiency by formula (15) is—

$$\epsilon = \frac{w_2 \left( c_2 \sin a_2 + \frac{r_1}{r_2} c \sin a \right) - w_2^2}{gh}.$$

When there is no load $E = 0$, and therefore—as $gh$ has a finite value—

$$w_2 \left( c_2 \sin a_2 + \frac{r_1}{r_2} c \sin a \right) - w_2^2 = 0,$$

whence the speed (velocity of rotation) without a load

$$w_2 = c_2 \sin a_2 + \frac{r_1}{r_2} c \sin a \, ;$$

the best speed is one-half of this quantity,

$$w_b = \tfrac{1}{2} \left( c_2 \sin a_2 + \frac{r_1}{r_2} c \sin a \right).$$

As a matter of fact the velocity of flow is not constant for all speeds of the motor, but in some cases varies considerably. For axial turbines, in which $a_1 = 0$, the above

formula gives correct results, since the velocity of flow in
such motors is constant.

The best speed is generally greater than the value above
given, but in designing turbines a somewhat different and
simpler method of calculating the best speed is followed,
which will be explained when the construction of turbines
is under consideration.

It is based on the obvious truth that, apart from
hydraulic resistances, the less the unutilized energy the
greater the efficiency of the motor, consequently the
absolute velocity of off-flow from the wheel or suction-
tube should be as small as possible. Unless, however,
combined with a suitable construction in other respects,
the fulfilment of this condition is in practice not sufficient
to insure maximum efficiency, as the losses from impact
or shock alone on the water entering the wheel-buckets
may exceed the whole of the loss due to the residual
velocity, and neutralize any advantage gained by reducing
this to a minimum. It follows that, simultaneously with
the reduction of the residual velocity to the lowest prac-
ticable value, the entry of the water into the wheel should
be effected without impact, that is, the water must enter
parallel with the direction of the extreme end of the
wheel-vane.

The efficiency of reaction turbines is reduced by all
methods of regulation when working otherwise than at
full power. A common method of regulating turbine
with suction-tube is by means of a sluice varying the
off-flow orifice. By partially closing the sluice the
quantity of water flowing through the turbine is reduced,
but at the same time the efficiency is reduced in much
greater proportion. The effect of diminishing the outflow
orifice on the quantity of water passing through the wheel
is not by any means proportional to the area of the orifice,

as might be supposed, while the work done decreases as the square of the velocity of flow.

Regulation by means of a throttle-valve and by a sluice in the head-race are equally bad.

The method of regulation now generally adopted, of closing a portion of the guide passages, is much less detrimental to the efficiency than any of those above mentioned; this part of the subject will be more fully investigated in dealing with the construction and design of turbines.
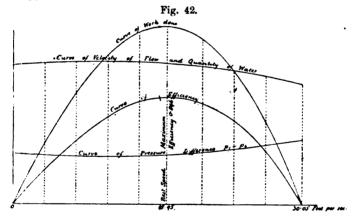
The foregoing formulæ and considerations apply to reaction turbines under perfectly general conditions, no assumptions tending to simplify the demonstration having been made. The water enters the wheel, except at one particular speed, at an angle relative to the latter which does not correspond with the inclination of the guide-vanes at the point of entry, and leaves it with an absolute velocity which varies in amount and direction with the speed of the wheel.

Let it be supposed that the wheel is at first held fast by imposing on it a load greater than the action of the water on the vanes is able to overcome, and that then the load is reduced in stages, so as to allow the turbine to revolve at successively increasing speeds. As the speed increases, the following results will take place in the case of an inward-flow turbine :—

(1) The force exerted by the water on the vanes will *decrease ;*

(2) The velocity of flow of the water passing through the turbine will first *increase* and afterwards *decrease,* and with it the quantity of water and available total energy ;

(3) The difference between the pressure at the entrance and that at the outlet of the wheel-buckets will *decrease,* and

(4) The work done will *increase* up to a certain point and afterwards *decrease ;*

(5) The efficiency will *increase* up to a certain point and afterwards *decrease.*

The point of maximum efficiency does not exactly coincide with that of maximum work, for the reason that the available energy varies with the speed, owing to a greater or less quantity of water flowing through the wheel.

Fig. 42.



The relations existing between the various quantities are illustrated graphically by Fig. 42, which shows the curves representing velocity of flow $c$ and other quantities proportional with it ($Q$ and $E$), the difference of pressure $p_1 - p_2$, the work done $W$, and the efficiency $E$.

The values of these are plotted as ordinates to corresponding values of $w_2$ as abscissæ ; they have been calculated for an inward-flow turbine. It will be seen by an inspection of the diagram, that while the increase and decrease of the velocity of flow $c$ is very gradual, the efficiency rises and falls very rapidly, so that a com-

paratively slight deviation from the best speed materially reduces the efficiency. This fact has been amply confirmed by experiment.

For radial outward-flow and axial turbines the course of the various curves above referred to is somewhat different, and in the following particulars :—

(1) The velocity of flow increases continuously with the speed, or remains constant.

(2) The difference between the pressure at the entrance and that at the outlet of the wheel-buckets decreases continuously as the speed increases, or remains constant. With respect to this latter value, it has been found better that it should be positive, that is, $p_2$ less than $p_1$, although this tends to cause loss by forcing water out through the space between the guide and wheel-buckets. It may be negative from the commencement, if the ratios which $A$, the sectional area of outflow from the guide passages, bears to the areas of other portions of the apparatus, be small.

The following results obtained with the experimental turbines of Rittinger, previously mentioned, p. 67, illustrate the connection between speed and efficiency :[1]—

*Turbine No.* 1.

Number of revolutions = 160, 147, 140, 122, 110, **99**, 93, 80.

Efficiency = 0, 0·326, 0·455, 0·626, 0·659, **0·673**, 0·667, 0·631.

*Turbine No.* 2.

Number of revolutions = 242, 201, 177, 150, **135**, 127, 102, 84.

[1] The thicker figures denote the best results.

Efficiency          =          0,     0·452, 0·601, 0·687,
**0·697**, 0·695, 0·650, 0·586.

*Turbine No. 3.*

Number of revolutions = 268,     224,     198,     168,
151,     142,     113,     91.
Efficiency          =          0,     0·405, 0·543, 0·622,
0·632, 0·630, 0·587, 0·519.

*Turbine No. 4.*

Number of revolutions = 289,     186,     171,     **164,**
156,     141,     126,     112.
Efficiency          =          0,     0·693, 0·711, **0·715,**
0·711, 0·705, 0·674.

*Turbine No. 5.*

Number of revolutions = 289,     184,     176,     167,
159,     150,     143,     115.
Efficiency          =          0,     0·623, 0·636, 0·641,
0·645, 0·639, 0·618, 0·575.

*Turbine No. 6.*

Number of revolutions = 586,     540,     493,     400,
354,     **308,**     285,     216.
Efficiency          =          0,     0·231, 0·410, 0·633,
0·687, **0·700,** 0·694, 0·626.

As a further illustration of the same fact, the following
figures giving the results of experiments with a Jonval
turbine, carried out by Mr. G. Meissner, and quoted in
his work on turbines, may be studied :—

*Jonval Turbine.*

Number of revolutions = 159,     103,     94,     **82,**
71,     65,     53,     41,     31,     15,     0.

Efficiency       =       0,   0·692, 0·735, 0·743,
0·704, 0·688, 0·605, 0·525, 0·440, 0·257,   0.

In this instance the experiments extended to the whole range of efficiency from zero when the wheel was held fast, to zero when it was running without a load. It will be noted that in every case the *best* speed is greater than one-half of that corresponding to the un-loaded condition, but it must be borne in mind that in practice it is impossible for the wheel to run absolutely without resistance, as there is always a certain amount of friction of the bearings and between the surface of the wheel and the water to be overcome, so that the *hydraulic* efficiency at the maximum actual number of revolutions would be, not 0, but 3 to 5 per cent.

Referring once more to the general expression for the efficiency $\epsilon = \dfrac{W_u}{W_u + L}$, it is easy to see that for a given value of $L$, the sum of all the losses of energy of the water during its passage through the turbine, the greater $W_w$ the work done, the greater the efficiency $E$.

If it were possible to reduce $L$ in the same proportion as $W$ when the latter diminished, then the efficiency would remain the same for all values of $W$. In fact, however, not only does $L$ *not* decrease, but it actually increases as $W$ becomes less, until, when the turbine is held fast, or runs without a load (the two extreme cases in which no work is done), the whole available energy is absorbed by friction and other resistances, or carried off in the kinetic form.

This is shown by the equation

$$h = W_u + L_1 + \frac{c_4^2}{2g};$$

when in this      $W_n = 0$

$$h = L_1 + \frac{c_4^2}{2g}.$$

In fact, for given dimensions, $c_4$ the velocity of off-flow does not differ very greatly whether the wheel be running at its best speed or standing still; in the latter case the available energy is swallowed up chiefly by friction, impact, and similar resistances within the apparatus.

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# CHAPTER IV.

## LOSSES OF ENERGY IN REACTION TURBINES.

Enumeration of losses, character, and mathematical expression.—Losses from pipe friction.—Examples.—Loss in guide passages.—Co-efficients.—Loss from shock.—Example.—Loss by leakage.—Loss in wheel-buckets.—Co-efficients.—Loss from resistance of sluice.—Loss from residual velocity of flow.—Numerical values.—Summaries of losses.—General summary of facts relating to reaction turbines.

## *Losses of Energy in Reaction Turbines.*

IN considering the losses of energy to which the water on its passage through the turbine and its adjuncts is subject, it will be best to follow the course of the fluid from the reservoir or conducting pipe over the wheel to its off-flow under the sluice at the bottom of the suction-pipe, where one is used.

The first loss incurred is that due to friction in the pipe or casing conducting the water to the turbine. If the diameter of the supply pipe be denoted by $d_o$, the length corresponding to this diameter by $l_o$, the velocity of flow by $c_o$, then the loss of energy is expressed by

$$l_o = \zeta_r \frac{l_o}{d_o} \frac{c_o^2}{2g},$$

G

or, in·terms of $c$, the velocity of outflow from the guide passages,

$$L_0 = \zeta_r \frac{l_0}{d_0} \left(\frac{A}{A_0}\right)^2 \frac{c^2}{2g} \qquad (a)$$

where $\zeta_r$ is an experimental co-efficient.

The next loss takes place in entering, passing through, and leaving the guide passages, and is made up, (1) of the resistance offered by the upper edges of the guide-vanes, causing a sudden deflection of portions of the stream and an abrupt contraction, (2) of the friction between the water and the surface of the guide passages, (3) of the loss on the water leaving the orifices of the guide passages owing to internal friction and friction of the vane edges, (4) of the loss resulting from the curved form of the passages.

It is usual to include all these in one expression of the form

$$L_1 = \zeta_1 \frac{c^2}{2g}. \qquad (b)$$

The loss due to friction in the guide passages is, owing to the shortness and comparatively great area of the latter, extremely small.

When the water leaves the guide passages and enters the wheel-buckets, it does not—as already stated—under general conditions, take the direction of the tangent to the vane curve at the point of entry, relative to the wheel, and in consequence impinges against the vane instead of moving parallel with it, the result being a sudden change of velocity causing a loss of energy.

If the relative velocity of flow *after* entering the wheel be $c_1$, this loss of energy is represented by

$$L_2 = \frac{1}{2g} \left[ (c \cos a - c_1 \cos a_1)^2 + (c \sin a - w_1 + c_1 \sin a_1)^2 \right].$$

$$(c)$$

A reference to the diagram, Fig. 43, will show that the quantities in the brackets are the differences between the components of the absolute velocities of flow $c$ and $c_a$ before and after entering the wheel-buckets, parallel with and at right angles to the axis of the wheel. The losses of energy in each of these directions is represented by

Fig. 43.



the squares of those differences; parallel with the axis, or radially, according to the type of the wheel, the velocity has been suddenly reduced from $O\,a$ to $O\,b$, the difference $b\,a = c \cos a - c_1 \cos a_1$; at right angles to the axis, the reduction is from $a\,d$ to $a\,e$, the difference

$$c\,d = c \sin a - (w_1 - c_1 \sin a_1);$$

hence $L_2 = \frac{1}{2g} \left[ (a\,b)^2 + (d\,e)^2 \right] = \frac{1}{2g} \left( f\,d \right)^2$,

the quantity within the brackets being easily determined by the graphic method, as evident from Fig. 43. In general the difference $c \cos a - c_1 \cos a_1$ will be small, owing to the condition of continuity, when the clear distance

or, in terms of $c$, the velocity of outflow from the guide passages,

$$L_0 = \zeta_r \frac{l_o}{d_o} \left(\frac{A}{A_o}\right)^2 \frac{c^2}{2g} \qquad (a)$$

where $\zeta_r$ is an experimental co-efficient.

The next loss takes place in entering, passing through, and leaving the guide passages, and is made up, (1) of the resistance offered by the upper edges of the guide-vanes, causing a sudden deflection of portions of the stream and an abrupt contraction, (2) of the friction between the water and the surface of the guide passages, (3) of the loss on the water leaving the orifices of the guide passages owing to internal friction and friction of the vane edges, (4) of the loss resulting from the curved form of the passages.

It is usual to include all these in one expression of the form

$$L_1 = \zeta_1 \frac{c^2}{2g}. \qquad (b)$$

The loss due to friction in the guide passages is, owing to the shortness and comparatively great area of the latter, extremely small.

When the water leaves the guide passages and enters the wheel-buckets, it does not—as already stated—under general conditions, take the direction of the tangent to the vane curve at the point of entry, relative to the wheel, and in consequence impinges against the vane instead of moving parallel with it, the result being a sudden change of velocity causing a loss of energy.

If the relative velocity of flow *after* entering the wheel be $c_1$, this loss of energy is represented by

$$L_2 = \frac{1}{2g} \left[ (c \cos a - c_1 \cos a_1)^2 + (c \sin a - w_1 + c_1 \sin a_1)^2 \right].$$ $\qquad (c)$

A reference to the diagram, Fig. 43, will show that the quantities in the brackets are the differences between the components of the absolute velocities of flow $c$ and $c_a$ before and after entering the wheel-buckets, parallel with and at right angles to the axis of the wheel. The losses of energy in each of these directions is represented by

Fig. 43.



the squares of those differences; parallel with the axis, or radially, according to the type of the wheel, the velocity has been suddenly reduced from $O\,a$ to $O\,b$, the difference $b\,a = c \cos a - c_1 \cos a_1$; at right angles to the axis, the reduction is from $a\,d$ to $a\,e$, the difference

$$c\,d = c \sin a - (w_1 - c_1 \sin a_1) ;$$

hence $L_2 = \dfrac{1}{2g} \left[ (a\,b)^2 + (d\,e)^2 \right] = \dfrac{1}{2g} (f\,d)^2,$

the quantity within the brackets being easily determined by the graphic method, as evident from Fig. 43. In general the difference $c \cos a - c_1 \cos a_1$ will be small, as, owing to the condition of continuity, when the clear distances

between the guide-vanes and wheel-vanes measured in
the direction of rotation are equal, $c \cos a = c_1 \cos a_1$,
except in so far as there is a difference in the width of
guide and wheel vanes, or in their thickness, or both.
This will be clear on referring to the diagram Fig. 44,

Fig. 44.



where $a\ b$ and $b\ c$ represent
the terminations of the two
guide-vanes and two wheel-
vanes respectively of equal
pitch $t$, measured on the
corresponding circumference.
The depths and consequently
areas of the passages of the
guide and wheel are propor-
tional to the cosines of the
angles $a$ and $a_1$, while the
velocities $c$ and $c_1$ must be in
the inverse ratio of the former, provided continuity is
maintained.

When a turbine is working as it ought to do, the water
entering the wheel-buckets without impact, the source of
loss under consideration disappears.

The next loss to be determined is that due to leakage
through the clearance space between the guide-casing
and wheel; it depends theoretically on the pressure
difference inside and outside the clearance space. As
the author does not consider that any reliance is to be
placed on a detailed calculation of the loss from the
leakage in question, he has followed Redtenbacher's
method of including it with the other losses incurred
during the passage of the water through and out of the
wheel-buckets; these are due to similar causes to those
sustained in the guide passages, and all are included in
an experimental co-efficient, and expressed in the form

$$L_3 = \zeta_2 \frac{c_2^2}{2g} = \zeta_2 \left(\frac{A}{A_2}\right)^2 \frac{c^2}{2g}. \qquad (d)$$

The absolute velocity $u$ with which the water leaves the wheel-buckets, as a rule, is not the same either in value or direction as that with which its flow in the suction-tube, immediately below the wheel, is continued. In consequence of this sudden change—in every case from a higher to a lower velocity—there is a loss of energy of the same kind as that previously denoted by $L_2$.

The water leaves the wheel-buckets with an *absolute* velocity $u$, which, as previously stated, is the resultant of the *relative* velocity $c_2$ of outflow and the speed of rotation of the bucket at the point in question. It may be assumed that on its passage downwards through the suction-tube the water takes a spiral course, this latter and the corresponding velocity immediately on leaving the wheel-buckets being represented in the diagram, Fig. 45, by $O\,x$, while the absolute resultant velocity of outflow $u$ is denoted by $O\,d$; the vertical component of $O\,x$ is $O\,b$, that of $O\,d$ is $O\,a$; the sudden change in velocity, in axial or radial direction, is therefore $O\,a - O\,b$, and the square of this is proportional to the loss of energy incurred; similarly, in a circumferential direction the change of velocity is $a\,d - b\,x = d\,y$, and the square of this corresponds to the loss of energy. The latter is small, as the circumferential component of velocity $a\,d$ probably remains nearly unchanged, so that it will be sufficient to consider only the axial (or radial) loss.

Fig. 45.



If $c_3$ be the velocity in a radial or axial direction

immediately after the water leaves the wheel, then the loss of energy in question,

$$\dot{L_4} = \frac{1}{2g}\,(c_2\cos a_2 - c_3)^2 = \frac{1}{2g}\,[a\,b]^2, \qquad (e)$$

where, as is clear from the diagram, $O\,a = c_2\cos a_2$ and $O\,b = c_3$.

It is assumed that $c_3$ is also the velocity in the suction-tube immediately below the wheel, or, in the case of an inward-flow radial turbine, it comes to the same thing if it be supposed that the absolute velocity of outflow remains unchanged after leaving the buckets until the course of the water has been deflected from a horizontal to a vertical direction, or, more strictly speaking, from a direction at right angles to the axis of rotation to one parallel with the axis of the suction-tube (*vide* Fig. 46).

Fig. 46.



In passing through the suction-tube the water meets with resistance from the friction of the sides; the loss due to this cause is expressed in the same way as that already considered under (*a*) and denoted by $L_0$; hence its value is

$$L_5 = \zeta_r\,\frac{l_3}{d_3}\,\frac{c_3^3}{2g} = \zeta_r\,\frac{l_3}{d_3}\left(\frac{A}{A_3}\right)^2\frac{c^2}{2g}. \qquad (f)$$

In finally leaving the suction-tube under a sluice the water encounters a resistance offered to its passage by the edge of the sluice; the amount of this depends very materially on the form of the sluice, whether the edge is sharp or well rounded. The loss incurred from this cause may be expressed by

$$L_6 = \zeta_4\,\frac{c_4^2}{2g} = \zeta_4\left(\frac{A}{A_4}\right)^2\frac{c^2}{2g} \qquad (g)$$

The last loss to be considered is that which is represented by the energy contained in the water when it leaves the suction-tube; this is obviously

$$L_7 = \frac{c_4^2}{2g} = \left(\frac{A}{A_4}\right)^2 \frac{c^2}{2g}. \qquad (h)$$

It is convenient to combine $L_6$ and $L_7$ together in the form

$$(L_6 + L_7) = (1 + \zeta_4) \left(\frac{A}{A_4}\right)^2 \frac{c^2}{2g}.$$

Having enumerated the various losses and briefly explained their character, it will be necessary to examine somewhat more closely into their nature and relative proportions.

*Loss in Supply-pipe.* The first loss $L_0$, due to friction in the supply-pipe or channel, is, in many cases, for moderate falls very small; for a Jonval or inward-flow turbine working with a suction-tube and placed tolerably high above the tail-water level, the water above the wheel is contained in a tank or reservoir of such dimensions that the loss from friction in it is quite inappreciable. On the other hand, the loss $L_5$ from friction in the suction-tube of course disappears where no suction-tube is employed, as in the majority of cases.

The co-efficient $\zeta_r$ will be the same in both instances where the material used for the tubes is similar, and for practical purposes with ordinary velocities the value $\zeta_r = 0\cdot03$ may be taken.

In reality $\zeta_r$ varies with the velocity of flow, as shown by the following table :—

For $c_3 = 0\cdot82$    1·64    3·28    6·56    16·40    feet.
   $\zeta_r = 0\cdot0350$    0·0289    0·0246    0·0216    0·0189.

The velocity in the parts of the turbine in question

will be within the above values, under ordinary circum-
stances.

In cases where the diameter of the suction-tube varies
the ratio $\dfrac{l}{d}$ will be different at various points; the velocity
of flow will of course also vary. This may be taken into
account, where the tube is conical, by introducing the
*mean diameter* and *mean velocity* of flow into the calcu-
lation.

As an example taken from practice, suppose the head
of water to be 15 feet, the length of the suction-tube
10 feet, the diameter 8 feet, and the velocity of flow
3 feet per second. According to the above table the
co-efficient $\zeta_r$ suitable to this case would be, say, 0·025,
hence the loss of energy

$$L_5 = 0·025 \, \frac{10}{8} \, \frac{3^2}{64} = 0·0312 \times 0·109$$

$$L_5 = 0·0034 \text{ feet,}$$

expressed as *head* of water; this is so small a proportion
of the total available head of 15 feet as to be practically
negligible.

With a tube of twice the above length, 20 feet, and
double the residual velocity of flow, 6 feet, the value of
$L_5$ would be 8 times as great as the preceding, but even
then would only amount to 0·0272 foot, and if the head
were, say, 24 feet (almost as small as possible with 20 feet
of suction-tube), the percentage of loss would be only
0·11 per cent.

*Loss in Guide Passages.* The resistance offered to the
water in flowing through the guide passages may be
represented by a single co-efficient, the value of which
does not vary much for the usual construction of this part
of a turbine. The lower portions of the guide-vanes are

generally made parallel with each other in axial turbines for a considerable length above the orifices, so that there is no contraction; also the passages are so formed that in flowing through them the stream of water undergoes no sudden change of section. The conditions under which the outflow takes place are therefore peculiarly favourable. Under these circumstances, if the surface of the vanes were smooth and polished, the co-efficient $\zeta_1$ (in the expression (*b*) for $L_1$) would be from 0·042 to 0·025, or otherwise expressed, the velocity of flow would range from 0·96 to 0·975 of the value it would have were there no resistance. In fact, the surfaces of the vanes are comparatively rough, so that a somewhat higher co-efficient must be used, say, 0·053, corresponding to a ratio of the actual to theoretical velocity of 0·95.

A very usual value for the velocity of flow $c$ in a reaction turbine of the Jonval type is $c = 0\cdot 7 \sqrt{2gh}$, corresponding to about half the actual head $h$. Taking this velocity to be 0·95 of that which would occur were there no resistance in the guide passages, the ratio of the energy due to the former velocity to that due to the latter would be as $(0\cdot95)^2 : 1$, or 0·90, corresponding to a loss of 10 per cent.; but as the velocity of flow only represents one-half of the available head, the actual loss as compared with the head would be 5 per cent.; with polished surfaces it might be 3 to 4 per cent.

Weisbach found by experiment the value of $\zeta_1$ to be from 0·05 to 0·10, Hänel gives $\zeta_1 = 0\cdot10$ to 0·20, and Francis 0·20 and over; for practical purposes $0\cdot125 = \frac{1}{8}$ may be taken as an average value, but is rather high for a good turbine. In impulse turbines $\zeta_1$ has been found to range from 0·11 to 0·17, and there is no reason why it should be different for reaction wheels.

*Loss due to Impact.* This loss $L_2$, it has been already stated, does not occur to any appreciable extent in a turbine working at the best speed with properly constructed vanes, in which the water enters the wheel-buckets parallel with the surface of the vanes. There will, it is true, even under the latter conditions, be a certain loss of this kind, owing to two causes: (1) the slight excess of width of the wheel-buckets over that of the guide passages, and (2) the leakage through the clearance space between guide passages and wheel; in consequence of these, the velocity relative to the wheel is suddenly retarded on entering the latter. This source of loss will however be included under the losses in the wheel-buckets for practical calculations. For an example of the calculation of the loss $L_2$, the following data, taken from an actually constructed turbine, may be used: $a = 73° 30'$, $a_1 = 90°$, $sin\ a = 0·96$, $cos\ a = 0\ 284$, $c = 20$ feet per second, $w_1 = 29·2$ feet per second, the proper speed being 20 feet; allowing for an excess of width in the entrance to the wheel-bucket and leakage through the clearance space, $c_1$ may be taken at 5·34 feet instead of $5·68 = c\ cos\ a$; ($cos\ a_1 = cos\ 0° = 1$).

Hence, using formula ($c$) (approximately)

$$L_2 = \frac{1}{64}\ (0·1156 + 100) = \frac{100·1156}{64} = 1·564 \text{ foot}$$

expressed as head.

The total head for the turbine in question is 14·75 feet; the loss therefore amounts to 10·6 per cent., and is due to the wheel running at a speed much greater than the best.

It will be found that for $w_1 = 19·2$ the second quantity in the brackets becomes *nil*.

There is good reason to believe that the loss from

shock as determined by the preceding method is in excess of that actually taking place, especially when the deviation from the best speed is considerable. Taking an extreme case, when a stream of water impinges at right angles against the surface of a vane, according to the formula given the *whole* of the energy represented by the velocity of the stream would be lost; this in fact does not happen, as a portion of the water itself forms a guide for the rest of the stream, interposing between the latter and the vane, and causing a deflection of the fluid over a *curved surface* instead of an abrupt stoppage. It should also be mentioned here that the theoretical conditions for avoidance of impact are not always those corresponding to the minimum loss. This will be further considered in the sequel.

*Loss by Leakage through Clearance Space.* It is obvious that to avoid friction between the wheel and the guide apparatus there must be a certain amount of clearance, and the excess of pressure inside the turbine at the points where the water leaves the guide passages over that outside will force it out through the annular space between the wheel and guide passages. If the pressure inside should be less than outside—which is possible —water would be forced into the wheel through this space; in practice this is rarely the case, and it will be assumed, for the purpose of investigation, that the pressure $p_1$ inside is greater than $p_2$ the pressure outside. The quantity of water which will leak out is obviously the product of the velocity of flow through the clearance space and the area of the latter. The velocity is that due to the head $p_1 - p_2$ multiplied with a co-efficient of discharge, which may be taken as 0·7.

The clearance for turbines of moderate dimensions cannot, for the present purpose, be safely taken as less than

⅛ inch, as there is always a tendency for it to increase
with the wear of the footstep.  If $D$ be the diameter of
the wheel where the water enters, then the quantity
leaking through the clearance space must be

$$q = 0\cdot70 \sqrt{(p_1 - p_2)\, 2g} \times 2\, D\, \Pi\, t$$

where $t$ is the clearance.

It has been shown that $p_1 - p_2 = h - (1 + \zeta^1)\dfrac{c^2}{2g}$;

hence $q = 0\cdot70\sqrt{2g\left(h - (1 + \zeta^1)\dfrac{c^2}{2g}\right)} \times 2\, D\, \Pi\, t.$

The loss of energy from leakage is clearly proportional
to $q$, and the percentage of loss increases as the square
root of the head $h$, and is inversely proportional to the
total quantity of water flowing through the turbine.  It
is impossible to determine exactly what occurs at the
clearance, since the conditions are very complicated; as
a portion of the water leaks out before reaching the
wheel-buckets, the conditions of continuity *in the guide
and wheel passages* are not strictly fulfilled, and in con-
sequence some interference with the assumed theoretical
state of affairs occurs, which must react on the pressures
$p_1$ and $p_2$, and modify the leakage in a manner which it
is impossible to calculate.
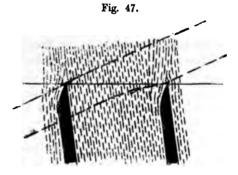
*Loss in Wheel.*  This includes, in addition to the loss
due to friction and the curved form of the passages, that
resulting from the obstruction offered by the edges of the
wheel-vanes to the free flow of the water.

Its effect is to cause a more or less sudden contraction
of the stream entering the buckets.  If the edges of the
buckets were flat with sharp corners, the loss would be
proportional to the square of the difference between the
absolute velocities of flow before and after entering the
buckets.

In practice, the edges of the wheel-vanes at the inflow are tapered off, as shown in Fig. 47, and the loss due to the obstruction caused by them is slight.

The loss in the wheel-buckets is similar in character to the loss incurred in the guide passages, and its ratio or percentage to the total head is about the same.

For practical purposes it would be a too laborious process to attempt to calculate separately all the losses of energy sustained by the fluid from the moment it leaves the guide passages to that at which it issues from

Fig. 47.



the wheel-buckets, and the uncertainty of the phenomena actually occurring is too great to give the results much value even if this were done. As already indicated, all the losses occurring during the period referred to, which for convenience will be termed *loss in the wheel*, may be combined and represented by a single co-efficient $\zeta_2$.

Meissner is of opinion that the sum of the losses in the guide passages and in the wheel, expressed in percentage of the total head, are about the same for all reaction wheels, as the causes which tend to increase the resistance in the one diminish it in the other, and *vice versâ*. A high velocity in the guide passages is accompanied by

a relatively low velocity in the wheel-buckets, while a high rate of flow in the latter corresponds to a relatively low rate in the former.

According to Hänel $\zeta_2 = 0{\cdot}10$ to $0{\cdot}20$.

Experiments made by Professor Fliegner tend to show that $\zeta_2$, contrary to what might be expected, decreases as the excess of pressure $p_1 - p_2$ increases; but a sufficient number of results are not available to be of any practical use.

## Loss from Resistance of Sluice.

This depends almost entirely on the shape of the lower edge of the sluice, and the co-efficient in formula $(g)$ may range from 4 to 1, as the edges of the sluice are sharp or well rounded off; there is, however, little doubt that it may be reduced much below this value by giving the lower end of the sluice-gate a suitable form.

If the co-efficient 1 be applicable, the loss $L_6 = \zeta_4 \dfrac{c_4^2}{2g}$ is equal to that due to the residual velocity $c_4$, and the latter may be kept within small limits by giving the sluice-gate a large opening.

## Loss from Residual Velocity of Off-flow.

The velocity with which the water leaves a turbine, whether direct from the wheel-buckets or under the sluice of a suction-tube, ranges in practice, as a rule, from $\dfrac{1}{5}$ to $\dfrac{1}{3{\cdot}5}$ of the velocity due to the total head $(\sqrt{2gh})$; for high falls it may be lower, but for large

turbines and moderate - falls $\frac{1}{5}\sqrt{2gh}$ may be considered the lowest limit.

If $u$ be the absolute velocity of off-flow, the corresponding loss is expressed by $\frac{u^2}{2g}$, and for $u = \frac{1}{5}\sqrt{2gh}$ will be $\left(\frac{1}{5}\right)^2 = \frac{1}{25}$ of the whole available head, or 4 per cent.; for $u = \frac{1}{4}\sqrt{2gh}$ the loss would be $\frac{1}{16}$, or 6·25 per cent.

One of the losses enumerated disappears when a turbine works at its proper speed, viz. $L_2$ the loss from impact; others occur only with a suction-tube, viz. $L_5$ and $L_6$.

Meissner gives the following summary of the usual losses in a Jonval turbine :—

| | | |
|---|---|---|
| (1) Sum of losses by friction and impact in guide apparatus and wheel . . . . . | 10¼ to 14 % | |
| (2) Loss due to residual velocity . | 6 „ | 6 „ |
| (3) Loss by leakage through clearance space. . . . . | 4½ „ | 4½ „ |
| (4) Friction in air and water and of bearings . . . . | 2 „ | 3½ „ |
| Total | 23 „ | 28 „ |

A careful estimate of the average losses in a turbine, made by Mr. Lehmann from an analysis of experiments on thirty-six turbines varying from 1 to 500 h.p., is the following :—

| Loss per cent. due to | Axial-flow Turbine. | Outward-flow Turbine. | Inward-flow Turbine. |
|---|---|---|---|
| Hydraulic resistances . | 12 | 14 | 10 |
| Unutilized energy . | 3 | 7 | 6 |
| Shaft friction . . | 3 | 2 | 2 |
| | — | — | — |
| Total | 18 | 23 | 18 |
| Efficiency | 0·82 | 0·77 | 0·82 |

In estimating the *hydraulic* efficiency, which has hitherto been understood by the term *efficiency*, the shaft friction is not taken into account; but in practice, unless otherwise stated, the expression denotes the ratio of the work as measured on a brake—exclusive of shaft friction—to the available energy.

Mr. Lehmann's estimate of the efficiency is for ordinary wheels, and for the purpose of design rather too high.

## *Summary.*

(1) Every part of the apparatus forming a reaction turbine must be full of water, and consequently there must be continuity of flow, the whole arrangement being comparable to a system of pipes of varying section.

(2) The available energy is the product of the total head and the weight of water flowing through the apparatus per unit of time (second or minute).

(3) The useful work done is the available energy less the various losses arising from hydraulic resistances (friction, impact, &c.), leakage, and the unutilized energy due to the remaining velocity of the water on leaving the apparatus.

(4) The efficiency is the ratio of the useful work done to the available energy, or—what comes to the same thing —the ratio of the useful work to the sum of the useful work and the various losses.

(5) The use of a suction-tube with a reaction turbine enables the wheel to be placed within a certain limit at any height above the tail-water level without diminishing its efficiency; the theoretical limit of this height at which the outflow orifices of the wheel may be placed, is that of a column of water equivalent to the pressure of the atmosphere, 33·9 feet, but in practice it is considerably lower, and varies according to the proportions of the suction-tube.

(6) The work done in turning any turbine wheel depends for a given speed on the change in the momentum in the direction of rotation, produced by the deflecting action of the wheel-vanes on the water passing through them.

Apart from losses, the work done is equal to the difference between the energy of the water entering and that leaving the wheel.

(7) The available energy of the water entering a reaction wheel is due partly to its velocity and partly to its excess of pressure over the atmosphere. This excess may be negative; its amount depends on the proportions of the turbine under any given conditions as to head and height of the wheel in the suction-tube.

(8) The difference between the pressure of the water entering and that leaving the wheel is independent of the position of the latter in the suction-tube.

(9) The velocity of flow in any one part of a reaction turbine determines that in all other parts. It is convenient for purposes of comparison to deal with the velocity with which the water leaves the guide passages. This velocity is primarily dependent on the head of water above the guide passage orifices and the excess of pressure in the latter, but it can be shown that it is unaffected by the position of the wheel in the suction-tube.

(10) The velocity of flow is determined for a given turbine by the total head of water and the speed of the wheel.

According to the construction of the wheel, the manner in which the velocity of flow varies with the speed is different.

For outward-flow turbines in which the angle $a_1$ is positive or $0°$, as is usually the case, the velocity of flow increases continuously with the speed. For a radial inward-flow turbine the velocity of flow at first increases, and subsequently decreases as the speed becomes greater, or decreases continuously.

For an axial turbine, according to the angle $a_1$, the velocity of flow increases, remains constant, or decreases with an increase of speed.

(11) The velocity of flow from the guide passages may be either greater or less than the velocity due to the head $V = \sqrt{2gh}$, according to the proportions between the various parts of the turbine, by which the pressure $p_1$, against which the water issues, is affected.

This velocity decreases as the ratio of the effective sectional area of the guide passages to that of the outflow orifices of the wheel increases.

(12) The efficiency of a reaction turbine of given dimensions varies with the speed, and is greatest for a velocity of rotation somewhat in excess of *half* that at which the wheel would run without a load.

The extent to which the sluice of the suction-tube is opened affects the efficiency very materially; by partly closing the sluice the efficiency is diminished; other methods of regulation, by closing some of the guide passages for instance, have a similar effect.

# CHAPTER V.

## THE DESIGN OF REACTION TURBINES.

Simplifying assumptions.—Mathematical conditions.—Relations between pressures and velocities of flow.—Velocity of flow from guide passages.—Another method of calculation.—Efficiency, various formulas.—Connection between various dimensions of turbines.—Effective and measured areas of passages.—Methods of determining areas.—Radii of wheels.—Width and depth of buckets.—Number and thickness of vanes.—Summary of formulas.—Application to axial turbines.—Usual data for designing turbines.—Influence of relative angle of outflow on efficiency.—General rules regarding suction-tube.—Alternative data.—Vanes of turbines.—Usual construction for axial turbines : guide-vanes ; wheel-vanes.—Back-vanes. — General remarks. — Vanes of inward-flow wheels. — Absolute path of water.

HITHERTO, in dealing with the theory of reaction turbines, the construction of the latter has been taken as a given quantity, and their actions investigated under perfectly general conditions, that is, at various speeds and with no special assumptions as to the mode of construction.

In applying the theory to the design of turbines, the problem to be solved is : Under given conditions, what must be the proportions and speed of a turbine constructed so that it may work to the greatest advantage, that is, with the maximum efficiency ?

It has been shown that when all the dimensions of an existing turbine are known, it is possible to calculate, for a particular fall or head of water, the speed of the wheel

at which the greatest efficiency is realized. The various quantities entering into the calculation are, however, so numerous, and their relations so involved, that it is not practicable to apply the formula used to the determination of the best proportions for a new turbine, and for the latter purpose some assumptions have to be made which materially simplify the calculations, and at the same time give a close approximation to the true result.

Under ordinary circumstances a turbine is required to run at a constant speed, and this speed should be so chosen that, under normal conditions, it is that at which the motor works with the maximum efficiency; in comparing, therefore, the merits of various systems of turbines, or the effects of varying proportions on the efficiency of any particular class of turbine, this comparison will always be made on the assumption that the wheels are running at the best speed.

It is evident that the maximum efficiency must be attained when the sum of all the losses of energy is reduced to a minimum.

All the losses, except two, depend *directly* only on the velocity of flow of the water in the particular part of the turbine in which they occur. When, therefore, the velocity of flow increases, these losses also increase, but the work done increases in the same ratio, and consequently the efficiency is not directly affected. The velocity of flow itself depends to a certain extent on the speed of the wheel, but the variations in the former due to changes in the latter do not influence materially the ratio of such losses to the total available energy.

Two of the losses, as above stated, are however *directly* influenced by the speed of the turbine. These are, (1) the loss due to impact or shock of the water on entering the wheel-buckets, and (2) the loss from unutilized energy

due to the residual velocity of the water leaving the turbine ($L_7$).

If therefore the speed is so determined that these two losses are reduced to a minimum, it will be a sufficiently close approximation to the best speed.

As previously indicated, the loss (1) will disappear if the water enters the wheel-buckets with a relative direction parallel with that of the end of the wheel-vanes at the points of entry, while loss (2) will be smaller the less the residual velocity of off-flow $u$ from the wheel.

Fig. 48.



The absolute velocity of flow at the moment the water enters the wheel-buckets is the resultant of the velocity of rotation of the wheel and of the velocity of flow relative to the latter at the point of entry; hence this absolute velocity of flow must coincide with that of the water leaving the guide passages, and the angles of the guide-vanes and wheel-vanes at the point of inflow of the water be made to fulfil these conditions, if the fluid is to enter without impact.

Let $O B$, Fig. 48, represent as regards value and direction the absolute velocity of flow $c$ of the water leaving the guide passages, and $O A$ the speed of rotation $w_1$ of the wheel at the point of outflow $O$. Join $A B$; then the

latter represents by its length and direction the relative velocity of flow $c_1$, and the tangent $O\,D$ to the curve of the wheel-vane at the point $O$ should be parallel with $A\,B$, while $O\,B$ gives the inclination of the guide-vanes where the water leaves them. $B\,O\,X$ is the angle $a$ formed by the guide-vanes at the point $O$, with a line $O\,X$ at right angles to the direction of rotation $O\,A$, and in the plane defined by $O\,A$ and the tangent $O\,D$ to the wheel-vane at the point $O$. In radial turbines $O\,X$ is radial, in axial turbines parallel with the axis. $D\,O\,X$ is the angle $a_1$ formed by the wheel-vanes at the point $O$ with $O\,X$.

Expressed algebraically the conditions for entrance of the water without impact are the following :—

$$c_1 \cos a_1 = c \cos a. \qquad (A)$$
$$w_1 - c_1 \sin a_1 = c \sin a. \qquad (B)$$
$$c^2 = c_1^2 + w_1^2 - 2\,c_1\,w_1 \sin a_1. \quad (C)$$
$$\frac{w_1}{c} = \frac{\sin\,(a+a_1)}{\cos a_1} \qquad (C_1)$$
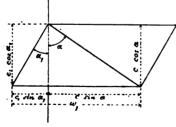
Their derivation will be clear from an inspection of the diagram, Fig. 49.
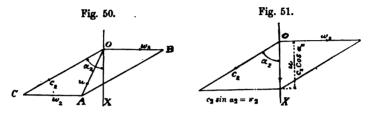
Fig. 49.



With respect to the loss from unutilized energy, it has been shown that the absolute residual velocity $u$ with which the water leaves the wheel-buckets is the resultant of the relative velocity $c_2$ with which the water issues from the latter and the velocity of rotation $w_2$ at the corresponding point (Fig. 50). For any given direction of $c_2$ a little consideration will show that $u$ is a minimum when the water leaves the wheel

parallel with $O\,X$, that is, radially in a radial turbine and axially in an axial turbine, *vide* Fig. 51, so that $c_2$ is the hypothenuse of a right-angled triangle of which $u$ and $w_2$

<table>
<tr><td>Fig. 50.</td><td>Fig. 51.</td></tr>
</table>



are the sides. If $a_2 = 90°$, then $u = 0$, but this is for practical reasons impossible, as in that case the water could not get away from the wheel. In fact $a_2$ ranges from 80° to 65°.

Algebraically the condition for radial or axial off-flow is expressed by

$$w_2 = c_2 \sin a_2. \qquad (D)$$

The two fundamental assumptions, therefore, from which the best speed is in practice determined, are: (1) Entrance of water without impact, and (2) Radial or axial exit from wheel.

These assumptions tend to simplify very considerably all the various expressions previously developed for the quantities relating to reaction turbines.

## Work.

As the water leaves the wheel at right angles to the direction of rotation, it has no component of velocity in that direction, and the momentum with which the fluid enters the wheel is reduced at the time of leaving to *nil*, and therefore the force exerted on the vanes is represented

solely by this momentum $m\ c\ sin\ a$, and the work done per unit of weight is

$$W_u = \frac{1}{g}\, w_1\, c\, sin\ a.$$

Now $w_1 = \frac{r_1}{r_2}\, w_2 = \frac{r_1}{r_2}\, c_2\, sin\ a_2$, since $w_2 = c_2\, sin\ a_2$

$c_2 = c\, \dfrac{A}{A_2}$ by the conditions of continuity;

hence

$$W_u = \frac{1}{g}\, \frac{r_1}{r_2}\, \frac{A}{A_2}\, c^2\, sin\ a\, sin\ a_2 \tag{ii}$$

or $W_1 = \dfrac{G}{g}\, \dfrac{r_1}{r_2}\, \dfrac{A}{A_2}\, c^2\, sin\ a\, sin\ a_2$ for the total work. (iiA)

## *Pressure Difference.*

The pressure of the water leaving the guide passages expressed in " head " is $p_1$, and by the time it issues from the wheel-buckets this is changed to $p_2$; the difference between these two pressures is determined (1) by the change in the relative velocity of flow from $c_1$ at the inflow to $c_2$ at the outflow from the wheel; (2) by losses in the wheel, including leakage through the clearance space; and (3) by the centrifugal action resulting from the change in the velocity of rotation as the water flows outwards or inwards in a radial turbine; hence

$$p_1 - p_2 = \frac{1}{2g}\, (c_2{}^2 - c_1{}^2 + \zeta_2\, c_2{}^2 + w_1{}^2 - w_2{}^2) \tag{iii}$$

where $\zeta_2\, \dfrac{c_2{}^2}{2g}$ is the loss in the wheel as previously defined.

In an axial (or parallel flow) turbine the term $w_1{}^2 - w_2{}^2$ representing the centrifugal action disappears, since $w_1 = w_2$, but as the inflow is higher than the outflow, the difference

of level ( = height of the buckets) $h_o$ appears in formula (iii), which becomes

$$p_1 - p_2 = \frac{1}{2g}(c_2^2 - c_1^2 + \zeta_2 c_2^2) - h_o \qquad \text{(iiiA)}$$

It will be seen from (iii) that for an inward-flow turbine the pressure difference is increased by the change in the velocity of rotation (as $w_1 > w_2$), while in an outward-flow turbine the reverse is the case.

The available energy of the water on leaving the guide passages is that due to the difference of pressures $p_1 - p_2$, plus that corresponding to the velocity of flow $c$, or $\frac{c^2}{2g} + p_1 - p_2$; hence available energy for wheel, by substitution,

$$E_w = \frac{1}{2g}(c^2 + c_2^2 - c_1^2 + \zeta_2 c_2^2 + w_1^2 - w_2^2).$$

This must equal $W_u$ the work done plus the unutilized energy due to the velocity $u$ and the loss in the wheel,

$$E_w = W_u + \frac{u^2}{2g} + \zeta_2 \frac{c_2^2}{2g}.$$

From this relationship the velocity of flow can be calculated, but it is more convenient for practical purposes to effect the calculation in a different way.

As already shown (p. 54, formula (6)), when $c$ is known the pressure-difference can be determined by the formula

$$p_1 - p_2 = h - (1 + \zeta')\frac{c^2}{2g},$$

and this is generally the more convenient method. As a rule the determination of the pressure-difference is not required in practice.

## Velocity of Flow.

The velocity of flow $c$ is generally determined from the equation

$$h = W_u + L_u + \frac{u^2}{2g}.$$

By (ii) $W_u = \dfrac{1}{g} \dfrac{r_1}{r_2} \dfrac{A}{A_2} c^2 \sin a \sin a_2.$

For practical purposes the loss $L_u$ may be divided into two parts : one made up of the losses in the wheel itself, and expressed by $\zeta_2 \dfrac{c_2^2}{2g}$; the other of the loss in the guide passages and other parts of the motor, expressed by $\zeta_1 \dfrac{c^2}{2g}$.

Where there is a long supply-pipe, suction-tube, and sluice, the losses caused by these must also be included in $L_u$.

The velocity of off-flow $u$, as can be easily seen by reference to Fig. 51, is

$$u = c_2 \cos a_2.$$

By substituting these values in the general equation, there follows :—

$$h = \frac{1}{g} \left[ \left( \frac{r_1}{r_2} \frac{A}{A_2} \sin a \sin a_2 \right) c^2 + \zeta_1 \frac{c^2}{2} + \zeta_2 \frac{c_2^2}{2} + \frac{c_2^2 \cos^2 a_2}{2} \right]$$

and since $c_2 = \dfrac{A}{A_2} c,$

$$h = \frac{c^2}{g} \left[ \frac{r_1}{r_2} \frac{A}{A_2} \sin a \sin a_2 + \frac{1}{2} \zeta_1 + \frac{1}{2} \zeta_2 \left( \frac{A}{A_2} \right)^2 + \frac{1}{2} \left( \frac{A}{A_2} \right)^2 \cos^2 a_2 \right];$$

from this by suitable transposition

$$c = \sqrt{ \frac{2gh}{2 \frac{r_1}{r_2} \frac{A}{A_2} \sin a \sin a_2 + \zeta_1 + \left( \frac{A}{A_2} \right)^2 (\zeta_2 + \cos^2 a_2)} } \qquad (iv)$$

$$\text{Let}\sqrt{\cfrac{1}{2\cfrac{r_1}{r_2}\cfrac{A}{A_2}\sin a \sin a_2 + \zeta_1 + \left(\cfrac{A}{A_2}\right)^2(\zeta_2 + \cos^2 a_2)}} = K_1,$$

then
$$c = K_1\sqrt{2gh}.$$

For ordinary practice the value of $K_1$ is confined within comparatively narrow limits, but by altering the proportions of certain parts of a turbine, it may be made to vary through a wide range.

The values of $K_1$ for a variety of cases will be found in Table I. In this table for a given value of $a_2$ the co-efficient $K_1$ has been determined, which corresponds to different values of $a$, $\dfrac{A}{A_2}$, and $\dfrac{r_1}{r_2}$; the co-efficients $\zeta_1$ and $\zeta_2$ have been taken as constant; although in all probability $\zeta_2$ varies with the dimensions of the turbine, there are not sufficient data to accurately determine this point, and for practical purposes a constant value may be used.

From the table the effect of various proportions between the parts of the turbine on the velocity $c$ can be studied. It will be seen that for given ratios between $r_1$ and $r_2$ and $A$ and $A_2$ the co-efficient $K_1$ *increases* as $a$, the angle at which the water leaves the guide passages, *decreases;* for example, for $a_2 = 75°$, $\dfrac{A}{A_2} = 2$ and $\dfrac{r_1}{r_2} = \frac{5}{4}$, with $a = 75°$, $K_1 = 0\cdot424$, while with $a = 60°$, $K_1 = 0\cdot444$; similarly for $\dfrac{A}{A_2} = 0\cdot5$ and the same ratio $\dfrac{r_1}{r_2}$, with $a = 75°$, $K_1 = 0\cdot864$, and with $a = 60°$, $K_1 = 0\cdot906$.

The effect of varying $a$ alone is evidently not very great within practical limits.

With respect to the influence of $\frac{r_1}{r_2}$, reference to the table shows that with decreasing values of this ratio $K_1$ increases; with $a = 75°$ ($a_2$ being also $75°$) and $\frac{A}{A_2} = 1$; for $\frac{r_1}{r_2} = \frac{3}{2}, \frac{5}{4}$, and 1, $K_1 = 0\cdot566, 0\cdot614$, and $0\cdot685$ respectively.

The most important factor as regards its effect on the velocity of flow is the ratio $\frac{A}{A_2}$; for $a_2 = 75°, a = 75°$, and $\frac{r_1}{r_2} = \frac{5}{4}$; $K_1$ varies from $0\cdot424$ with $\frac{A}{A_2} = 2$ to $0\cdot864$ with $\frac{A}{A_2} = 0\cdot5$.

The smaller the velocity of flow from the guide passages, the greater is the proportion of available energy existing in the form of pressure in the fluid at the moment it enters the wheel-buckets. Some writers call this proportion the *degree of reaction* with which the turbine works, so that the less the velocity of flow with a given head the greater the degree of reaction. It will be seen from the figures above given that this degree of reaction depends chiefly on the ratio of $\frac{A}{A_2}$, but also to a less extent on the other dimensions of the turbine.

## *Another Method of calculating the Value of $K_1$.*

For ordinary practical purposes the best method of determining $K_1$ is based upon the following considerations:—

The useful work done can be expressed as $W_u = \epsilon\, E_u$,

and substituting for $W_u$ and $E_u$ their values from formula (ii) p. 104, and p. 50, respectively there results

$$\frac{c^2}{g} \frac{A}{A_2} \frac{r_1}{r_2} \sin a \sin a_2 = \epsilon\, h$$

or
$$c^2 = \frac{\epsilon\, 2gh}{2 \dfrac{A}{A_2} \dfrac{r_1}{r_2} \sin a \sin a_2}$$

whence
$$c = \sqrt{\frac{\epsilon\, 2gh}{2 \dfrac{A}{A_2} \dfrac{r_1}{r_2} \sin a \sin a_2}} = K_1 \sqrt{2gh}.$$

Now for the efficiency $\epsilon$ in the above expression some value may be assumed which experience has proved to be probable, with a sufficient margin to allow for slight variations, so that the velocity of flow may not be too high, and consequently the area for the guide passages insufficient.

For the purpose in question $\epsilon$ may be taken as $0.81$ in ordinary cases, and then

$$K_1 = 0.9 \sqrt{\frac{1}{2 \dfrac{A}{A_2} \dfrac{r_1}{r_2} \sin a \sin a_2}}.$$

It is almost superfluous to remark that where special circumstances exist tending to increase the losses, such, for instance, as a long supply-pipe or channel, these must be taken into account, and a correspondingly lower efficiency assumed; it is better, in such cases, to use the first method of calculating $K_1$ and add the necessary co-efficients, for the particular kind of additional resistance occurring, in the denominator.

A third expression for $K_1$ very usually adopted for Jonval turbines is—

$$K_1 = 0.9 \sqrt{\frac{\cos a_1}{2 \sin (a + a_1) \sin a}}.$$

This is in fact identical with the preceding formula, and is arrived at in the following way :—

As before, $\qquad W_u = \epsilon\, E_u$

and from the formula on p. 104,

$$W_u = \frac{1}{g}\, c\, w_1 \sin a;$$

now by $C_1$ (p. 102) $\qquad \dfrac{w_1}{c} = \dfrac{\sin (a + a_1)}{\cos a_1};$

whence $\qquad w_1 = c\, \dfrac{\sin (a + a_1)}{\cos a_1},$

and by substitution

$$W_u = \frac{c^2}{g}\, \frac{\sin (a + a_1)\, \sin a}{\cos a_1},$$

$$\frac{c^2}{g}\, \frac{\sin (a + a_1)\, \sin a}{\cos a_1} = \epsilon\, h;$$

and therefore

$$c^2 = \frac{\cos a_1}{\sin (a + a_1)\, \sin a}\, \epsilon\, gh,$$

or $\qquad c = \sqrt{\dfrac{\epsilon\, 2gh\, \cos a_1}{2 \sin (a + a_1)\, \sin a}},$

and $\qquad K_1 = \sqrt{\dfrac{\epsilon\, \cos a_1}{2 \sin (a + a_1)\, \sin a}}.$

For the ordinary form of Jonval turbine, with stereotyped values of $a$, $a_1$ and $a_2$, the above formula is very convenient; but it is, in the author's opinion, unsuited for general use. In the first place, the angle of outflow $a_1$ depends on all the other leading proportions of the

turbine, $\dfrac{A}{A_2}$, $a_2$ and $\dfrac{r_1}{r_2}$, as well as $a$, and cannot therefore

be assumed without involving these.   The ratio $\dfrac{A}{A_2}$ is the

proportion which has more influence, for a given class of motor, than any other on the performance of a reaction turbine, and it ought therefore to form the starting-point in designing one.   In the second place, a slight alteration in $a_1$ very materially affects the value of $K_1$ when calculated by the above formula, while in reality,

*provided* $\dfrac{A}{A_2}$ *is not altered,* the influence of such a modifi-

cation on the velocity of flow is unimportant.   Of course the theoretical assumption is that if $a_1$ is varied, all the other dimensions are changed to correspond, so that the conditions of the inflow without shock and vertical or radial outflow are always fulfilled; in practice, however, it often occurs that a turbine has not been designed strictly in accordance with these principles, and then, if the formula in question is employed, the calculated result will be further from the truth than if the expression previously given be used, in which $a_1$ does not appear,

but in which the correct values of $\dfrac{A}{A_2}$ and $a_2$ are sub-

stituted.   In such cases a deviation of the value of $a_1$ from that corresponding to inflow without shock implies, when used to determine $K_1$, a simultaneous modification of the other dimensions which does not really take place.

For these reasons, in comparing experimental with theoretical results, for $K_1$ the formula

$$K_1 = \sqrt{\dfrac{\epsilon}{2\,\dfrac{A}{A_2}\dfrac{r_1}{r_2}\,sin\ a\ sin\ a_2}}$$

is always to be preferred to any other, since it gives results which approximately agree with experiment, even when the assumed conditions on which it is based are not accurately fulfilled.

## *Velocity of Rotation (Speed).*

For vertical or radial discharge from the wheel (*vide* (*D*) ) the velocity of rotation at the outflow must be

$$w_2 = c_2\ sin\ a_2$$

and the velocity at the inflow

$$w_1 = \frac{r_1}{r_2}\,w_2 = \frac{r_1}{r_2}\,c_2\ sin\ a_2$$

or since $\qquad c_2 = \dfrac{A}{A_2}\,c$

$$w_1 = c\,\frac{A}{A_2}\frac{r_1}{r_2}\,sin\ a_2 \qquad\qquad (v);$$

but $\qquad\qquad c = K_1\sqrt{2gh}$

Therefore $\qquad w_1 = \left(\dfrac{A}{A_2}\dfrac{r_1}{r_2}\,sin\ a_2\right) K_1\sqrt{2gh}$

If the quantity $\left(\dfrac{A}{A_2}\dfrac{r_1}{r_2}\,sin\ a_2\right) K_1$ be denoted by $K_2$, then the velocity of rotation can be written

$$w_1 = K_2\sqrt{2gh} \qquad\qquad (\text{vA})$$

$K_2$ may be termed the *co-efficient of speed;* like $K_1$, for a given type of turbine of normal proportions, its value lies within narrow limits.

## *Efficiency.*

This, as has been previously shown, may be generally expressed by the ratio of the useful work done to the useful work *plus* losses from all causes, or

$$\epsilon = \frac{W}{W + L_1 + L_2}, \ \&c.$$

Under the special assumed conditions as regards the inflow and outflow, this takes the form—

$$\epsilon = \frac{2\, c^2\, \dfrac{r_1}{r_2}\dfrac{A}{A_2}\, sin\ a\ sin\ a_2}{c^2 \left\{ 2\, \dfrac{r_1}{r_2}\dfrac{A}{A_2}\, sin\ a\ sin\ a_2 + \zeta_1 + \left(\dfrac{A}{A_2}\right)^2 (\zeta_2 + cos\ ^2 a_2) \right\}}$$

from which $c^2$, being a factor both in the numerator and denominator, disappears, and hence

$$\epsilon = \frac{2\, \dfrac{r_1}{r_2}\dfrac{A}{A_2}\, sin\ a\ sin\ a_2}{2\, \dfrac{r_1}{r_2}\dfrac{A}{A_2}\, sin\ a\ sin\ a_2 + \zeta_1 + \left(\dfrac{A}{A_2}\right)^2 (\zeta_2 + cos\ ^2 a_2)} \quad \text{(vi)}$$

This expresses the fact that the efficiency of reaction turbines, designed in such a manner that the water enters without impact and leaves at right angles to the direction of rotation, is independent of the velocity of flow, and therefore the efficiency may be as great or greater in a turbine with a low velocity of flow from the guide-wheels as in one where this velocity is high. When $c$ is small, in comparison with $\sqrt{2\,gh}$, the subsequent relative acceleration due to decrease of pressure is greater than when $c$ has a relatively high value, this acceleration being due directly to the water having to pass through passages of diminishing area.

I

Where a suction-tube is used, the last quantity in the denominator of $\epsilon$, representing the absolute velocity $u$ with which the water leaves the wheel, is replaced by $c_4{}^2 = \left(\dfrac{A}{A_4}\right)^2 c^2$, so that including resistance of sluice

$$\epsilon = \cfrac{2\,\dfrac{r_1}{r_2}\dfrac{A}{A_2}\,\sin a \,\sin a_2}{2\,\dfrac{r_1}{r_2}\dfrac{A}{A_2}\,\sin a \,\sin a_2 + \zeta_1 + \zeta_2\left(\dfrac{A}{A_2}\right)^2 + \zeta_6\left(\dfrac{A}{A_4}\right)^2} \qquad \text{(viA)}$$

where $\zeta_6 = 1 + \zeta_4$, vide p. 87.

These somewhat elaborate formulas for the efficiency are given for the reason that they show how it is affected by the leading proportions.

The simplest expression for $\epsilon$ is

$$\epsilon = \frac{W^u}{h} = \frac{w_1\,c\,\sin a}{gh} = \frac{w_1\,V_1}{gh}; \qquad \text{(viB)}$$

but to make use of this, $c$ and $w_1$ must first have been determined.

If $e_1$ and $e_2$ denote respectively the clear width of the buckets at inflow and outflow, then approximately

$$\frac{A}{A_2} = \frac{r_1\,e_1\,\cos a}{r_2\,e_2\,\cos a_2}$$

By substituting this value in equation (vi) for the efficiency, an expression for the latter is obtained from which the influence of the proportion between the radii $r_1$ and $r_2$ can be studied.

$$\epsilon = \cfrac{2\left(\dfrac{r_1}{r_2}\right)^2\left(\dfrac{e_1}{e_2}\right)\sin a \,\cos a \,tang\,a_2}{2\left(\dfrac{r_1}{r_2}\right)^2\dfrac{e_1}{e_2}\sin a \,\cos a \,tang\,a_2 + \zeta_1 + (\zeta_2 + \cos {}^2a_2)}$$

$$\left(\frac{r_1}{r_2}\right)^2\left(\frac{e_1}{e_2}\right)^2\left(\frac{\cos a}{\cos a}\right)^2$$

In a similar way the velocity of flow may be written

$$c = \sqrt{\frac{2gh}{2\left(\frac{r_1}{r_2}\right)^2 \frac{c_1}{c_2} \sin a \cos a \ tang \ a_2 + \zeta_1 + \left(\frac{r_1}{r_2}\right)^2 \left(\frac{c_1}{c_2}\right)^2}}$$
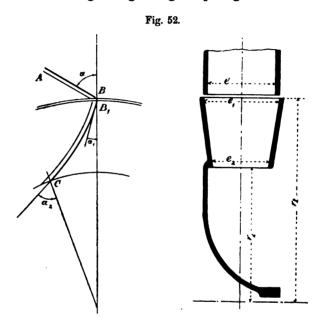
$$\left(\frac{\cos a}{\cos a_2}\right)^2 (\zeta_2 + \cos {}^2 a_2)$$

## Practical Application of Theory.

In order to apply the theory as hitherto developed to the determination of the leading dimensions, it will first be necessary to examine in detail the connection between the various quantities introduced into the calculations. So far, these have consisted of the three angles $a$, $a_1$ and $a_2$; the sectional areas of the stream at various points, $A$, $A_1$ and $A_2$; the radii at the points of entrance and exit, respectively $r_1$ and $r_2$.

Let $A$ $B$, Fig. 52, represent the course followed by a thin stream during its passage along the last portion of a guide-vane, and $B_1$ $C$ the course of the same in a wheel-bucket; the width of the stream—*vide* Fig. 52—at $B$, $B_1$ and $C$ is $e$, $e_1$ and $e_2$ respectively, $e$ being the width of the guide passages at outflow, and $e_1$ and $e_2$ the widths of the wheel-buckets where the water enters and leaves respectively. If $x$ be the depth of the stream at the point $B$, then —Fig. 53—it is easy to see that $x = \overline{Bb} \cos a$, provided $\overline{Bb}$ the portion of the circumference intercepted between $A$ $B$ and $a$ $b$ be so small that it can be considered as a part of a straight line. The sectional area of the thin stream at $B$ is evidently $e$ $x$; and as $e$ is constant for the whole

circumference, if $B D$ be that portion of the latter con-
tained between two vanes, then the *total* sectional area of
the water flowing through one guide passage is $\overline{B D} e \cos a$;

Fig. 52.
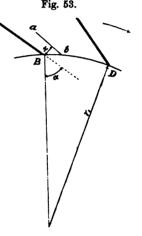


for the whole circumference the sectional area $A_1$ is the
preceding value multiplied by the number of guide
passages. If the vanes were extremely thin, the value of
$A_1$ would be (whole circumference) $\times e \times \cos a$; as
however each vane has a certain thickness, $t$, the aggregate
thickness of all the vanes must be deducted from the
above, whence *the measured area*

$$A^1 = 2 r_1 \Pi e \cos a - z t e = z \overline{B D}_{\wedge} \cos a,$$

where $z$ is the number of guide-vanes. When $(r_1)$ the radius
of the wheel where the water enters differs considerably

from the radius at which the water leaves the guides, the latter value must be used in the above formula; this occurs when the sluice-gate passes between the guide apparatus and wheel, so that a large clearance is necessary. For purposes of approximate calculation, $e_1$ the width of the wheel-buckets where the water enters may be taken as equal to $e$ the width of the guide orifices, although in practice $e_1$ is slightly greater than $e$ to allow for play and inaccuracy in setting the wheel. $A_1^1$ and $A_2^1$, the *measured* inflow and outflow areas of the buckets, are determined in a manner similar to that applied to $A_1$, hence

Fig. 53.



$$A^1 = 2 r_1 \Pi e \cos a - z t e \qquad (b)$$
$$A_1^1 = 2 r_1 \Pi e_1 \cos a_1 - z_1 t_1 e_1 \qquad (c)$$
$$A_2^1 = 2 r_2 \Pi e_2 \cos a_2 - z_1 t_2 e_2 \qquad (d)$$

where $t_1$ and $t_2$ are the thicknesses of the wheel-vanes at the points of entrance and outflow respectively, and $z_1$ the number of vanes in the wheel.

If the depth of the guide passages measured on the circumference—$B\,D$, Fig. 53—be denoted by $s$, and the corresponding dimensions for the wheel passages at inflow and outflow by $s_1$ and $s_2$, then the sectional areas may also be written—

$$A^1 = z\,s\,\cos a; \quad A_1^1 = z_1\,s_1\,\cos a_1; \quad A_2^1 = z_1\,s_2\,\cos a_2,$$

and hence $\dfrac{A^1}{A_1^1} = \dfrac{e\,z\,s\,\cos a}{e_1\,z_1\,s_1\,\cos a_1}; \quad \dfrac{A^1}{A_2^1} = \dfrac{e\,z\,s\,\cos a}{e_2\,z_1\,s_2\,\cos a_2}; \quad$ and

for very thin vanes, or when $z\, s = z_1\, s_1$,

$$\frac{A}{A_1} = \frac{\cos a}{\cos a_1};$$          (vii)

also approximately          $\dfrac{z\, s}{z_1\, s_2} = \dfrac{r_1}{r_2};$

whence          $\dfrac{A}{A_2} = \dfrac{r_1}{r_2}\dfrac{c_1 \cos a}{c_2 \cos a_2}.$          (viii)

These values are sufficiently close for first approximations, and more convenient to use than the accurate formulas.

For axial turbines it is merely necessary to substitute everywhere for $r_1$ and $r_2$ the *mean* radius $r$, in which case

$$\frac{A}{A_2} = \frac{c_1 \cos a}{c_2 \cos a_2}.$$

In practice the simplest plan is to measure off the clear depths (perpendicular to the direction of flow) $s \cos a$, $s_1 \cos a_1$, and $s_2 \cos a_2$ from a drawing.

The quantity of water $Q$ passing through the wheel per second is generally given, and when the velocity of flow $c$ has been found, the area $A$ is determined by it, since $Q = A\, c$, whence $A = \dfrac{Q}{c}.$

It is very evident from the method in which the relative velocity $c_1$ and direction of the water entering the wheel are determined—so that no shock shall occur—that when $w_1$, $c$ and $a$ are fixed, $a_1$ the angle at which the water enters relative to the wheel must also be fixed. To calculate $w_1$ and $c$, the angles $a$ and $a_2$ must be known or assumed, and hence, if two of the angles are given, the third necessarily follows. It is easiest to find $a_1$ graphically (*vide* Fig 61). The algebraical expression is found from the condition that the speed of the wheel

stands in a certain definite relation both to the angle at
which the water enters and that at which it leaves :—

$$w_1 = c \sin a + c_1 \sin a_1,$$

and also
$$w_1 = \frac{r_1}{r_2} w_2 = \frac{r_1}{r_2} c_2 \sin a_2.$$

By substituting the values above found, so as to give $c_1$
and $c_2$ in terms of $c$, $A$, $A_1$ and $A_2$, it is easy to prove that

$$tang\ a_1 = \left(\frac{r_1}{r_2}\right)^2 \frac{c_1}{c_2} tang\ a_2 - tang\ a, \qquad (ix)$$

approximately, or accurately—

$$tang\ a_1 = \frac{A}{A_2} \frac{r_1}{r_2} \frac{\sin a_2}{\cos a} - tang\ a. \qquad (ixA)$$
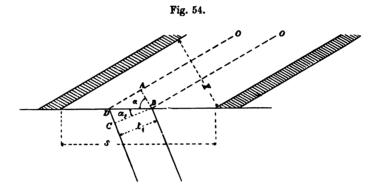
The values of $A_1$ and $A_2$[1] given in formulas (b) and (d)
are the sectional areas of the guide and wheel passages,
but not the actual effective sections of the stream itself.
These will in fact be less, for the following reasons :—The
edges of the wheel-vanes passing before the outlets of
the guide passages obstruct the latter and diminish the
area available for the flow of the water; there is a
momentary diminution of this obstruction during the
time that a guide-vane covers wholly or partially a
wheel-vane, but it is *safe* to assume that the interference
is equally great at all times, and in that case it is easy to
prove that the area of the guide passages is diminished

by $z_1 t_1 \frac{\cos a}{\cos a_1} e_1$. In the case of a Jonval turbine, where

the guide-vanes are parallel for a portion of their length
near the outflow orifices, there is no contraction of the
stream, and therefore the effective area is represented

by $A^1 - z_1 t_1 \frac{\cos a}{\cos a_1} c = A$. In practice this can be

most simply measured from a drawing; in Fig. 54, $A B$ represents the depth or thickness of the obstructed portion of the stream, and this multiplied by the width $e$ and the number of vanes $z_1$ gives the total area to be deducted from $A^1$.

Fig. 54.



In radial inward- and outward-flow turbines, in addition to the reduction of the constructive area from the cause just explained, there is a further reduction owing to the contraction of the stream from the convergent or divergent direction of the vanes, and the value $A^1$ must be further multiplied with a co-efficient of contraction; for the latter the value $0.9$ may be taken.

As regards the effective value $A_2$ of the outflow orifices of the wheel-buckets, there need be in a Jonval turbine or axial turbine no contraction, but owing to the leakage through the clearance space, less water passes through the wheel than comes out of the guide passages. Meissner, in his work on turbines, takes this loss into account in determining $A_2$, but there is so much uncertainty as to its amount that it appears to the author a waste of labour to do this. Hence, when there is no

contraction, $A_2 = A_2^1$, and has the value given in formula $(d)$. In a radial outward- or inward-flow turbine, however, contraction of the stream occurs in leaving the wheel-buckets, and, as in the previous instance, the co-efficient $C$ may be taken as 0·9.

The true value of the ratio $\dfrac{A}{A_2}$ resulting from the above considerations would therefore be :—

$$\frac{A}{A_2} = \frac{A^1 - z_1 t_1 \dfrac{\cos a}{\cos a_1} \, r}{A_2^1} .$$

Where there is contraction the same co-efficient occurs in both numerator and denominator, except in some mixed-flow wheels, so that it does not affect the proportion.

In determining $A_1$, $A_1^1$ or $A_2^1$ there is no necessity for introducing any trigonometrical quantities at all into the calculation; referring to Fig. 54, the distance $x$ ($= s \cos a$) between two guide-vanes can be measured directly from a drawing, and then the area of each passage is expressed by $x\,e$, and of the whole of the passages by

$$A_1 = z\, x\, e.$$

An exactly similar method may be followed for $A_1^1$ and $A_2^1$, the distance between the vanes, or *depth* of the passages, being respectively $x_1$ and $x_2$.

In the case of the guide orifices, if the thickness $A\,B$, Fig. 54, be denoted by $y$, then the effective area $A$, apart from contraction, is

$$A - z\, x\, e - z_1\, y\, e = e\, (z\, x - z_1\, y).$$
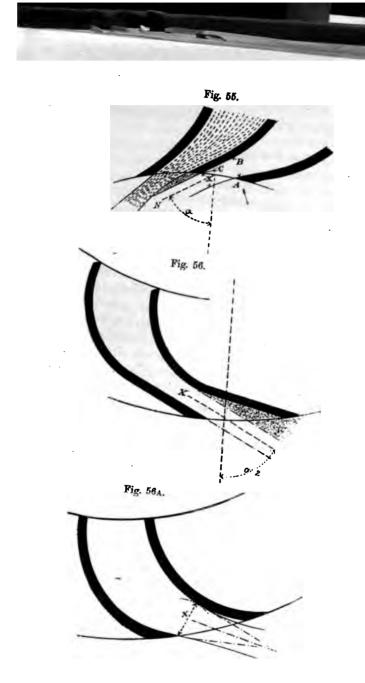
For radial turbines, the distance between two vanes $s$ measured *on the circumference* may be set out on a straight

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[1] That is, the effective area of the stream equals that of the wheel-bucket.

line, and the vane-angles measured relatively to a perpendicular to this line, so that both vane surfaces become parallel. The mode of proceeding then becomes just the same as for a parallel-flow turbine, except that contraction has to be allowed for.

$$\frac{v}{l}$$

## *Another Method of Computing $A$, $A_1$, $A_2$.*

The preceding method of determining the outflow areas from the guide passages and buckets is based upon the assumption that all the consecutive ideal filaments of water follow, relatively to the wheel, *similar* paths. In radial-flow turbines this condition is not necessarily always fulfilled. If the vanes were extremely thin, so that on leaving the orifices the streams from two adjacent passages or buckets were practically in contact, the assumption in question would probably be correct, but in practice the thickness of the vanes divides the water issuing from two neighbouring orifices into two distinct streams, which, for an appreciable distance after leaving the orifices, are free to follow their own course without mutual interference. Each stream will naturally take the path of least resistance, and escape by the largest area available, provided this does not involve a sudden change of direction. Such a path is not always that corresponding to the hypothesis of a similar course for each filament, as will be clear from a consideration of Fig. 55, which represents an inward-flow turbine in which the last portion of each vane at the outflow is straight. In this case, if a section along $A$ $B$ be considered approximately at right angles to the general direction of flow, it is clear that the portion of the stream which has passed this section can deviate freely in the direction indicated by the arrow, *away* from the convex vane surface, but is prevented by the latter from deviating

Fig. 55.

Fig. 56.

Fig. 56A.

in the opposite sense. It is therefore compelled on one side to follow the convex surface as far as $C$, where the stream may be said to finally leave the wheel. The result is, that the general mean direction of the stream as it issues from the wheel is parallel with the convex side of the bucket (or passage), and the outflow angle $a$ or $a_2$ is as indicated. After passing $A\ B$, a contraction of the stream, somewhat as shown in the illustration, occurs at $X$, where the effective area would have to be measured if this were possible.
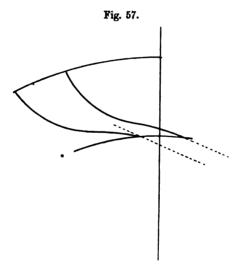
The measured outflow area is that corresponding to the shortest distance $A\ B$ between two adjacent vanes.

There might, at first sight, appear to be some question as to the effective outflow radius corresponding to the method of measurement above described, but on consideration there is little doubt that the *constructive* outflow radius—measured to the inner or outer circumference of the vanes, as the case may be—should be introduced into the calculations. At any rate, this method is sufficiently accurate for practical purposes. Of course the angle of flow and velocity of flow must be taken at the same mean radius. The area of outflow thus obtained is *greater* than that resulting from the assumption of similar paths as previously given. The *mean* angle of outflow for each stream is under these circumstances *not* the vane angle, but the angle formed by the centre line $X\,N$ of the stream with a radial line drawn to the point of intersection with the outflow circumference; this angle is smaller than the vane angle. Where the outflow end of each vane is *not* straight, a similar state of things may occur, each stream issuing in a direction parallel to the tangent to the vane-curve on the *convex* side, or when the vane-curve is a volute, with inflection, parallel to the tangent at the extremity of the concave side (Fig. 57).

For outward-flow turbines in which the vanes are con-

structed as shown in Fig. 56, it is clear that the effective outflow area is that corresponding to $X$, while the effective angle of outflow is as shown, the mean direction of flow being parallel with the concave sides of the vanes. In such cases no allowance is necessary for contraction if the various parts of the stream become parallel by the time they reach $X$.

When on the other hand the vanes—instead of being straight at the ends—are curved up to the outer circumference, as in Fig. 56A, the outflow takes place probably somewhat in the manner illustrated, and it is necessary to make allowance for contraction.

Fig. 57.



The dimensions of the radii of a turbine are purely arbitrary, fixed solely with reference to practical convenience, except, of course, in so far that the wheel must be sufficiently large to contain passages of the area necessary for the flow of the water at the required velocity.

The widths of the buckets ($e_1$ and $e_2$) have no fixed relation to one another, but must be determined with reference to the areas $A$, $A_1$ and $A_2$, as is evident from ($b$), ($c$) and ($d$).

The depth of the buckets in radial turbines is obviously $r_2 - r_1$ or $r_1 - r_2$, according to whether $r_2$ is greater than $r_1$, or *vice versâ*; it must be sufficient to allow of a gradual curvature of the vanes, but is in itself of little importance; with greater length there must be, it is true, increased friction in the buckets, but under all ordinary conditions this is so trifling that it may be left out of account.

The numbers of buckets and vanes $z$ and $z_1$, and their thickness, are empirical, practical experience being the sole guide. The same remark applies to the depth of buckets in axial turbines, which is not limited by the radii as in radial wheels.

The following is a summary of the formulas applying to reaction turbines as used in determining their construction :—

## Assumed Conditions.

(1) Inflow without shock :
$$c_1 \cos a_1 = c \cos a \qquad (A)$$
$$w_1 = c \sin a + c_1 \sin a_1 \qquad (B)$$
$$c^2 = c_1^2 + w_1^2 - 2 c_1 w_1 \sin a_1 \qquad (C)$$
$$\frac{w_1}{c} = \frac{\sin (a + a_1)}{\cos a_1} \qquad (C_1)$$

(2) Radial or axial outflow :
$$w_2 = c_2 \sin a_2 \qquad (D)$$
$$u = c_2 \cos a_2 \qquad (E)$$

(3) Continuity of flow :
$$Q = A\,c = A_1\,c_1 = A_2\,c_2 = A_3\,c_3, \text{ \&c.} \qquad (F)$$
$$G = Q\,\lambda \;(\lambda = \text{weight of cubic foot of water}).$$

## Fundamental Relations between Dimensions.

$$w_1 = \frac{r_1}{r_2} w_2 \qquad (a)$$

$$A^1 = 2\, r_1\, \Pi\, e\, \cos a \; - z\, t\, e \; = e\, z\, s\, \cos a \; = z\, x\, e \quad (b)$$

$$A_1^1 = 2\, r_1\, \Pi\, e_1\, \cos a_1 \; - z_1\, t_1\, e_1 = e_1\, z_1\, s_1\, \cos a_1 = z_1\, x_1\, e_1 \quad (c)$$

$$A_2^1 = 2\, r_2\, \Pi\, e_2\, \cos a_2 \; - z_1\, t_2\, e_2 = e_2\, z_1\, s_2\, \cos a_2 = z_1\, x_2\, e_2 \quad (d)$$

$$A = C\left(A^1 - z_1\, t_1\, \frac{\cos a}{\cos a_1}\, e\right) vide \text{ p. 121.} \qquad (e)$$

$A_2 = C\, A_2^1$ where $C = 1$ for axial and $0{\cdot}9$ for radial inward-flow turbines $\qquad (f)$

$$E_1 = Gh = \left(W_u + L_u + \frac{u^2}{2g}\right) G \qquad (i)$$

$$W_u = \frac{1}{g}\, w_1\, c\, \sin a = \frac{1}{g}\, \frac{r_1}{r_2}\, \frac{A}{A_2}\, c^2\, \sin a\, \sin a_2 \qquad (ii)$$

$$W_1 = \frac{G}{g}\, \frac{r_1}{r_2}\, \frac{A}{A_2}\, c^2\, \sin a\, \sin a_2 \qquad (iiA)$$

$$p_1 - p_2 = \frac{1}{2g}\left(c_2{}^2 - c_1{}^2 + \zeta_2\, c_2{}^2 + w_1{}^2 - w_2{}^2\right) \qquad (iii)$$

$$p_1 - p_2 = \frac{1}{2g}\left(c_2{}^2 - c_1{}^2 + \zeta_2\, c_2{}^2\right) - h_o \text{ for axial turbines (iiiA)}$$

$$c = \sqrt{\dfrac{2gh}{2\dfrac{r_1}{r_2}\dfrac{A}{A_2}\sin a\, \sin a_2 + \zeta_1 + \left(\dfrac{A}{A_2}\right)^2 (\zeta_2 + \cos{}^2 a_2)}} \qquad (iv)$$

$$c = K_1\sqrt{2gh} \qquad (ivA)$$

$$c = \sqrt{\dfrac{\epsilon\, 2gh}{2\dfrac{A}{A_2}\dfrac{r_1}{r_2}\sin a\, \sin a_2}} \qquad (ivB)$$

$$c = \sqrt{\dfrac{\epsilon\, \cos a_1}{2\sin(a + a_1)\sin a}} \qquad (ivC)$$

where $\epsilon$ may be taken as $0{\cdot}81$

$$w_1 = c \, \frac{A}{A_2} \frac{r_1}{r_2} \, sin \, a_2 \tag{v}$$

$$w_1 = K_2 \sqrt{2gh} = \left( \frac{A}{A_2} \frac{r_1}{r_2} \, sin \, a_2 \right) K_1 \sqrt{2gh} \tag{vA}$$

$$\epsilon = \frac{2 \, \frac{r_1}{r_2} \frac{A}{A_2} \, sin \, a \, sin \, a_2}{2 \, \frac{r_1}{r_2} \frac{A}{A_2} \, sin \, a \, sin \, a_2 + \zeta_1 + \left( \frac{A}{A_2} \right)^2 (\zeta_2 + cos \, {}^2a_2)} \tag{vi}$$

$$\left\{ \epsilon = \frac{2 \, \frac{r_1}{r_2} \frac{A}{A_2} \, sin \, a \, sin \, a_2}{2 \, \frac{r_1}{r_2} \frac{A}{A_2} \, sin \, a \, sin \, a_2 + \zeta_1 + \zeta_2 \left( \frac{A}{A_2} \right)^2 + \zeta_6 \left( \frac{A}{A_2} \right)^2} \right.$$

with suction-tube $\tag{viA}$

$$\epsilon = \frac{W_u}{h} = \frac{w_1 \, c \, sin \, a}{gh} \tag{viB}$$

$$\frac{A}{A_1} = \frac{z \, s \, cos \, a}{z_1 \, s_1 \, cos \, a_1} = \text{approximately} \, \frac{cos \, a}{cos \, a_1} \tag{vii}$$

$$\frac{A}{A_2} = \frac{c \, z \, s \, cos \, a}{c_2 \, z_2 \, s_2 \, cos \, a_2} = \text{approximately} \, \frac{r_1 \, c_1 \, cos \, a}{r_2 \, c_2 \, cos \, a_2} \tag{viii}$$

$$tang \, a_1 = \frac{A}{A_2} \frac{r_1}{r_2} \frac{sin \, a_2}{cos \, a} - tang \, a \tag{ix}$$

## *Application of Formulas to Axial Turbines.*

To apply the preceding formulas to axial turbines, it is merely necessary to make $\frac{r_1}{r_2} = 1$, except where otherwise stated. *Strictly* speaking, the formulas under these conditions are correct only for a *very narrow* turbine, but they are usually employed with reference to the mean radius for wheels of the proportions actually occurring

in practice. In this sense $w$ $(= w_1 = w_2)$ is the velocity of rotation of the circumference taken at the middle of the width of the buckets.

It is evident that if the conditions of entrance without impact and axial exit, for given angles, are fulfilled for the mean radius, this cannot be the case for any other radius, because the velocity of rotation at any other distance from the centre must be different from that calculated, which alone complies with the assumptions made. At the outer radius the velocity will be *greater*, at the inner *less* than the calculated value. Referring to the diagram, Fig. 58, let $O\ a$ represent the value and direction of the absolute velocity

Fig. 58.

$c$, which is assumed to be the same at every point in the width of the buckets; $a\ b$ the mean velocity of rotation as determined by calculation; then for the mean
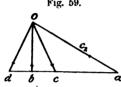


radius, $O\ b$ gives the proper inclination of the wheel-vanes at the inflow, which coincides with the relative direction there taken by the fluid.

At the inner radius the velocity of rotation is only $a\ c$, *less* than $a\ b$; the relative direction of the water entering the wheel is therefore $O\ c$, which it is clear is not the same as that of the vane. Similarly if $a\ d$ be the velocity at the outer radius, *greater* than $a\ b$, then $O\ d$ represents the relative direction taken by the entering fluid. Hence, for every other than the mean radius, the water must enter with *shock* under the assumed conditions. As the radius is either less or greater than the mean, so the impact is in the same sense as or opposed to the motion of the wheel.

What happens at the outflow is similar.

Let $O\ a$, Fig. 59, be the relative velocity $c_2$ and direction

K

with which the water issues from the buckets, it being
assumed that this is the same for
every radius; $a\,b$ the mean velo-
city of rotation, for which the
water leaves the wheel in the
absolute direction $O\,b$ parallel
with the axis.  The velocities of
rotation at the inner and outer
circumferences are respectively $a\,c$ and $a\,d$, and the
corresponding absolute velocities of outflow $O\,c$ and $O\,d$,
both of which it is evident deviate from the assumed
condition of vertical outflow.

Fig. 59.



The effect of these deviations from theoretical accuracy
is to reduce the efficiency by causing a loss in the first
place from impact or shock, and secondly by increasing
the absolute residual velocity, which is least when its
direction is axial.   To what extent this evil may be avoided
by a suitable construction of the vanes will be shown in a
subsequent chapter.

In ordinary practice the angles $a$ and $a_2$ are not
constant for the whole width of the wheel, as the vanes
have helical surfaces with constant pitch, so that those
angles increase with the distance from the axis (except for
vertical surfaces).   It is easy to modify the construction
shown in Fig. 59 to suit a helical vane surface.   Instead
of the constant angle $a\,O\,b$, the angles at the outer, mean,
and inner circumferences are all different; when one of
these (generally that for the mean circumference) is given,
the others are easily found by construction, and for each
the corresponding residual velocity can be ascertained in
the usual way.

## *General Data and Method for Design.*

In a large proportion of the cases which occur in actual practice, the head and the quantity of water available under average conditions are given, that is, $h$ and $Q$.

A turbine has to be constructed which will utilize to the best advantage the power available $E_a = (Weight\ of\ water\ per\ second) \times h$, in foot-pounds per second, or

$$\frac{(Weight\ of\ water) \times h \times 60}{33,000}$$

expressed in horse-power, and if $\lambda$ be the weight of a cubic foot of water, then

$$E_{hp} = \frac{Q \times \lambda \times h \times 60}{33,000} = \text{available Horse-power.}$$

Now certain dimensions or dimension-ratios can, within reasonable limits, be arbitrarily chosen, being independent of each other. Different writers have followed various methods in the choice of these quantities, and upon this choice depends the course of the subsequent calculations.

The author has for various reasons taken the following independently selected quantities, (1) the angle $a$ at which the water leaves the guide passages, (2) the angle $a_2$ at which the water leaves the wheel-buckets, and (3) the ratio $\dfrac{A}{A_2}$ of the outflow-area of the guide passages to the outflow-area of the wheel-buckets, determined as previously explained.

It is usual to fix arbitrarily the ratio between the inner and outer radius, but this may also be varied to suit convenience.

When $a$, $a_2$, $\dfrac{A}{A_2}$ and $\dfrac{r_1}{r_2}$ are fixed, it is clear from formula (viii) that the ratio $\dfrac{e_1}{e_2}$ of the width of the buckets at the

entrance to that at the outflow is also determined. The
difference between the number and thickness of the
vanes in the guide apparatus and wheel gives a slight
margin for variation, as may be seen from formulas (*b*)
and (*d*), but this may in practice be neglected, as the
numbers of vanes and their thickness are determined
purely by practical considerations—the thickness, more
especially, should be as small as possible.

If, as is generally the case in Jonval turbines, the
width of the buckets is the same throughout, so that
$\frac{e_1}{e_2} = 1$, then the ratio $\frac{A}{A_2}$ can no longer be indepen-
dently fixed, but is absolutely determined by $a$ and $a_2$,
since

$$\frac{A}{A_2} = \frac{cos\ a}{cos\ a_2} \text{ approximately,}$$

but there is no reason why the width should be constant
if it is desirable to vary it, and in many cases this has
been done.

The influence of the various dimensions on the effi-
ciency of reaction turbines will be subsequently studied,
but as regards the choice of $a_2$ it is easy to see (from
the triangle of velocities, Fig. 59, for instance) that,
from a theoretical point of view, the larger this is the
better, as the residual velocity of the water decreases
with an increase of $a_2$. The extreme value for $a_2$ in prac-
tice is about 80°, $a$ ranges from 50° to 80° for different
classes of turbines.

$\frac{A}{A_2}$ ranges from 1·8 to 0·5, but for Jonval turbines, to
which class belong the majority of European reaction
wheels, it is near 1 as a rule. For an inward-flow tur-
*bine (to* which it may, for the sake of greater generality,
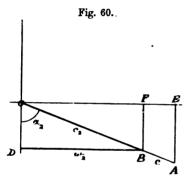
be assumed that the present investigation applies) a possible value for $\frac{A}{A_2}$ would be 1·3.

Having chosen $a$, $a_2$, and $\frac{A}{A_2}$, and taking for $\frac{r_1}{r_2}$ some value between $\frac{1}{4}$ to $\frac{3}{4}$ (for inward flow), the ratio of the widths $\frac{e_1}{e_2}$ can be calculated from (viii) $\frac{e_1}{e_2} = \frac{A}{A_2} \frac{r_2}{r_1} \frac{\cos a_2}{\cos a}$.

Suppose $\frac{A}{A_2} = 1\cdot3$, $a = 75°$, $a_2 = 75°$, and $\frac{r_1}{r_2} = \frac{2}{3}$,

then $\frac{e_1}{e_2} = 1\cdot3 \times \frac{2}{3} \times 1 = 0\cdot866,$

that is, the width of the buckets is less at the outer than at the inner circumference. This calculation is merely necessary at the present stage in order to be satisfied that $e_1$ and $e_2$ do not assume impracticable proportions.

After these preliminary operations, the first quantity to be ascertained is the velocity of flow $c$; it has been shown that $c = K_1 \sqrt{2gh}$; referring to the table (I.) it will be found that for $a_2 = 75°$, $a = 75°$, $\frac{r_1}{r_2} = \frac{3}{4}$ and $\frac{A}{A_2} = 1\cdot3$, the corresponding value of $K_1$ is about 0·48, hence $c = 0\cdot48 \sqrt{2gh}$. Had the dimensions been differently chosen, $K_1$ and consequently $c$ would have had a different value; by making, for instance, $\frac{A}{A_2} = 1$ instead of 1·3, $K_1$ would be 0·566, and therefore $c$ proportionately greater. To utilize the same head and quantity of water per second, it is evident that turbines may be constructed which vary very considerably in their details. It has already been demonstrated that

the efficiency is not directly dependent on the velocity of flow $c$, but as the various losses are affected by the velocities in different parts of the apparatus, and these again depend on the proportions of the latter, it is clear that to some extent the efficiency must be influenced by the same factors which determine $c$.



Fig. 60.

It is sufficient for the present to state that in fact the efficiency is only slightly affected by very considerable variations in the *degree of reaction* or velocity of flow from the guide passages, and consequently great latitude is admissible in the choice of dimensions without much practical modification of the results.
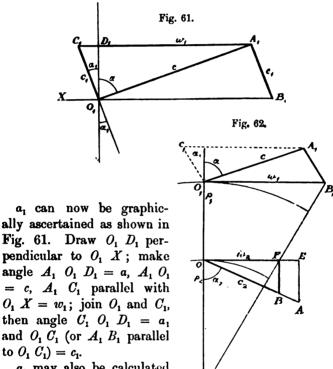
Having calculated $c$ the velocity of flow, the next step is to determine $w_1$ the best speed of rotation at the inflow.

By (v)        $w_1 = c \, \dfrac{A}{A_2} \, \dfrac{r_1}{r_2} \, \sin a_2 = K_2 \sqrt{2gh};$

but this value can very easily be found graphically in the following way: Set out $O\,D$, Fig. 60, perpendicular to $O\,E$, and draw $O\,A$ forming with $O\,D$ the angle $A\,O\,D$ equal to $a_2$; make $O\,A = c$ to any scale, $O\,F =$ the area $A$, and $O\,E = A_2$ (the unit in the latter cases being the square foot, to any scale, as only the ratio is concerned); join $A$ and $E$—$A\,E$ not being necessarily perpendicular to $O\,E$ as shown—and draw $F\,B$ parallel to $E\,A$ intersecting $O\,A$ in $B$; then $O\,B = c_2$ the relative velocity of outflow. From $B$ draw $B\,D$ parallel with $O\,E$; then

$B\,D = w_2$ the velocity of rotation at the outflow; $w_1 : w_2 = r_1 : r_2$, from which $w_1$ is easily determined by construction.



Fig. 61.



Fig. 62.

$a_1$ can now be graphically ascertained as shown in Fig. 61. Draw $O_1\,D_1$ perpendicular to $O_1\,X$; make angle $A_1\,O_1\,D_1 = a$, $A_1\,O_1 = c$, $A_1\,C_1$ parallel with $O_1\,X = w_1$; join $O_1$ and $C_1$, then angle $C_1\,O_1\,D_1 = a_1$ and $O_1\,C_1$ (or $A_1\,B_1$ parallel to $O_1\,C_1$) $= c_1$.

$a_1$ may also be calculated by formula (ix)

$$tang\ a_1 = \frac{r_1}{r_2}\ \frac{A}{A_2}\ \frac{sin\ a_2}{cos\ a} - tang\ a;$$

The two diagrams, Figs. 60 and 61, are shown combined in Fig. 62, which requires no further explanation except that $C\,O_1 = r_1$ and $C\,O = r_2$.

By ($F$) the quantity of water flowing through the turbine per second $Q = A\,c$, whence $A = \dfrac{Q}{c}$, and from this the effective area $A$ can be calculated.

From ($e$) and ($b$)

$$A = C \left( 2\,r_1\,\Pi\,cos\;a - z\,t - z_1\,t_1\,\frac{cos\;a}{cos\;a_1} \right) e\,;$$

of the quantities in this expression $r_1$, $z$, $t$, $z_1$ and $t_1$ may be empirically determined, while $cos\;a$ and $cos\;a_1$ and the rest are known.

For the former the following rules can be used :

$$r_1 = 1\cdot 25 \text{ to } 1\cdot 5\;\sqrt{A}\,;$$

$$z = \frac{\text{Circumference corresponding to } r_1}{\text{pitch of guide-vanes}}$$

Pitch of guide-vanes $= \dfrac{\text{Diameter of wheel}}{7\cdot 5 \text{ to } 9}$ (*vide* Table of rules and formulas) ;

$t = \frac{1}{2}$ to $\frac{5}{8}$ inch for cast-iron ; $\frac{1}{4}$ to $\frac{3}{8}$ for wrought-iron ;

$$t_1 = \quad ,, \qquad ,, \qquad ,, \qquad ,, \qquad ,, \qquad ,,$$

$$z_1 = z \text{ to } 1\cdot 2\,z\,;$$

$c$ the width of the guide passages can now be calculated by ($c$)

$$c = \frac{A}{C \left( 2\,r_1\,\Pi\,cos\;a - z\,t - z_1\,t_1\,\dfrac{cos\;a}{cos\;a_1} \right)}$$

$$= \frac{A}{C\,(z\,x - z_1\,y)}\,; \quad \text{where } C = 0\cdot 9,$$
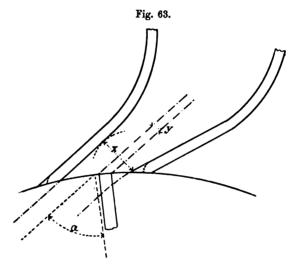
$$t_1\,\frac{cos\;a}{cos\;a_1} = y \text{ and } \frac{1}{z}\,(2\,r_1\,\Pi\,cos\;a - z\,t) = x,$$

*vide* Fig. 54. $x$ and $y$ can be measured from a drawing.

In practice the latter method must always be adopted when the outflow areas are computed as described at p. 122,

*vide* Fig. 63. By rounding off the corners of the vanes at the concave side, it is certain the contraction of the stream might be much reduced, if not altogether avoided. The author is, however, not aware that this is ever done.

Fig. 63.



$A$ being known, $A_2$ follows from the assumed ratio $\left(\dfrac{A}{A_2}\right)$, (which was taken at 1·3 for example :)

$$A_2 = \frac{A}{1\cdot3};$$

and by ($f$) and ($d$)

$$c_2 = \frac{A_2}{(2\,r_2\,\Pi\,\cos a_2 - z_1\,t_2)\,C}$$
$$= \frac{A}{z_1\,x_2\,C}; \qquad \text{(where } C = 0\cdot9\text{)}.$$

To the calculation of $e_2$ exactly the same remarks apply as to the calculation of $e$.

When $x$ or $x_2$ and $y$ are measured as shown in Fig. 63 the formulas to be used are obviously

$$e = \frac{A}{C(z\,x - z_1\,y)}$$

and
$$c_2 = \frac{A_2}{C\,z_1\,x_2}$$

As a matter of fact the co-efficient of contraction $C$ must vary according to the convergence of the vanes, but no data applicable to orifices of the form occurring in turbines are available.

$r_2$ follows from $r_1$, the ratio $\dfrac{r_1}{r_2}$ having been assumed at the commencement.

$t_2$ as a rule equals $t_1$, $z_1$ is known.

The correct value of $\dfrac{c_1}{c_2}$ can now be calculated and compared with the approximate value first determined.

As a check on the accuracy of the calculation, $c_2$ the relative velocity of flow from the buckets may be ascertained from the formula

$$c_2 = \frac{Q}{A_2}, \text{ and if } c_2 = w_1\,\frac{r_1}{r_2}\,\frac{1}{\sin a_2},$$

the construction is correct. This can also be done graphically; it simply amounts to ascertaining whether the absolute outflow from the wheel is radial or not.

All the elements necessary for the construction of the turbine itself have now been calculated. The next step is to determine the form of the vanes ; how this is done will be subsequently described.

Where a suction-tube is employed, it is usually made of about the same diameter as the wheel where the water leaves the latter, and the residual velocity of outflow $u$ is suddenly changed to that corresponding to the area of

the suction-tube; the latter is greater, as generally constructed, than that of the wheel, and consequently there is an abrupt retardation of flow and corresponding loss of energy which might be utilized if the transition were gradual, as then the loss from unutilized energy is only that due to the velocity with which the fluid leaves the tube. There is no reason why there should be a sudden enlargement of sectional area when the water leaves the wheel; by making the suction-tube annular at the top, and of a section exactly corresponding to that of the wheel, and gradually enlarging this section towards the lower end, this evil might be avoided. The orifice through which the water leaves the suction-tube should be as large as practicable, but not sufficiently so to reduce the velocity of the water below 3 feet per second, as this is necessary to carry it off. There is, of course, no absolute rule on this point, and what the lowest admissible velocity may be must depend on circumstances, among others the sectional area and fall of the tail-race, for instance.

Instead of the available quantity of water being fixed, it may occur that the supply of water for a certain fall is practically unlimited, and that a turbine has to be constructed which with that fall will develop a certain effective power. The data are then Effective Horse-power and Head $h$. If—as is generally the case—the effective power is measured at the point where it is given off direct from the wheel-shaft without intermediate gearing, then to the effective power need only be added an allowance for shaft friction to arrive at the work to be done by the turbine. From the work expressed in horse-power per minute, $W_1$, that to be done per second is easily determined. The simplest and safest plan is then to assume a certain efficiency well within the limits of probability, say 70 to 75 per cent., and substitute this value for $\epsilon$ in formula (viii)

$\epsilon = \dfrac{W_1}{Gh}$; from this the weight of water required per

second can be determined, $G = \dfrac{W_1}{\epsilon h}$, and then $Q = \dfrac{G}{\lambda}$ the

quantity in cubic feet per second. After this the method of calculation is the same as before, both $Q$ and $h$ being now given.

## CONSTRUCTION OF THE VANES OF TURBINES.

With given angles of entrance and exit at the ends of the vane, it has been shown that the intermediate form of the vane-curve is a matter of indifference, provided the curvature is not too abrupt, and that there are no sudden changes of sectional area in the passages and buckets.

*Guide-Vanes.* In an axial or parallel-flow turbine the water generally speaking enters the guide apparatus in a direction parallel with the axis of the wheel, and the tangent to the vane-curve at the point of entry should be parallel with this direction. In Fig. 64, *A B* and *C D* represent the curves of the vanes at their intersection with a cylindrical surface of the mean diameter in the middle of the width. Assuming the axis of the turbine to be vertical, then the first portions at *A* and *C* of the vane-curves should also be vertical. At the lower end, where the water leaves the guide passages, the vanes should form the angle *a* with a vertical line, or strictly speaking with a direction parallel to the axis, and the lower end of the curve should be continued as a straight line for a

certain length $B\,5$ and $D\,4$, so that the surfaces of the vanes run parallel with each other for some distance in order to guard against contraction of the stream.

In an inward-flow radial turbine this is not possible, as the vane-curves terminate at the points $B$ and $D$, &c., on a circle, and if they form equal angles with the radii at those points must of course converge ; in an outward-flow turbine they will diverge.

Supposing, as is often the case, that the width of the passages is uniform, and that $M\,N$ represents the mean course of the stream, then the depth of the latter, measured
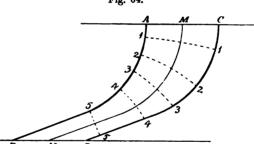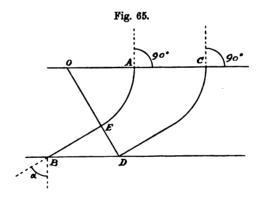
Fig. 64.



normally to the direction of flow at all points as shown by 1 1, 2 2, 3 3, &c., at any point is proportional to the sectional area. The curves $A\,B$ and $C\,D$ should be so chosen that the lengths 1 1, 2 2, 3 3, &c., *gradually and uniformly* diminish down to the shallowest part.

A simple method of construction, Fig. 65, is to describe a portion of a circle with some radius $O\,A$ from the centre $O$ on the continuation of the line $A\,C$, and draw a tangent to this circle at $E$, forming the angle $a$ with a direction at right angles to $A\,C$, parallel with the axis. It is usual to make the construction such that $E$ is the point of intersection of a line joining $O$ and $D$, and the method of pro-

ceeding is as follows:—Draw *B E* at the proper inclination
*a*; set out *B D* the pitch of the vanes on the mean cir-
cumference; from *D* draw *D O* at right angles to *B E*,
intersecting the latter in *E*, and the continuation of *A C*—
the level of the top of the guide-vanes—in *O ;* from *O* as
centre with the radius *O E* describe the arc of the circle
*A E*. Of course each vane-curve is exactly similar. It is
often preferable, although involving a little more trouble,
to draw the curves by hand, trying various forms until
that which gives the most gradual change in the area of
the stream is found.
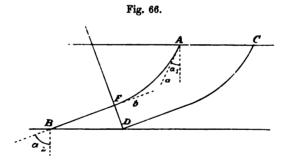
Fig. 65.



The usual construction of the surface of the vanes is
such that a series of radial lines drawn from any points of
the curve at the mean circumference—in the middle of
the passage—to the axis, touch the surface for its whole
width; in other words, the surfaces are *helical*. In this
case it is clear that the angles at the inner and outer cir-
cumferences are not the same as those at the mean cir-
cumference, and the greater the width of the passages the
greater will be the difference between the angles. There
is, however, no reason whatever why the angle should not

be the same throughout the whole width of each vane, the lower portion of the surface of the latter, instead of being helical, is then simply a plane surface, that is, the part of it which is parallel with the adjacent vane, of the length represented by $EB$ in Fig. 66.
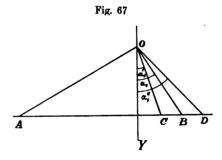
*Wheel-Vanes.* Similar remarks to those respecting the guide-vanes apply to the wheel-vanes. The angles $a_2$ and $a_1$ being determined, the lower end of the curve $Bb$, Fig. 66 (for the mean circumference), is set out at the proper angle $a_2$; $D$ being the end of the next vane-curve, $DE$ is drawn at right angles to $Bb$, and from some centre

Fig. 66.



on the prolongation of $DE$ an arc of a circle touching $Bb$ in $E$ may be drawn; the line $Aa$ at the upper end of the curve should be inclined at the angle $a_1$ and form a tangent to the circle previously referred to. As in the case of the guide-vanes, the curvature is best found by trial. Many designers use a parabolic curve, but there is no special merit in this, and in some cases it is unsuitable.

In arranging the curves so as to insure a gradual change in the sectional area of the passages, any increase or diminution in the width of the buckets must be taken into account; as a rule axial turbines are of equal width

throughout,.but in the variety known by the name of
Henschel, the width is increased towards the outlet.

As was before pointed out, with given values of $a$ and
$a_1$, if the vanes are designed for entry of the water without
impact at the mean circumference, this condition will not
be fulfilled for the inner and outer circumferences; it is,
however, possible to overcome this difficulty by giving the
vanes varying angles at the points of entry. The mean
velocity being fixed, the velocities at the inner and outer
diameters are thereby determined, and must be in the
same ratio as the latter.

Fig. 67



Let $O A$, Fig. 67, represent the value and direction of
the velocity $c$ with which the water leaves the guide pas-
sages, $A B$ the velocity $w$ of the wheel at the mean circum-
ference; then $O B$ is the relative velocity $c_1$ with which
the water enters the wheel-buckets at that circumference,
and $B O Y$ the suitable angle for $a_1$. At the inner cir-
cumference, instead of $A B$, the velocity of rotation is
$A C$, and if the water is to enter without impact there
also, $O C$ must be the direction of inflow, and the angle
$a_1{}^1 = C O Y$; similarly $A D$ is the velocity at the outer
circumference, and $D O Y$ the proper angle $a_1{}^{11}$ for the
*vane.* By varying the angles $a_1$ according to the diameter

in the manner explained, it would be possible to insure the entry of the water without impact at all points of the width.

The differences in the speed at different diameters also affects, as previously explained, the absolute velocity of outflow $u$, so that with a constant angle $a_2$ for the whole width of the buckets, the direction of outflow is vertical only at the middle, for which the calculations were made, and the velocity of outflow is consequently greater at the inner and outer circumference than at the middle, thus causing a loss of energy in excess of that assumed.

With a helical vane surface the angle $a_2$ will be less at the inner and greater at the outer circumference than at the middle.
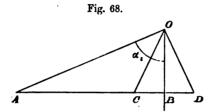
The losses arising out of the deviations from the assumed conditions, both as regards the entry and exit of the water, are inconsiderable when the width of the buckets is moderate in proportion to the diameter of the wheel, but for wide buckets may have a very appreciable influence on the efficiency.

The problem then is, how to construct the turbine vanes so that both these sources of loss may be partially or wholly avoided.

It is evident that the pressure $p_1$ at which the water leaves the guide passages does not vary at various diameters, as otherwise there would be a motion of the water *across* the buckets, a state of things which could not continue, and must almost instantly correct itself. The sum of the energy due to the pressure $p_1$ plus that represented by the velocity of flow $c$, is under any given circumstances a fixed quantity depending on the head of water and the proportions of the wheel; hence if $p_1$ is constant, so must also be $c$.[1] The velocity of flow has

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[1] The ratio $\dfrac{c_2}{c}$ is not modified by the fact that the relative velocity

therefore to be taken as invariable for all parts of the width of a guide passage, and may be calculated in the way shown with the angles chosen for the mean circumference.

Fig. 68.



$c_1$ of inflow varies at different points, *provided* the inflow takes place at all of them without shock, since the area of any narrow segmental strip of the orifice varies inversely as the relative velocity of flow.

If, for instance, $d\,A_1$ be the area of any segmental strip of the inflow orifices $A_1$, and $d\,A_2$ that of the corresponding strip of the outflow orifice $A_2$, the relative velocity of inflow at any diameter

$$c_1 = \frac{Constant}{d\,A_1},$$

since $d\,A_1\,c_1 = $ Constant ; further,

$$d\,A_2\,c_2 = d\,A_1\,c_1 = \text{Constant},$$

and therefore

$$d\,A_2\,c_2 = \text{Constant}.$$

The only *workable* assumption is that the relation between $c$ and $c_2$ is governed by the *total* areas $A$ and $A_2$, so that in effect

$$\frac{d\,A}{d\,A_2} = \frac{A}{A_2},$$

and $$c_2 = \frac{d\,A}{d\,A_2},\ c = \frac{A}{A_2}c.$$

In practice the ratio $\frac{d\,A}{d\,A_2}$ at different diameters varies only slightly from $\frac{A}{A_2}$, and when, as is frequently the case, $a$ and $a_2$ have the same value, $\frac{d\,A}{d\,A_2}$ is absolutely constant. It is assumed throughout the preceding investigation that all particles of water entering at a certain diameter leave at the same diameter in a Jonval wheel of constant *width.*
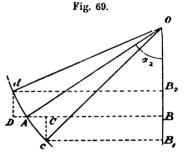
The relative angles of exit at the inner and outer circumferences have now to be so determined that the water there shall leave the wheel parallel with its axis, as is the case at the mean diameter, *if possible.*

Let $O A$, Fig. 68, represent the value and direction of the relative velocity of outflow, and $O B$ the absolute velocity of the water leaving the wheel—at the mean circumference, parallel with the axis. If $A C$ and $A D$ be respectively the speeds at the inner and outer circumferences, then $O C$ and $O D$ are the corresponding absolute velocities of outflow. It will be seen that these are divergent and both greater than $O B$, in other words, only at the mean circumference is the speed of rotation the best.
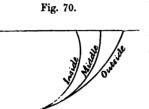
With a helical vane surface for which the angles at the different diameters vary, the divergences in the directions of outflow are not so great as when $a_2$ is constant.

Fig. 69.



The corrections required to insure a parallel outflow at all diameters are effected as follows: Let $O A$, Fig. 69, as before denote the value and direction of the relative velocity of outflow $c_2$, and $A B$ the velocity of rotation $w$ at the mean circumference. (Since $c$ is constant, $c_2$ must be so as well, as the two velocities have a fixed ratio to each other.) The absolute velocity of outflow for the mean circumference is $O B = u$.

Make $B C$ and $B D$ respectively equal to the speed of the inner and outer circumference; draw $C c$ and $D d$ parallel with $O B$ until they intersect a circle described from the centre $O$ with the radius $O A$ in $c$ and $d$ respec-

tively ; join $O c$ and $O d$; then $B O c$ and $B O d$ are the appropriate angles for the end of the vane at the inner and outer circumferences, and the corresponding absolute residual velocities are $O B_1$ and $O B_2$, both vertically directed. When the correc-

Fig. 70.



tions just described are applied to the angles of inflow and outflow, $a_1$ and $a_2$, the vane-curves at the inner, mean, and outer circumference will have the form shown in Fig. 70.

The whole of the formulas as applied to the design of turbines are based upon the assumption that the water leaves the wheel at right angles to the direction of rotation and enters without impact, and if these conditions are not fulfilled the results obtained from them are more or less vitiated; on the other hand, varying the angles in the manner indicated also causes a deviation from the assumed conditions, as may be seen by reference to the Table I. for $K^1$, which varies for different angles $a_2$, but not to a very great extent. This would affect the value of the velocity of flow $c$, but in using the angle at the mean diameter for determining the latter no error of practical importance is incurred. Similar remarks apply to the ratio $\dfrac{A}{A_2}$, which is also influenced from the same cause.

*Note.*—Meissner in his work on turbines, in dealing with the correction of the vane angles, treats the velocity of flow at different diameters, as though the wheel were actually divided by concentric divisions into separate parts, and calculates the velocity of flow independently for each, so that it varies with the diameter according to the angles of the vane. This method the author

considers incorrect, as, for the reasons given, *c* must be at any rate *approximately* constant for the whole width of the guide-passage orifices.

### Vane Curves and Angles.

The effect of the angles $a$ and $a_2$ on the efficiency of a turbine has been already considered, and the angle $a_1$ is fixed when the former have been determined on.

It was shown that for a given ratio $A : A_2$ efficiency is promoted by making $a$ and $a_2$ as great as possible, but that they may vary through a very considerable range without material detriment to the performance of the motor.

*Axial Turbines.* For Jonval turbines with constant width of bucket, when $a$ and $a_2$ are equal, the relative direction of entrance of the water is vertical or parallel to the axis, and the vane angle $a_1 = 0°$. For purposes of manufacture it is very convenient to adopt this arrangement, and from a theoretical point of view there is little to be said against it. When the angles at which the water leaves the guide-vanes and wheel-vanes respectively are equal, the ratio $\dfrac{A}{A_2}$ is approximately 1. The best value for that ratio, as apparent from the Table No. II. of efficiencies, is about 0·75, but the difference is so small between the efficiency for the latter value and that for $\dfrac{A}{A_2} = 1$, as not to be worth practical consideration, so that the proportions in question may be adopted without hesitation.

The usual limits to the values of $a$ and $a_2$ are stated in the summary of rules and formulas for the design of Jonval turbines; but it is worthy of notice that the

greater the angles the larger must be the turbine for a given quantity of water.

In parallel-flow wheels with buckets of constant width the ratio $\dfrac{A}{A_2}$ may be varied by making $a_2$ greater or less than $a$; in the former case $a_1$ is positive, in the latter negative. In turbines of the Henschel type, in which the buckets are wider at the outflow than at the inlet, $a_1$ may be negative even when $a_2$ is greater than $a$; this is evident from the formula

$$tang\ a_1 = \frac{e_1}{e_2}\ tang\ a_2 - tang\ a,$$

where, provided $\dfrac{e_1}{e_2}$ is a sufficiently small fraction, $tang\ a_1$ may always become negative.

For radial inward-flow wheels $a_1$ is greater than for parallel-flow wheels with the same angles $a_2$ and $a$; the formula in this case is

$$tang\ a_1 = \frac{e_1}{e_2}\left(\frac{r_1}{r_2}\right)^2 tang\ a_2 - tang\ a,$$
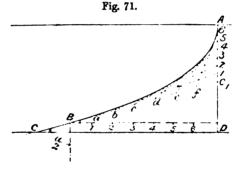
where $\dfrac{r_1}{r_2} > 1$.

Under ordinary circumstances with Jonval turbines the simplest plan is to make $a$ and $a_2$ equal, but there are cases in which for some reason it may be desirable to have as high a speed of wheel as possible, and to effect this a high ratio of $A$ to $A_2$ is required, which renders it necessary to make $a_2$ greater than $a$. Where the buckets are not restricted to a constant width, the same end may be attained without this expedient by a suitable ratio $\dfrac{e_1}{e_2}$ between the widths at the inflow and outflow.

For axial turbines the vane-curves are frequently constructed in the manner shown in Fig. 71.

Let $A D$ be the depth of the wheel—or guide passages—parallel with the axis; $C D$ the circumferential length of a vane.

The outflow end of the vane is made straight for a certain length $B C$, and set out at the proper angle $a_2$ (or $a$). $C B$ is continued to $C_1$, its point of intersection with the line $A C_1$; $A C_1$ being the direction of inflow, $B C_1$ and $A C_1$ are then each equally divided into an equal number of parts by the points $a, b, c, d, e, f$ and

Fig. 71.
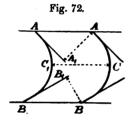


1, 2, 3, 4, 5, 6 respectively, and the lines $1a$, $2b$, $3c$, $4d$, $5e$, and $6f$ are drawn. These lines form tangents of the desired vane-curve, which can then be filled in.

In the illustration the angle $a_1$ is taken as $0°$, but the construction remains the same with any other value, only then $A C_1$ is not perpendicular to $C D$.

The ratio of the horizontal length $C D$ to the depth $A D$ may be from $1\frac{1}{2}$ to 2. As elsewhere explained, the vane-curves must always be drawn with due regard to as gradual a change as possible in the sectional area of the streams passing between them; and, apart from this, there is little merit in any particular form of curve.
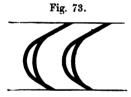
*Back Vanes.*   In cases where the relative angle of entrance is negative, the form of the passage between the vanes of a turbine may be such as to cause great and sudden changes in the section of the stream.   This will be clear on reference to Fig. 72.   Assuming the width of the turbine to be uniform, the area of the passages is proportional at various points to the normal depth, and the latter increases from $A A_1$ at the entrance to $C C_1$ in the middle, and decreases again at the outflow to $B B_1$.   To avoid these extreme variations, the buckets

Fig. 72.



have, under such circumstances, been made with so-called *back vanes*, as illustrated in Fig. 73.   This construction amounts simply to giving the vanes a variable thickness to suit the desired depth of the passages; but to avoid excessive weight and thickness of metal, the vanes are cored out, and are, so to speak, double.

Fig. 73.



This construction is now very unusual, as the relative angle of entrance $a_1$ is generally positive for reaction turbines, and in impulse wheels back vanes have no object.

Where back vanes are used the two surfaces forming the sides of a passage are of different shape, and the courses taken by the particles of fluid at various points of the stream are not parallel with each other nor similar in form.   This leads to varying velocities and disturbances within the buckets resulting in internal friction, whirling motion, and consequent loss; back vanes should therefore be avoided.

In turbines intended to work sometimes with free deviation and sometimes drowned, according to the tail-water level, back vanes are necessary.

*Radial Turbines.* With radial turbines the difficulties occurring in axial turbines owing to the varying speeds at different points of the vanes are absent; all points at
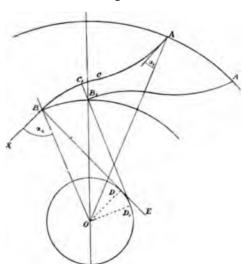
Fig. 74.



which the water enters move with the same speed, and all points at which the water leaves have an equal velocity.

A different drawback, already referred to, is incidental to the use of inward-flow wheels, and that is the contraction of the streams issuing from the buckets; to obviate this to some extent the form of wheel-vane shown in Fig. 74 has been devised with double curvature. The result of this is that the passage between the vanes remains constant in depth for some time before the inner circum-

ference is reached; this of course does not prevent the convergence of the streams after leaving the various buckets and the resulting loss of energy due to the increased pressure at the orifices. The form of vane in question is used in the well-known "Vortex" turbine, and was originally suggested by Professor James Thomson.

The construction of the vane-curves in question is shown in Fig. 74. $A B$ represents the vane-curve, the tangent $B X$ to which at $B$ forms the angle $a_2$ with the radius $O B$; the portion $B C$ of this curve is a volute, described by development from a circle of the radius $O D$; to find $O D$, from $B$ draw $B E$ at right angles to the tangent $B X$ of the vane-curve, and from the centre $O$ of the wheel draw $O D$ at right angles to $B E$, which will be the desired radius of a circle touching $B E$ in $D$.

The true curve would be described by the end of a very fine thread as it is unwound from the circumference of the circle $O D$ and kept in tension during the process, so that in all positions it forms a tangent to the circle. It is easy to determine any number of points on the curve on paper by imitating this construction. The point $C_1$ of the volute-curve is such that a tangent drawn from it to the circle $O D$ passes through the end $B_1$ of the next vane curve $A_1 B_1$; the normal depth of the stream here is $B_1 C_1$, and it is evident that as far as both volute-curves of the adjacent vanes are intersected by the same tangent to the circle $O D$, the normal depth of the passage between them remains constant, and there is no contraction.

The two points $B_1$ and $C_1$ of the same imaginary thread both describe similar volutes parallel with each other and separated by the distance $B_1 C_1$. The volute $B C$ should be carried to $C$ a short way beyond $C_1$ to insure the parallel motion of all parts of the stream, and the vane *then* continued by a curve of another form terminating in

*A* with the inclination $a_1$. Care must be taken that the transition is gradual.

The guide-vanes of the "Vortex" turbine are constructed in such a manner that the passages between them are long, and of a gradually convergent form. Thus the velocity of flow gradually increases from the outer towards
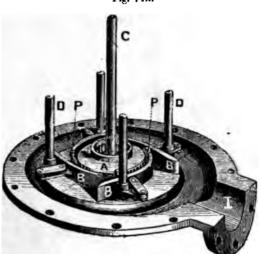
Fig. 74A.



the inner circumference, causing the water to take the form of a vortex.

The construction is illustrated in Fig. 74A—in which the turbine is shown with the cover of its outer casing removed—and in 74B, showing the exterior of the casing complete.

*A* is the wheel keyed to the shaft *C ; B, B,* &c. are the guide-vanes, pivoted at *P, P,* &c., nearer to their inner than to their outer ends; *D, D,* &c., shafts and bell-cranks connecting the guide-vanes with outside bell-cranks and

coupling-rods *E, E,* &c. Suitable gear acting on one of
the shafts, *D,* actuates simultaneously all the guide-vanes,
increasing or diminishing the outflow area of the guide
passages in a manner sufficiently clear from the illustration.
*I* is the supply-pipe; *H* the wheel-cover; *G* a bracket and
screw for adjusting the pivot (not shown in the illustration).

The absolute angle of outflow *a* from the guide passages
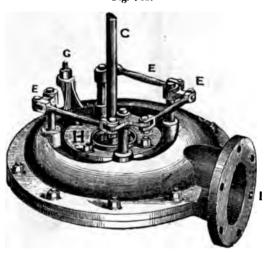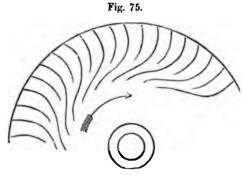
Fig. 74B.



is very large, so that the water enters the wheel almost
tangentially, and very few guide-vanes are necessary. It
is, however, a drawback to this form of turbine, that owing
to the great length of the guide-vanes the dimensions of
the casing for large wheels become excessively great.

In order to obtain, with a given diameter of wheel, a
greater outflow area than would otherwise be possible,
every alternate wheel-vane is made shorter than the rest,
so that it occupies only the outer half of a bucket where

the passages are widest, as shown in Fig. 75. This construction also tends to reduce the loss by friction in the passages.

In other systems of radial inward-flow turbines the curves of guide and wheel vanes both terminate in straight lines at the outflow (*vide* Fig. 22). The outer extremities of the guide-vanes are very often radial. The radial height of the guide passages in such turbines is then about the same as that of the wheel-buckets, and the guide-casing has a much smaller diameter relatively to the

Fig. 75.
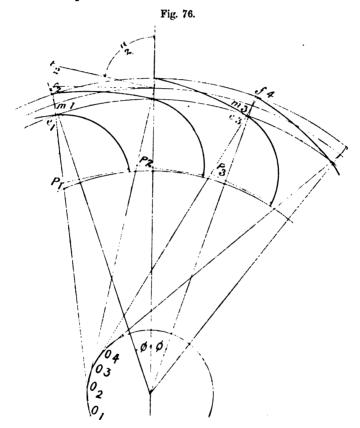
wheel than is required for the "Vortex" turbine; on the other hand, a much greater number of vanes is necessary in order to give the flow the desired direction. Judging by results, as far as efficiency is concerned, there appears to be no difference between the two systems, with both of which admirable results may be obtained.

For the vanes of radial outward-flow turbines the construction shown in Fig. 76 is convenient, and may be applied to either guide or wheel vanes.

Let $m_1$, $m_2$, $m_3$, &c., be the points at which the vanes are pitched on the mean circumference of outflow. Taking the point $m_2$—for example—set out $m_2 t_2$, forming the

outflow-angle $a_2$, with a radial line from the centre of the
wheel to $m_2$.

Fig. 76.



Perpendicular to $m_2\,t_2$ draw $m_2\,O_3$, and on either side
of $m_2$ set out—on $m_2\,O_2$—the length

$$m_2\,e_2 = m_2\,f_3 = \tau_2\,cos\,a_2\,tang\,\frac{\phi}{2},$$

where $\phi$ is the angle subtending the pitch.

From the adjacent pitch-point $m_3$, draw $m_3$ $O_4$ in the same way as $m_2$ $O_3$.

$m_2$ $O_3$ and $m_3$ $O_4$ intersect in $O_3$, and $O_3$ is the centre from which the arc $e_2$ $f_2$, forming the outer portion of the vane-curve, is described.

From $O_4$, with the radius $O_4$ $e_3 = O_3$ $e_2$, the corresponding part $e_3$ $f_3$ of the adjacent vane-curve is also drawn. The centres $O_1$, $O_2$, $O_3$, &c., for all the outer vane-curves, lie in a circle. If the construction is correct, the arcs drawn through the points $e_1$, $e_2$, $e_3$, &c., must pass through the corresponding points $f_1$, $f_2$, $f_3$, &c., lying on the outer circumference.

The inner portions of the vane-curves are also arcs described from points $P_1$, $P_2$, $P_3$, &c., as centres, on the radii $O_2$ $m_1$, $O_3$ $m_2$, $O_4$ $m_3$.

The result of this construction is, that the tangents to the adjacent vane-curves at $e_1$ and $f_2$, $e_2$ and $f_3$, &c.—where the outflow area is least—are *parallel*, and thus there is no *contraction* of the streams issuing from the buckets.
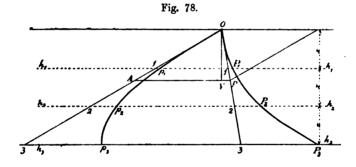
It is generally easy to find by trial the proper position for each of the centres $P_1$, $P_2$, $P_3$, &c., in such a manner that the relative angle of inflow is correct. The inflow end of each vane may terminate in a straight line, if necessary, forming a tangent to the inner portion of the vane-curve. When the distances $e_1$ $f_2$, $e_2$ $f_3$, &c., are determined as described, and the thickness of the vanes is settled, the clear depth of the outflow passages is of course fixed, and to obtain the desired outflow area, the *width* of the passages must be calculated accordingly.

### General Remarks on the Construction of Vanes.

In designing and constructing the vanes of a turbine everything possible should be done to avoid unnecessary

$O$ to $X$, $X$ has advanced to $X_1$, so that the latter is the last point of the absolute path.

If the shape of the buckets is such that the velocity with which the water traverses the wheel, parallel with the axis, remains *constant*, it is a very simple matter to construct the curve of the absolute path. On entering the wheel the water has the absolute velocity $O\,A$, Fig. 78; the vertical component is $O\,V$, and this, by hypothesis, must remain the same at every point during the passage through the buckets. $A\,P$ is the velocity of rotation of the wheel, so that while the water moves vertically through a distance $O\,V$, the wheel has advanced by $A\,P$.

Fig. 78.
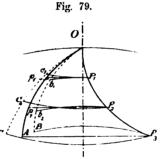


If a series of lines $h_1\,h_1$, $h_2\,h_2$, $h_3\,h_3$, parallel with $O\,X$, be drawn at equal distances $x$ apart, the interval 1 1 intercepted on $h_1\,h_1$ by its intersection with $O\,A$ and $O\,P$ represents the distance traversed by the wheel while the water has passed vertically through $x$ and arrived at the point $P_1$ on the vane-curve; similarly 2 2 on $h_2\,h_2$ is the length by which the wheel has advanced while the fluid has traversed the vertical distance 2 $x$ and arrived at the point $P_2$; and at the point $P_3$ the corresponding advance *is 3 3.* To determine the points of the absolute path

it is only necessary to mark off from $P_1$, $P_2$, and $P_3$ the distances $P_1 \, p_1$, $P_2 \, p_2$, and $P_3 \, p_3$ equal respectively to the intervals 1 1, 2 2, and 3 3; then $p_1$, $p_2$, and $p_3$ are the points required.

In constructing the absolute path for a radial turbine it is necessary to take into account the fact that the speed of the wheel varies at different points of the vane according to the distance from the axis of rotation. The result of this on the curve of the absolute path, as compared with that of an axial turbine having the same velocity of rotation at the inflow, is to produce a greater deflection for an inward-flow and a less deflection for an outward-flow wheel. Let $O \, A$, Fig. 79, be the absolute path for an



Fig. 79.

axial turbine, then for an inward-flow wheel the path will be $O \, B$, for an outward-flow wheel $O \, C$. The distance $P_1 \, p_1$ which is traversed by the point $P_1$ of the vane while the water moves relatively from $O$ to $P_1$ in an axial turbine, is reduced to $P_1 \, b_1$ in the case of the inward-flow turbine, $P_1 \, b_1$ bearing to $P_1 \, p_1$ the ratio of the radius of $P_1$ to that of the starting-point $O$; similarly $P_2 \, b_2$ is to $P_2 \, p_2$ as the radius at $P_2$ to the radius at $O$; as the radii of $P_1$ and $P_2$ are less than that of $O$, $P_1 \, b_1$, and $P_2 \, b_2$ are smaller than $P_1 \, p_1$ and $P_2 \, p_2$.

In the case of an outward-flow wheel where $c_1$ and $c_2$ are points of the absolute path, $P_1 \, c_1$ and $P_2 \, c_2$ are greater than $P_1 \, p_1$ and $P_2 \, p_2$ respectively.

# CHAPTER VI.

## THE DESIGN OF REACTION TURBINES (*continued*).

Influence of angle of outflow from guides and area ratio on efficiency.—Methods of regulating turbines.—Correct system of regulation.—The distribution of water in turbines.—Helical vanes. —Constant angles of outflow.—Loss from unutilized energy.—Loss by shock.—Comparison of axial with radial turbines.—Reasons for theoretical superiority of inward-flow wheels.—Practical drawbacks. —Mixed-flow turbines.—On the use of the suction-tube and Boyden diffusor. — Best shape for suction-tube.—Boyden diffusor.—Necessary adjuncts.—Losses in tubes and diffusors.—Experiments.

## *Effect of Proportions and Dimensions on the Performance of Reaction Turbines.*

IT has been already shown that under otherwise similar conditions the efficiency is greater the larger the relative angle of exit $a_2$, as the residual velocity of off-flow decreases, the more nearly the relative direction with which the water leaves the buckets approaches to the direction of rotation. It is therefore always desirable to make the angle $a_2$ as great as practicable in all kinds of reaction turbines. If, however, the direction of outflow approach too closely to that of rotation, it is obvious that the depth of the passages becomes very small, and to allow a given quantity of water to pass through the wheel, either the width must be very great or the diameter large, so that the number of buckets may be increased. In practice

$a_2$ rarely if ever exceeds 80°, and is generally less; it is not desirable to exceed a certain proportion between the width and diameter of the wheel, and an increase in the diameter means increase of cost and of space required. If the residual velocity of the water as it leaves the wheel is not low enough, the efficiency can be improved by the use of a *diffusor* or suitably constructed *suction-tube.*

In considering the effect of the other dimensions and proportions on the efficiency of turbines, it will be convenient to investigate first the case of axial turbines and afterwards to compare these with those of the radial type.

Before this can be done, some assumptions must be made as to the co-efficients of resistance, and the value of any conclusions arrived at necessarily depends on the accuracy of these assumptions. It has so far been taken for granted that the co-efficients $\zeta_1$ and $\zeta_2$ for the resistances in the guide apparatus and wheel respectively are constant, but this has been done merely as a matter of convenience for practical purposes.

As regards $\zeta_1$, the co-efficient for the guide passages, there is no reason to suppose that under average conditions its value varies appreciably for surfaces of a given character and a given class of workmanship.

The loss in the guide wheel is made up—as has been stated—of the friction of the walls of the passages, the resistance of the bend, and the obstruction offered by the edges of the vanes to the entry of the water. The first depends on the nature of the internal surface of the passages and their proportions. For wheels of about the same power, but varying as regards the vane-angles and the ratio of the areas $A$ and $A_2$, the proportions of the passages will not differ sufficiently to seriously affect the co-efficient of resistance. The second, the resistance of the

bend, will also vary very slightly for unit velocity for different wheels. The third component of the loss, due to the obstruction of the vanes, depends on the proportion which the obstructive area bears to the total area of the stream entering the guide passages; this proportion is not necessarily affected in any way by the angles $a$ and $a_2$ nor by the ratio $\dfrac{A}{A_2}$, so that as far as that is concerned a constant value of $\zeta_1$ is justified. On the whole, therefore, for purposes of comparison, a constant value of $\zeta_1$ is a reasonable assumption where turbines not differing very widely in dimensions are concerned.

The losses from the moment the water leaves the guide passages to that at which it quits the wheel have been hitherto all included under the co-efficient $\zeta_2$. In reality they are of a very complicated character, which it would be impossible to determine theoretically with any approach to accuracy. It is quite certain, however, that the value of $\zeta_2$ in fact is not constant. It includes, among other factors, the loss by leakage through the clearance space between guide passages and wheel, and from this it might be supposed that its value would increase with an increasing ratio $\dfrac{A}{A_2}$, that is, the greater the degree of reaction and the higher the pressure at the mouth of the guide passages, since the amount of leakage must—it would be inferred—depend on that pressure.

Experiments which have been carried out and calculations made by Professor Fliegner of Zürich, tend to show that this is not the case, but that, on the contrary, with increasing reaction and greater pressure at the mouth of the guide passages, the co-efficient $\zeta_2$ decreases. The number of data available on this subject is, however,

very limited, and quite insufficient to form any basis for rules or formulas.

For the present the assumption of a constant value of $\zeta_2$ will be adhered to, and the conclusions to which it leads compared with practical results.

The efficiency of a reaction turbine is expressed from (viii) by :

$$\epsilon = \frac{2 \dfrac{r_1}{r_2} \dfrac{A}{A_2} \sin a_2 \sin a}{2 \dfrac{r_1}{r_2} \dfrac{A}{A_2} \sin a_2 \sin a \; + \; \zeta_1 \; + \; \left(\dfrac{A}{A_2}\right)^2 (\zeta_2 + \cos^2 a_2)} .$$

In Table No. II. the efficiencies corresponding to various values of $\dfrac{r_1}{r_2}$, $\dfrac{A}{A_2}$, $a_2$, and $a$ will be found. For an axial turbine $\dfrac{r_1}{r_2} = 1$, so that

$$\epsilon = \frac{2 \dfrac{A}{A_2} \sin a_2 \sin a}{2 \dfrac{A}{A_2} \sin a_2 \sin a + \zeta_1 + \left(\dfrac{A}{A_2}\right)^2 (\zeta_2 + \cos^2 a_2)} .$$

*Influence of a on Efficiency.* The numerator of the fraction expressing the value of $\epsilon$ is proportional to the effective work done *per unit of velocity of flow* by the wheel; the same quantity also appears in the denominator, but with other quantities added; for any given values of $\dfrac{A}{A_2}$ and $a_2$ these additional quantities are constant, and only the first term of the denominator changes with the angle $a$, to the sine of which it is proportional. The greater, therefore, the value of $2 \dfrac{A}{A_2} \sin a_2 \sin a$, the more

closely will the fraction $\epsilon$ approach unity, and hence the efficiency is increased by making $a$ larger.

This will be most clearly seen by reference to Table No. II.; for instance, with $\dfrac{A}{A_2} = 2$ and $a_2 = 70^\circ$, the efficiency increases from 0·674 for $a = 50^\circ$ to 0·722 for $a = 75^\circ$; with $\dfrac{A}{A_2} = 1$ the corresponding figures are respectively 0·764 and 0·804. It will be noted that the effect is not very marked even for a considerable variation in the angle, but still it is evident that to obtain the highest efficiency $a$ should be as large as possible.

This conclusion is quite independent of the value of the co-efficients of resistance $\zeta_1$ and $\zeta_2$.

In practice it is not usual to make $a$ greater than $75^\circ$, occasionally $80^\circ$.

*Influence of Ratio* $\dfrac{A}{A_2}$ *on Efficiency.* An inspection of the Table No. I. for $K_1$—which is proportional to the velocity of flow from the guide passages—shows that for a decreasing ratio $\dfrac{A}{A_2}$, with any given value of $a$ and $a_2$, the velocity of flow $c$ becomes greater; at the same time the relative velocity $c_2$ with which the water leaves the wheel-buckets decreases, but more rapidly than $c$ increases. The loss in the wheel, to which the co-efficient $\zeta_2$ corresponds, is assumed to be proportional to the square of the relative velocity $c_2$, and this is true also of the residual velocity of off-flow $u = c_2 \cos a_2$. On the other hand the work done depends only on the product of $c$ and $c_2$, and is therefore proportional only with the first power of $c_2$; consequently with a diminish-

ing ratio $\dfrac{A}{A_2}$ the work done decreases less rapidly than the losses in the wheel and from unutilized energy. The result is that the efficiency varies with the ratio $\dfrac{A}{A_2}$, and attains a maximum, as is evident from an examination of Table No. II. for a value of $\dfrac{A}{A_2}$ about 0·75. Between the efficiency corresponding to this value and that for $\dfrac{A}{A_2} = 1$, there is a practically unappreciable difference, and for axial turbines the most usual ratio is in the neighbourhood of these, ranging from somewhat above to somewhat below unity. The greatest occurring difference in the efficiency for values of $\dfrac{A}{A_2}$ between 2 and 0·25 is, however, only 0·096 or 9·6 per cent., the efficiency diminishing for an increasing ratio $\dfrac{A}{A_2}$ above 0·75. If it is taken into account that $\zeta_2$ tends to become less for increasing reaction, the actual efficiencies for high ratios of $A$ to $A_2$ would be greater than those contained in the table. The value 1·75 for $\dfrac{A}{A_2}$ has seldom been exceeded in practice, and for this the theoretical efficiency is 0·80, only 4·9 per cent. less than the maximum. In view of these facts it may safely be stated that for practical purposes the efficiency is only slightly affected by varying degrees of reaction.

This conclusion is fully borne out by experimental results, in so far as they are available for comparison ; the latter is rarely the case, for various reasons. In order to

compare two or more turbines as regards the influence of any particular proportion or dimension on their performance, they should in other respects be as nearly alike as possible, and the workmanship of all should be of the same quality; this condition can only be insured when the motors tested are by the same maker; otherwise the results may be affected by subordinate details to such an extent as to vitiate any conclusions drawn. In spite of this, the best performances recorded of a large number of turbines of different kinds varying very widely in dimensions and proportions, agree sufficiently well to confirm the theoretical deductions.

Three experimental turbines of Rittinger's—previously mentioned—afford an exceptionally good opportunity of observing the influence of various proportions of $A$ to $A_2$ on the performance of the motors. All three wheels were of the same outside diameter and approximately the same power, and were tested with as nearly as possible an equal head of water; the angles $a$ and $a_2$ differed somewhat, and the ratio $\dfrac{A}{A_2}$ very greatly in the three motors. The following are the necessary data and results :—

|  | $a$ | $a_2$ | $\dfrac{A}{A_2}$ | Efficiency. | No. of revolutions per minute. |
|---|---|---|---|---|---|
| (1) | 78° | 64° | 0·522 | 0·673 | 99 |
| (2) | 73° | 74° | 1·274 | 0·697 | 135 |
| (3) | 68° | 75° | 1·661 | 0·632 | 151 |

In each instance the speed given is the *best* speed. There is, it will be noted, a maximum variation of only 6·5 per cent. in the efficiencies, while between the efficiencies of motors (1) and (3), which differ most in construction, the variation is only 4·1 per cent.

It should be observed that in Rittinger's turbines the

differences in the ratio $\dfrac{A}{A_2}$ were obtained almost entirely by varying the angles, and the effect of the latter on the efficiency cannot be separated from that of the ratio in question; in general, however, this is the only method in which the proportion between the outflow areas is varied in axial turbines, for which the width of buckets remains constant.

### Methods of Regulating Reaction Turbines.

A reaction turbine only works to the best advantage when water is admitted to all the buckets; all systems of regulation, except one, reduce the efficiency of the turbine when not working full. This one correct method of regulation is attended with practical difficulties and is very seldom applied. It will be subsequently explained; but before this is done it is desirable to describe the other systems adopted, and show in what consist their disadvantages. These systems are the following :—

(1) Regulation by means of the head-race sluice.
(2)     ,,      ,,     ,,          tail-race sluice.
(3)     ,,      ,,     ,,     a throttle valve.
(4)     ,,      ,,   partially closing the guide passages.
(5)     ,,      ,,   means of concentric subdivision.

(I.) *Regulation by Head-race Sluice.* The orifice through which the water is admitted to the reservoir or space immediately above the turbine is altered by means of an ordinary sluice, thus allowing more or less water to pass through, means being provided for the passage of the surplus fluid by another channel when the whole available quantity is not required.

If by partially closing the sluice the quantity of water admitted is reduced to half that with which the wheel works at its full power, the result will be that the head will sink until it corresponds approximately to half the velocity of flow at full power, which—as the head is proportional to the square• of the velocity—would be ¼ of the original head. At this reduced velocity of flow the speed should be also reduced in proportion if the motor is to work to the best advantage, that is, to about half the speed at full power; as, however, it is generally necessary to run at a constant speed, this latter will under the assumed conditions be double as great as the best speed, and as this is approximately the velocity with which the wheel revolves without a load, doing no work, the efficiency is reduced to zero.

From this extreme case it will be seen that the method of regulation by the head-race sluice is very defective.

(II.) *Regulation by Tail-race Sluice.* This system is generally applied to turbines with suction-tubes. The sluice is usually a circular one, surrounding the lower end of the suction-tube, as shown in Fig. 24, p. 35. Suppose, again, in this case, that the sluice is so far closed that only half the quantity of water required at full power is allowed to pass. The velocity of flow will be reduced to one-half, but, as the work done per unit of weight is proportional to the *square* of the velocity, the efficiency will be only ¼ of that at full power with the same speed, and the *total* work done only ⅛.

By the regulating action of the sluice, therefore, the efficiency is very much reduced, and the system in question is consequently bad.

Apart from this, it has another drawback, which consists in the fact that a very considerable reduction in

the outflow orifice produces a comparatively slight effect on the velocity of flow. The effect of regulating by a sluice to the suction-tube is illustrated by the following figures, taken from the results of experiments made many years ago by Messrs. A. Koechlin and Co. of Mulhouse.

| Open height of sluice. | Quantity of water per second. | Efficiency. |
|---|---|---|
| 0·426 | 296·10 | 0·718 |
| 0·176 | 271·71 | 0·631 |
| 0·095 | 253·90 | 0·349 |
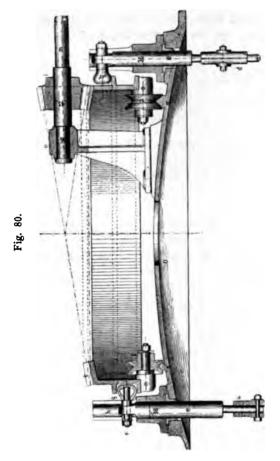
It is clear that as the opening of the sluice is changed the speed of greatest efficiency is altered in proportion as the velocity of flow is reduced, and it is assumed that for practical reasons the speed must be maintained constant under all conditions.

For raising and lowering the sluice of a suction-tube an arrangement of rack and pinion, acting on rods by which the sluice is suspended, is generally employed.

(III.) *Regulation by Throttle Valve.* The effect of a throttle valve applied either before the water enters the guide passages or in the suction-tube is similar to that resulting from the methods of regulation (I.) and (II.) previously described, and is therefore objection-able; owing to its shape it offers a greater resistance to the passage of the water than the sluices; the lower edge of a sluice may be rounded off with a curve of large radius, but with a throttle valve this is impracticable.

(IV.) *Regulation by partially closing Guide Passages.* This system is now most generally adopted for regulating reaction turbines. The mechanisms employed are various, but the result produced is in all cases much the same; the following is a description of some of the more import-ant arrangements used :—
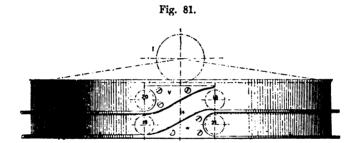
(1) **Vertical slides to each guide passage.** This construction is shown in Figs. 80, 81, and 82; every slide is



Fig. 80.

attached to a rod, which in some cases is worked independently, while in others several are coupled together and connected by a cross head to a single rod. Each rod

may be actuated by a hand wheel and nut acting on the
screwed upper end, or all the rods can be worked from
a sort of horizontal cam (*vide* also Fig. 30), which is fur-
nished with a groove on the outer circumference into
which project rollers carried on pins attached to the upper
ends of the slide rods; as the cam is made to revolve,
each of the rods, with the slides connected to it, is raised
or lowered in succession.   Referring to the illustrations,
Figs. 80 and 81 show the cam with the grooves formed
by projecting flanges $i_2$ and $i_3$; at $i_4$ the groove is inclined,

Fig. 81.
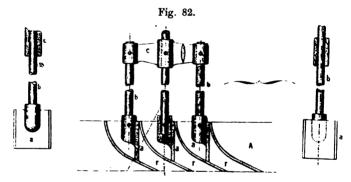
and as the cam revolves from left to right, a roller which
has arrived in the position indicated by 18, is forced down
or drops by its own weight into the relative position 19,
thus lowering the slides connected with it and closing the
corresponding guide passages.   18 and 20 show the posi-
tions of the extreme rollers when all slides are open, 19
and 21 the same when all are shut.

Fig. 82 shows the construction of the slides *a*, *a*, *a*,
attached to the rods *b*, *b*, *b*, which are connected by the
cross head *c*; the central rod is continued upwards, *vide*
Fig. 80, and at its upper end *e* is enlarged in diameter,
and carries the roller *g* revolving on the pin *f*. At the
top of the cam *i* is a toothed bevel ring *k*, which gears

with the pinion $l$ keyed to the shaft $m$, by means of which motion is imparted to the cam; $n$ is the bearing for the shaft $m$. The cam is supported by rollers $p$, which run on a circular rail $q$ cast on the top of the guide-wheel cover $o$. The dimensions given in the illustration are in millimètres.

In Fig. 83 another form of slide is shown, in which the faces $c$ are of cast-iron with curved wooden blocks $f$ on the back. The slides are worked by the rods 1, 2, and 3; $b$, $b$, $b$ are the guide, and $a$, $a$, $a$ the wheel vanes.

Fig. 82.
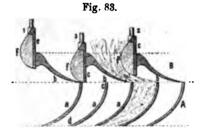


The backs of the slides are curved, in order that they may offer less resistance to the passage of the water beneath them.

This type of slide is unnecessary where the regulation is effected, as it should be, by *entirely* closing some of the guide passages.
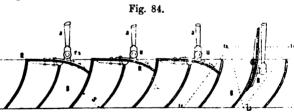
(2) **Horizontal slides on the top of guide passages.** The slides, each of which generally covers two or three guide passages, are moved radially inwards or outwards by a cam, in a manner similar to that previously described; as the mechanism is all under water, it is liable to become choked and otherwise interfered with by mud, sand, and other material carried by the water, and is on this account not to be recommended for practical purposes.

(3) **Pivoted flaps in the guide passages.** Fig. 84 illustrates this arrangement: two guide passages *B* are covered by one flap *R*, which is pivoted over the central vane between the two passages. Each flap is worked by a rod *a* attached by a pin at *u*. The rods can be actuated by a cam in the manner described under (1); another method of effecting this is by prolonging the spindle on which each flap turns through the

Fig. 83.
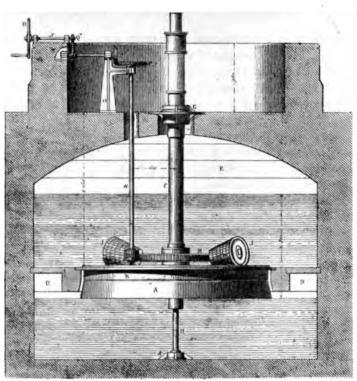


inner wall of the guide-wheel, and on the end of this prolongation fixing a crank, the pin of which projects into a groove in the surface of a cylinder or cam concentric with the turbine shaft. When this cam is made to revolve, the pins are forced to travel in succession up or down an inclined portion of the groove, thus closing or opening the flaps.

Fig. 84.
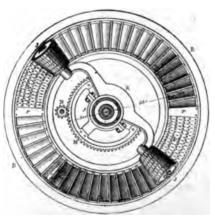


This latter arrangement has the drawback of being inaccessible. The use of flaps as described is open to the objection that they do not close tight, and are rather weak in case, as often happens, they should stick and require the application of considerable force to move them.

(4) **Scroll regulator.** This arrangement is shown in

N
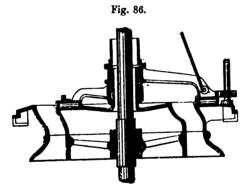
Fig. 85.



JONVAL TURBINE WITH SCROLL REGULATOR.
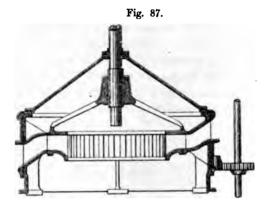
Fig. 85 ; it consists essentially of two scrolls or bands of leather or india-rubber *L L*, on the face of which a series of metal strips are riveted, wound upon conical rollers *J J.* The rollers are connected by a casting *K*, which turns loose about the turbine shaft *C*. Each band or scroll is fastened down to the guide apparatus at one end *r*, and when the casting *K* with the rollers is made to revolve in one direction, the bands are unrolled and laid down over the guide passages; when the motion is in the opposite direction they are wound up again and the passages un-

Fig. 86.



covered. The mechanism is actuated by the wheel *M* and pinion *N*, from the hand-wheel *R* through the spindle *u*. This method of regulation is over twenty-five years old, but is still used by some engineers, and has given very satisfactory results; leather has been found to be the best material for the scrolls.

(5) **Ring slide or sluice.** With this system two concentric half-rings of different diameters are employed, each of which is sufficient to cover one-half of the guide passages; the entrances to the latter are situated on two concentric annular surfaces, half the orifices being in one

of these and half in the other diametrically opposite.
When all the guide passages are open, the two slides

Fig. 87.



rest over those portions of the annular surfaces not occu-
pied by the guide orifices. The slides may be either flat,
conical, or cylindrical, according to circumstances. Fig. 86
shows this method of regulation as applied to an axial
turbine, while Fig. 87 illustrates its adaptation to an
inward-flow wheel.

(6) **Movable guide-vanes.** The late Professor James
Thomson introduced a system of regulation for his
inward-flow turbines which consists in attaching the guide-
vanes to pivots, so that by changing their inclination with
respect to the wheel the area of the guide passages can be
varied. This construction has already been described and
illustrated in connection with the " Vortex " turbine, Figs.
74A and 74B.

Various means of actuating the vanes are employed, the
simplest being by means of levers or cranks fixed on the
ends of the spindles about which the vanes turn.

The drawback to this method is, that in all positions of

the vanes except one the water enters the wheel with impact, and there is a corresponding loss of energy.

(7) **Circular slide between guide passages and wheel.** This system, shown in connection with "the Humphrey" turbine (*vide* Chapter XI.), is applied to radial inward-flow turbines, and consists of a ring or cylinder concentric with the wheel, fitted between the latter and the guide apparatus, and containing a number of openings corresponding to the orifices of the guide passages; the spaces between the openings are at least equal to the latter, so that by turning the regulator cylinder the guide passages can be covered. This arrangement necessitates a great thickness of the guide-vanes measured on the inner circumference of the guide-wheel, and in a reaction turbine interferes very much with the continuity of flow essential to its correct action.

(8) **Sluice inside wheel.** For Fourneyron (outward-flow) turbines a cylindrical sluice inside the wheel shutting off the guide passages vertically—that is, reducing the height of the passages all round—has been employed; this arrangement, of course, when the wheel is not working at its full power, allows the wheel-buckets to be only partially filled, and therefore interferes with the correct action of the turbine when working drowned; if, as sometimes occurs, the off-flow takes place above water, then the influence of the sluice would not be so detrimental, as in that case the motor becomes practically an impulse turbine with free deviation. A similar method of regulation is employed with some inward-flow wheels.

There are besides the mechanisms enumerated a large number of other arrangements and modifications, which it would occupy too much space to describe.

The system of regulation by partially closing guide passages reduces the efficiency, but not to the same extent

as when the exit of the water is controlled by a throttle-valve or sluice at the lower end of the suction-tube.

Theoretically it is essential that a reaction turbine should work *full* of water in every part, and by closing some of the guide passages this condition is of course interfered with, as water is only admitted to a portion of the wheel-buckets at a time. When, however, a good arrangement is adopted, the result on the efficiency is not as bad as might be anticipated. It is best to close a certain number of the guide passages entirely in such a manner that all the closed passages are contiguous, as this interferes less with the efficiency than when *every* passage in the turbine is *partially* closed. It has been found that to obtain good results air should be admitted to the closed passages, so that the water in the corresponding buckets of the wheel may be able to escape freely. This is done in some cases through tubes connecting the interior of the guide-buckets with the atmosphere. When the passages are closed the tubes are open.

The effects of regulation by the systems in question will be best illustrated by the examples following, selected from the results of actual experiment.

## *Jonval Turbine at Goeggingen.*

This turbine[1] is regulated by flaps hinged on the outside of the guide-wheel, each flap covering three passages, or $\frac{1}{12}$ of the whole area.

| Proportion of passages open. | Efficiency. | No. of revolutions. |
|---|---|---|
| *All.* | 83·2 per cent. | 45·52 |
| $\frac{3}{4}$ | 79·5 ,, | 45·29 |
| $\frac{7}{12}$ | 74·9 ,, | 44·35 |
| $\frac{1}{2}$ | 75·1 ,, | 45·63 |

[1] *Vide* Chap. XI.

*Humphrey Turbine at the Tremont and Suffolk Mills.*

The "Humphrey"[1] turbine is of the mixed-flow class, the water entering radially, flowing inwards, and leaving axially. In the wheel in question regulation took place by means of a circular slide between guide passages and wheel as described under (7).

| Proportion of passages open. | Efficiency. | No. of revolutions. |
|---|---|---|
| 100 per cent. | 81·9 per cent. | 47·42 |
| 82·5 ,, | 76·69 ,, | 51·60 |
| 77·24 ,, | 76·54 ,, | 48·00 |
| 52·90 ,, | 61·02 ,, | 50·64 |
| 40·66 ,, | 56·14 ,, | 45·59 |

A comparison of these results with those given in the preceding example demonstrates the superiority of the regulating arrangement adopted in the Jonval turbine; with 50 per cent. of the passages open the efficiency was reduced by 8·1 per cent. as compared with that obtained with all open, while in the case of the Humphrey turbine, with the whole area of the passages and with 52·9 per cent., the efficiencies were respectively 81·9 per cent. and 61·02 per cent., showing a reduction of 20·88 per cent.

*Fourneyron Turbine.*

The following figures are given by Professor Unwin as the result of experiments with a Fourneyron turbine regulated by a cylindrical sluice working out of water :—

[1] *Vide* Chap. XI.

| Opening of sluice. | Efficiency. |
|---|---|
| *Full* | 62 per cent. |
| $\frac{7}{8}$ | 60 ,, |
| $\frac{5}{8}$ | 43 ,, |
| $\frac{3}{8}$ | 30 ,, |

With the turbine drowned the efficiencies were lower.

(V.) *Regulation by concentric subdivision.* This method, which is applicable to various kinds of turbines, is generally adopted only in those of large dimensions; it consists, in the case of an axial turbine, in dividing the whole guide apparatus and wheel by one or more concentric partitions into two or more parts, each part forming, so to speak, a separate turbine. As less power is required these parts can be shut off in succession, and those remaining act each as a complete turbine, the efficiency of which is not affected by its neighbours being put out of action.
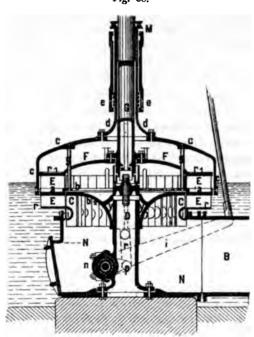
In radial turbines the subdivision for purposes of regulation is effected by partitions at right angles to the axis.
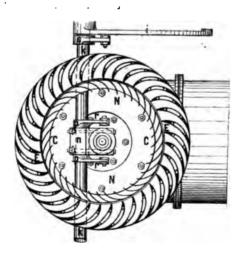
For large powers this system of regulation is one of the best in use; the efficiency is only slightly reduced by shutting off one of the divisions. In the case of a double Jonval turbine, of which particulars are given in Meissner's work on Turbines and Water-wheels, the efficiency was about 6 per cent. less with one part only in use than when the whole wheel was in action.

(VI.) *Correct system of regulation.* The only mode of regulating a reaction turbine which is theoretically correct, is by reducing the areas of both guide and wheel passages simultaneously, so that the proportions remain the same. This is not practicable in any way for axial turbines, but has been applied in exceptional cases to radial-flow turbines.

The turbine of Messrs. Nagel and Kämp is constructed on this principle. The wheel (Fig. 88) is of the outward-

Fig. 88.

flow (Fourneyron) type, the water being supplied to the guide apparatus from below through the tube $B$, from which it enters the chamber $N$. The vanes of the guide-wheel are attached to a movable cover $b_2$, which can be raised or lowered, while their lower portions are guided vertically by slots in the projection or flange $i\,i$ which extends round the whole inner circumference. The boss $y$ of the cover $b_2$ slides upon the column $D$. The wheel $E$ itself is rigidly connected through the plate or arms $F$ with the shaft $G$, and is regulated as follows :—Within the upper and lower wheel-casings $r$ and $r_1$,.there is placed between each pair of vanes a plate $a$, which fills the space between them, but fits sufficiently easily to slide up or down freely. All these plates $a$ are connected at the inner circumference to a ring $b$ of angle-iron, and at the outer circumference to a circular casing $c$. The ring $b$ is suspended by rods $g$ from the top of $c$. It is clear that when $c$ is raised or lowered the plates $a$ move with it and alter the depth of the wheel. The mechanism is so arranged that $c$ and $b_2$ rise or fall together, and the width of the wheel-buckets always corresponds with that of the guide passages.

The raising and lowering of $c$ and $b_2$ are effected through the lever $l$ and horizontal shaft $n$; to the latter are keyed levers $p$ and $s$ and connecting-rods $r$ and $D$; of these, two levers $p$ act through $r$ and $D$ on the guide-casing $b_2$, and two levers $s$ through connections not shown in the illustration, on the casing $c$.

The method of regulation described, although perfect in principle, appears somewhat too expensive and complicated for general practical use.
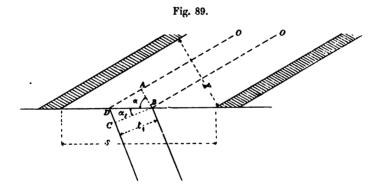
### The Distribution of Water in Turbines.

Not merely the effect of a given weight, of water acting on a turbine is influenced by the form of the vanes, but also the distribution of the water flowing through the guide passages and buckets, and this again may modify to some extent the efficiency, so that to ascertain the connection between the latter and the construction of the vanes, the distribution as well as the direction and velocity of flow of the water must be taken into account.

In radial inward- or outward-flow wheels, in which all the water enters or leaves at the same diameter, and the edges of the vanes are parallel with each other, while the angles of inflow or outflow are constant for the whole width of the vanes, the distribution is necessarily uniform, on the assumption that the velocity of flow does not vary with the width, but in a parallel-flow turbine the case is different. It may be supposed that a Jonval wheel is divided—by imaginary cylindrical surfaces—into a large number of very narrow concentric rings of equal width, and the quantity of water flowing through each of these with a given construction of vanes can then be ascertained; if through each ring the same quantity of water passes in a given time, then the distribution is *equal* or *uniform*, but if more (or less) flows through the outer than through the inner rings, then the distribution is *unequal*. Obviously, to avoid internal friction, the distribution should be similar in all parts of the buckets and guide passages.

The most usual form of vane surface both for guides and buckets is the *helical*, and the distribution of water resulting from this construction will therefore be first considered, beginning with the guide passages. With helical vane surfaces the angle *a* *decreases* from the outer

to the inner diameter, since the pitch of the helices defining the surface at all diameters is constant. If $s$ be the clear distance, Fig. 89, between two vanes measured parallel with the direction of rotation, then the normal depth $X$, or clear distance measured at right angles to the direction of flow, is $s \cos a$, and to this product the outflow area of any of the narrow concentric rings of equal width is proportional. The value of $s$, however, is proportional to the diameter or mean radius $r$ of the given ring, while $\cos a$ increases as the radius decreases; the effect of this

Fig. 89.



is, that the product $s \cos a$, for moderate widths of the passages, is nearly the same at all diameters, and therefore the distribution of the water almost equal.

As an example a turbine may be taken in which the width of the buckets is $\frac{1}{4}$ of the mean diameter—a rather large proportion; the inner, mean, and outer radii are then in the ratios $3:4:5$.

Assuming the angle $a$ at the mean diameter to be $70°$, those at the inner and outer diameters respectively will be $64° \ 15'$ and $74°$, and

$$cos\ 64^\circ\ 15' = 0\cdot4345$$
$$cos\ 70^\circ\ \ \ \ = 0\cdot3420$$
$$cos\ 74^\circ\ \ \ \ = 0\cdot2760$$

The distribution is then represented at the three diameters by

$$3 \times 0\cdot4345 = 1\cdot3035$$
$$4 \times 0\cdot342\ \ = 1\cdot368$$
$$5 \times 0\cdot276\ \ = 1\cdot380$$

the quantity of water passing through the wheel at the outside being very slightly greater than at the inside.

If, as is usual, the inflow into the buckets only takes place without shock at the mean diameter, then owing to the shock at other points the distribution will probably be somewhat disturbed, in a manner which it is impossible to accurately determine; but it is reasonable to assume that the relative velocity of inflow $c_1$ will be inversely proportional to the normal depth of the inflow orifices at various points, and with a constant velocity of flow $c$, a little consideration shows that the relative velocity of outflow $c_2$ must remain constant also, since *in any one of the narrow concentric rings* before referred to, $c_1$ is inversely proportional to $A_1$, and hence $A_1\, c_1 = $ constant.
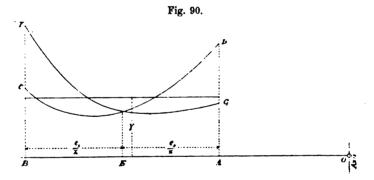
If this is not the case, then either a part of the water entering at one diameter is forced radially towards another or escapes through the clearance space between the guides and wheel.

When the angle $a_1$ is constructed at all points for entry without shock, the distribution of the water remains unchanged on leaving the guide passages.

It is of course evident that with a constant value of $c_2$, and with the angle $a_2$ very nearly the same as $a$, the distribution of the water on leaving the buckets is practically the same as on leaving the guides.

If, instead of helical vane surfaces, the angles $a$ and $a_2$ are constant for the whole width of the wheel, then the product $s \cos a$ (or $s_2 \cos a_2$) varies proportionately to the radius, and the quantity of water flowing through a given small width at the outer circumference is greater than at the inner; the distribution increases from the inside to the outside in the ratio of the diameters (or radii).

With the turbine of the dimensions before assumed, but with a constant angle $a$, the distribution of the water at the inner, mean, and outer diameters respectively would be as $3 : 4 : 5$.

Fig. 90.



In determining the loss from unutilized energy both the residual velocity and the distribution of the water must be taken into account. In any of the narrow rings into which the wheel is supposed to be divided, the total loss from unutilized energy is proportional to the square of the residual velocity $u$ and the quantity of water $q$ passing through that ring, that is, to the product $q \times u^2$.

If for every point of the width this product be determined, or a value proportional to it, and the results plotted as ordinates with the corresponding radii as abscissæ, a *curve is* obtained, and the area $A\ B\ C\ D$ (Fig. 90), of

which it forms one boundary, represents the total loss $\Sigma$ $(q\ u^2)$, and the *mean* ordinate $Y$ the average value of $u_m{}^2$, or average loss from unutilized energy per unit of weight.

When the distribution is uniform the ordinates of the curve are simply proportional to $u^2$ at each point, since $q$ is constant.

Under any given conditions as regards the vane angles, it is easy to find graphically the residual velocity at any point, and by multiplying this with a value proportional to the distribution at the same point, to determine the ordinates for the curve in question. It will be found convenient to take the quantity of water passing through the wheel at the mean diameter as *unity*.

For a helical vane surface the curve representing the unutilized energy has the form shown in Fig. 90 by the line $C\ D$, $O\ E$ being the mean radius, $O\ A$ and $O\ B$ respectively the inner and outer radius.

With a constant angle $a_2$, and consequent distribution proportional to the radius at any point, the curve of unutilized energy is shown by $F\ G$. The distribution of loss, it will be seen, differs very much in the two cases considered, but in each the total loss is very nearly the same.

The distribution of loss by shock over the width of a turbine might be determined by a method analogous to that followed in the case of the unutilized energy. For *uniform* distribution the loss *per unit of weight* of the fluid, on either side of the mean circumference, would be theoretically equal, so that the curve representing that loss would be symmetrical with respect to a vertical line through the middle of the width of a bucket, assuming that, as before, the abscissæ are measured parallel with the edge of a vane and the ordinates perpendicular to it.

In the absence of experimental data showing in what

way and to what extent theory and facts regarding loss by shock differ, it would be unsafe to draw any definite conclusions from an investigation of the kind indicated.

### *Comparison of Axial with Radial Turbines.*

An investigation of the relative merits of Axial as compared with Radial and Mixed-flow turbines when based on theoretical grounds is comparatively easy, but any attempt to form a judgment on this point from existing experimental results labours under disadvantages similar to those mentioned in discussing the effect of different proportions on the efficiency of turbines of the same class. The matter is if possible rendered more difficult by the fact that the manufacture of radial-flow turbines—more especially with inward flow—is confined almost entirely to America, where these motors are generally of the mixed-flow type, the water entering radially and leaving axially.

In dealing with the subject from a theoretical point of view, it will be assumed in the first instance that the co-efficients of resistance $\zeta_1$ and $\zeta_2$ are the same for radial as for axial turbines. In Table No. II. will be found the efficiencies of radial wheels for different ratios between the radii of inflow and outflow.

For an outward-flow turbine this ratio $\dfrac{r_1}{r_2}$ is *less*, for inward-flow *greater* than unity.

The expression $\dfrac{1}{g}\dfrac{A}{A_2}\dfrac{r_1}{r_2}\sin a\,\sin a_2$, which represents the work done on the wheel per unit of weight, is directly proportional to the ratio $\dfrac{r_1}{r_2}$, and consequently is greater for an inward-flow than for an outward-flow turbine. This

is explained by the fact that as a particle of water flows outwards in a radial wheel the successive parts of the vane surface with which it comes in contact have an increasing speed, and the deflection of the stream produced by any given curvature of the vane becomes less the greater the distance from the axis. With an inward-flow turbine the *reverse* of this is the case. *The same vane curve*—referred to a line normal to the direction of rotation—will cause a greater absolute deflection of the stream in an inward-flow wheel than in an axial wheel, and in the latter again a greater deflection than in an outward-flow turbine.

At different distances from the centre the speed with which, so to speak, the vane is *getting out of the way* of the water, varies, so that when the latter approaches the circumference that speed increases, and the obstruction to a straight course becomes less; when the water approaches the centre, the opposite occurs.

Provided the losses remain constant, the greater the work done per unit of velocity of flow the higher the efficiency of a turbine must be, hence *for the same values of* $\dfrac{A}{A_2}$, $a$ and $a_2$, the efficiency of a radial inward-flow wheel is greater than that of an axial turbine, and—apart from certain considerations to be afterwards discussed—increases with the ratio of the outer to the inner radius, or, for a given outside diameter, as the inner diameter diminishes. This is shown clearly in Table No. II.: with $\dfrac{A}{A_2} = 2$, $a_2 = 70°$, and $a = 75°$;

for $\dfrac{r_1}{r_2} = \frac{3}{2}$, the efficiency is $0·796$; for $\dfrac{r_1}{r_2} = \frac{5}{4}$, $\epsilon = 0·765$;

for $\dfrac{r_1}{r_2} = 1$, $\epsilon = 0·722$, and for $\dfrac{r_1}{r_2} = \frac{2}{3}$, $\epsilon = 0·634$.

A given angle of outflow $a_2$ in an inward-flow turbine is equivalent to a greater angle in a wheel with parallel

o

or outward-flow, because owing to the decreasing speed of the vane towards the centre a greater deflection of the stream is obtained.

Mathematically the highest efficiency would be attained by an inward-flow turbine with the inner radius *nil*. This of course is a practical impossibility, and it is easy to perceive that the reduction of the inner diameter must soon reach a limit. In the first place, after leaving the buckets the water has to be deflected through an angle of 90° degrees in order that it can flow off into the suction-tube or tail-race, and if this is too abruptly effected a loss of energy results which makes itself felt by retarding the flow. In the second place—apart from the above consideration—if the inner radius be too small, the converging streams leaving the buckets will interfere with each other, causing a disturbance of the water and choking the exit. In the third place, for a *constant width* of the buckets, the ratio $\dfrac{A}{A_2}$ varies as that of the *outer* to the *inner* radius, and if the latter is reduced, it becomes necessary to increase the width of the buckets at the outflow to maintain the same ratio $\dfrac{A}{A_2}$. This widening of the buckets cannot however be carried to any great length without loss, as it involves a change in the direction of flow of parts of the stream at right angles to its general course. On the other hand, if this be not done, and the ratio $\dfrac{A}{A_2}$ increased in proportion to $\dfrac{r_1}{r_2}$, then, as a reference to Table II. will show, the efficiency is diminished.

## Difference between Performance of Inward- and Outward-flow Turbines.

A radial outward-flow turbine, under given conditions, consumes more water than an inward-flow wheel of the same proportions and with the same angles of inflow and outflow. This is often ascribed to the action of centrifugal force, which in the one case is supposed to *accelerate* while in the other it *retards* the flow. That is an inaccurate way of explaining the phenomenon, and likely to lead to misunderstanding. The true explanation is, that owing to causes already stated, *less work is done on the wheel* by a given weight of water *per unit of velocity* in the outward-than in the inward-flow turbine, and consequently the fluid leaves the buckets of the former with a larger proportion of unutilized energy and correspondingly higher velocity. Centrifugal force, so called, cannot possibly produce any acceleration of flow, since the rotary motion of the fluid to which it is due is itself the result of the pressure caused by the head, and to credit the centrifugal force with increasing or reducing the velocity of the water is to confound cause with effect.

## Mixed-flow Turbines compared with Radial-flow Turbines.

In a mixed-flow turbine the two objections to a small inner diameter, previously noticed, are to some extent removed by continuing the buckets into that part of the wheel-casing where the water assumes a vertical course; the change in the flow from a horizontal to a vertical direction is thus made *during the passage through the*

*buckets*, and the exit of the water near the centre does not necessitate such an abrupt alteration of its course as when the wheel is of the purely radial type. Unfortunately these apparent advantages of the mixed-flow turbine are to a great extent neutralized by accompanying drawbacks; the chief of these is, that for a small mean outflow-radius $r_2$, except with extremely narrow buckets, the variation in the speed from the inner to the outer circumference of the points at which the water leaves the wheel, is so great as cause losses which cannot be materially mitigated by any modifications in the form of the vanes. The cause of these losses has already been explained in connection with axial turbines, but in the case of mixed-flow wheels they are more serious, owing to the greater ratio of the width of the buckets to the diameter of the wheel. Another point to be noticed as bearing upon the construction of inward-flow or mixed-flow turbines is, that where the difference between the inner and outer diameters is great, the mean angle $a_2$ cannot be made as large as when the diameters are more nearly equal, and this again reduces the efficiency.

Taking all these facts into account, it may be safely stated that there can be no advantage in mixed-flow turbines as generally constructed over a well-designed simple inward radial-flow wheel, with a moderate ratio of outer to inner radius. There are very few reliable experimental results available from which a comparison can be made between the various classes of turbines, and these show no marked difference between the efficiency of radial inward-flow, and axial- or parallel-flow wheels; radial outward-flow turbines are somewhat inferior to the latter. A large number of tests have been carried out in America, chiefly with mixed-flow turbines, but although useful for comparing these among themselves, there is reason to

believe that in some instances the co-efficients used for the calculation of the quantities of water consumed were too small, thus giving too high an efficiency. Many of the wheels in question of obviously irrational design are reported to have shown an efficiency only attained by very good European turbines.

In spite of this, however, a test carried out by Mr. Francis, a well-known American engineer and careful experimenter, with a "Humphrey" mixed-flow turbine, shows that an efficiency equal to that of the best Jonval turbines may be obtained by a motor of this class, although in the case in question the inner diameter, where the water leaves the wheel, is relatively to the outer diameter very small. The loss due to the deviation from the best speed towards the centre is up to a certain point compensated by the advantage arising from the difference in the diameters.

It is obvious that, as in the case of radial inward-flow wheels, to maintain a given ratio of $A$ to $A_2$, the less the mean outflow-radius $r_2$ the greater must be the width of the buckets, and therefore also the greater the divergence from the best speed, and a limit must soon be reached at which the values of $\dfrac{A}{A_2}$ and $a_2$ can no longer be chosen, but are restricted within a very narrow range.

As compared with axial or parallel-flow turbines, both inward-flow radial and mixed-flow wheels have a disadvantage in the fact that the water must change its direction of flow twice—once before entering the wheel from a vertical to a horizontal course, and again on leaving from the horizontal to the vertical.

Against this may be set the greater friction on the bearings of an axial turbine due to the vertical reaction

and pressure on the vanes, which tends to force the shaft downwards. In a mixed-flow turbine the downward reaction also exists, but is approximately balanced by the upward force exerted on the casing, arising from the deflection of the water in passing through the buckets, from a horizontal to a vertical direction.

As far as experience goes, it shows that advantages and disadvantages practically compensate each other in the different systems.

It seems not improbable that a mixed-flow turbine with comparatively narrow buckets, so that there should be no great variation from the best speed, and a moderately large ratio of outer to mean inner diameter, might give results considerably superior to those obtained with a parallel-flow turbine of otherwise similar proportions (that is, with the same values of $\frac{A}{A_2}$, $\sin a$ and $\sin a_2$).[1]

The question naturally presents itself to the practical man : Why have American engineers so generally adopted the mixed-flow type of turbine if there is no great advantage as regards efficiency to be derived from so doing ?

The explanation would appear to be partly given in the conditions under which water-power in the United States is generally available, namely, in the form of large rivers with an immense volume of water. Under such circumstances the power at the disposal of the engineer or manufacturer in any given case is vastly in excess of his requirements, and economy in consumption of water is not so much an object as low first cost of the motor. The aim of many turbine manufacturers has therefore been, not to produce an economical turbine, but one which, with a given outside diameter, will do the largest amount of

[1] Vide *American Turbines.*

work regardless of the quantity of water consumed per horse-power.

It is easy to ascertain what are the conditions necessary to obtain this result. The work done by a turbine can be expressed as—

$$W = Q \lambda \epsilon h = A c \lambda \epsilon h,$$

or substituting for $c$ its value from (via)

$$W = A \lambda \epsilon h \sqrt{\frac{2gh}{2\frac{r_1}{r_2}\frac{A}{A_2} \sin a \sin a_2 + \zeta_1 + \left(\frac{A}{A_2}\right)^2}}$$

$(\zeta_2 + \cos^2 a_2).$

Under given conditions the total work done is directly proportional to the velocity of flow $c$. Taking $r_1$ the outer radius (of an inward-flow wheel) and $A$ the area of the guide passages as fixed, it is clear that the *larger $A_2$* and the *smaller $\sin a$* and $\sin a_2$, the greater will be the value of $c$. On the other hand, as may be seen by reference to Table II., the efficiency decreases as $\sin a$ and $\sin a_2$ are reduced, and beyond a certain point this applies also to $A_2$. The *diminution of the efficiency*, however, is *less rapid* than the *increase* of the velocity of flow, so that, within certain limits, by sacrificing efficiency the total work obtained from a wheel of a given diameter and with a given area of the guide passages may be increased. The mixed-flow type of turbine appears to lend itself most readily to this purpose, more particularly to enlarging the area of exit $A_2$ without increasing the outer diameter at any part.

The preceding remarks are not intended to apply to the *best* American wheels, which have undoubtedly been shown by experiment to have a distinct advantage as

regards efficiency over the best Jonval wheels. This advantage, however, is chiefly due, not to the *mixed* flow, but to the fact that the flow is radially *inward;* with similar proportions a purely radial-flow wheel would probably have an equally high efficiency.

## On the Use of the Suction-Tube and Boyden Diffusor.

It has been previously demonstrated that, in addition to its primary function of enabling the turbine to be placed at any convenient height above the tail-water level, a suction-tube may also improve the efficiency of the motor by relatively reducing the velocity with which the water finally leaves the apparatus, as the loss from unutilized energy depends on this.

Suppose, in the first instance, a given wheel is used without a suction-tube; the water leaves the buckets with a certain absolute residual velocity $u$, which is entirely lost. If now a suction-tube be added, so constructed that from the moment the water leaves the wheel to that at which it finally issues from the tube there is a gradual enlargement of the area of the stream, the velocity after leaving the buckets will be gradually reduced from $u$ to $c_4$ in the inverse ratio of the sectional areas. The velocity $u$, however, after the suction-tube has been added, will not be the same as before, but *greater;* the gradual enlargement *increases the velocity of flow c*, and improves the efficiency by making the residual velocity less in proportion to $c$ than it was previously. Of course the increase in the velocity of flow requires a correspondingly greater *best speed.*

When *no* suction-tube is used

$$c^2 = \frac{2gh}{2\frac{A}{A_2}\frac{r_1}{r_2}\sin a \sin a_2 + \zeta_1 + (\zeta_2 + \cos^2 a_2)\left(\frac{A}{A_2}\right)^2}$$

when a suction-tube is used

$$c^2 = \frac{2gh}{2\frac{A}{A_2}\frac{r_1}{A_2}\sin a \sin a_2 + \zeta_1 + \zeta_2\left(\frac{A}{A_2}\right)^2 + \left(\frac{A}{A_4}\right)^2},$$

where $A_4$ is the area of the outlet from the tube. In the latter expression $\left(\frac{A}{A_4}\right)^2$ takes the place of $\left(\frac{A}{A_2}\right)^2 \cos^2 a_2$, all other quantities remaining the same; if, therefore, $\left(\frac{A}{A_4}\right)^2$ be less than $\left(\frac{A}{A_2}\right)^2 \cos^2 a_2$, then the velocity of flow will be increased by the use of a suction-tube.

The effect on the efficiency can be calculated from the formula

$$\epsilon = \frac{2\frac{A}{A_2}\frac{r_1}{r_2}\sin a \sin a_2}{2\frac{A}{A_2}\frac{r_1}{r_2}\sin a \sin a_2 + \zeta_1 + \zeta_2\left(\frac{A}{A_2}\right)^2 + \left(\frac{A}{A_4}\right)^2}$$

and compared with that obtained without a suction-tube as given by

$$\epsilon = \frac{2\frac{A}{A_2}\frac{r_1}{r_2}\sin a \sin a_2}{2\frac{A}{A_2}\frac{r_1}{r_2}\sin a \sin a_2 + \zeta_1 + \zeta_2\left(\frac{A}{A_2}\right)^2 + \left(\frac{A}{A_2}\right)^2 \cos^2 a_2}$$

Expressed in terms of the velocity of flow

$$\epsilon = \frac{c^2 \, 2\frac{A}{A_2}\frac{r_1}{r_2}\sin a \sin a_2}{2gh}$$

from which it is clear that the improvement in efficiency

will be exactly proportional to the square of the increased velocity $c$. In the preceding formula for the efficiency with suction-tube, it has been assumed that the additional friction and resistance caused by the tube is negligible.

As an example, assume $\dfrac{A}{A_2} = 1$, $\dfrac{r_1}{r_2} = 1$, $a = 65°$ and $a_2 = 65°$, then without a suction-tube

$$\epsilon = 0\text{·}765;$$

with a suction-tube having an outflow area four times as great as the area $A$ of the guide passages, so that $\dfrac{A}{A_4} = \tfrac{1}{4}$, the efficiency would be

$$\epsilon = 0\text{·}809,$$
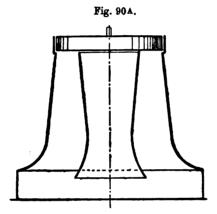
an improvement of 4·4 per cent.

Where the relative out-flow angle $a_2$ is large, say 75° to 80°, there is not much to be gained by a gradually enlarged suction-tube area, as the absolute velocity of the water on leaving the wheel is already low. The action of the tube is, however, better when thus formed than when of equal sectional area throughout, as the column of water in it is less liable to pulsations arising from the presence of air, which causes irregularities in the running of the wheel. If, as is generally the case, the sectional area of the suction-tube immediately below the wheel is much in excess of that of the stream leaving the buckets and is the same as the outflow area $A_4$, there is theoretically no gain of efficiency secured by its use. For a parallel-flow turbine a form of suction-tube similar to that indicated in Fig. 90A would probably be best.

It has been shown that the less the angle $a_2$ the smaller the width of the buckets for a given quantity of water, but at the same time the lower the efficiency. It may in some cases be convenient, when a suction-tube is used, to keep down the size of the wheel in the manner indicated,
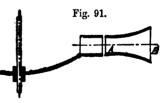
and compensate for the loss of efficiency by giving appropriate dimensions to the tube.

The so-called Boyden diffusor is an arrangement designed for use with radial outward-flow wheels, solely to improve their efficiency in a manner similar to that in

Fig. 90A.

which this is effected by an enlarged suction-tube. It has the form shown in Fig. 91, and consists of a circular casing, concentric with the wheel, having the cross-section (through the axis), represented at *A B*, of a bell-mouthed shape. It forms, so to speak, a continuation of the wheel-casing, but is stationary; at *A* the area of the inner orifice is the same as that of the wheel measured over the circumference, while at *B* it is greater. The effect is exactly similar to that of a gradually enlarged suction-tube, and can be calculated in the same way; for $A_4$, the
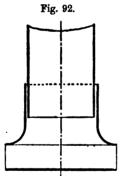
Fig. 91.

area at *B* through which the water leaves the diffusor must be substituted in the formulas.  It has been used to a considerable extent in America to improve the efficiency of Fourneyron turbines; a corresponding apparatus might, where necessary, be applied to Jonval wheels working without suction-tube in the tail-water, consisting of a short bell-mouthed tube below the wheel, but in many cases there would be no room for this without alterations to the tail-race.  For a well-designed turbine appliances of this kind are, generally speaking, unnecessary, or the improvement effected by them is not worth the increased first cost, but experimental results have proved their efficacy in many instances where the residual velocity of the water leaving the wheel was too high.

In designing the suction-tube of a turbine care should always be taken to round off the corners of the sluice-gate, or, better still, curve the lower end as shown in Fig. 92; sharp corners and sudden turns much increase the hydraulic resistance, and reduce the efficiency.

Fig. 92.



Another point of great practical importance when a suction-tube is employed, is the provision of an air escape-cock at the upper end, as the presence of air interferes with the proper action of the tube; it is also essential that the sluice should entirely and effectually close the outflow orifice when required, so that the tube before starting may be *quite filled* with water.  Owing to a neglect of these precautions, in some cases suction-tubes have given unsatisfactory results, and fallen into disrepute with many makers.

### *Loss in enlarged Suction-Tubes and Diffusors.*

It has been thus far assumed that in flowing through a gradually enlarged tube or diffusor the water suffers no loss of energy other than that due to the friction of the walls; this is, however, not strictly correct. When water flows out of a bell-mouthed or inverted conical tube, under a certain head measured to the centre of the orifice, the velocity of flow from the latter is *theoretically* the same whatever may be the form of the tube or mouthpiece. If the area of the outflow orifice is, for instance, four times as great as that of the smallest part of the tube, the velocity of flow through this latter must—if continuity is preserved —also be four times as high as in the orifice.

It is clear, however, that there must be an absolute limit to the velocity of flow, since this depends primarily on differences of pressure, and with a given head attains its maximum when the pressure at the corresponding point is *nil*, that is, with a vacuum in the smallest section of the tube.

If, therefore, the ratio between the area of the orifice and that of the smallest section is so great that the maximum possible velocity of flow would be exceeded if continuity were maintained, the assumed conditions no longer hold good; the water, instead of filling out the mouthpiece, flows through it as an independent stream surrounded by *dead* water, and the case is practically similar to that of a sudden enlargement of the area of the tube, and is accompanied by a corresponding loss of energy.

This is an extreme instance, but it has been found by experiment, that unless the enlargement of the tube is very gradual and the velocity of flow small, a very con-

siderable loss results from the decrease of the latter, and this loss may become as great as, or greater than that which would be theoretically due to a *sudden* change from the least to the greatest sectional area of the tube.

The effect of losses of energy of this kind on the action of a Boyden diffusor or enlarged suction-tube is to reduce the advantage obtained by their use.

Francis found by experiment that the efficiency of a Fourneyron turbine was increased 3 per cent. by the addition of a Boyden diffusor, theoretically the advantage should have been 4 per cent.

Venturi, who was among the first to make accurate observations on the discharge of water through a diverging tube, found that with an angle of divergence of $4°\ 27'$ the velocity through the smallest section was increased in the ratio of 2 to 1 as compared with the discharge through the same orifice when no tube was employed. According to theory (Bernoulli's), the increase should have been in the proportion of 3·03 to 1, showing a very considerable loss of energy, proportional to $(3\text{·}03)^2 - (2)^2$ or $9\text{·}18 - 4$, over 50 per cent.

Experiments carried out by Francis on the same subject demonstrated that the greater the ratio of the final discharge orifice to the smallest section, the greater the loss. Fliegner's investigations, subsequently to be described, confirm this result, and also prove that beyond a certain point a greater head (and consequently increased velocity) is accompanied by increased loss.

Francis noted that the losses occurred in spite of the tubes being quite full and the theoretical conditions apparently complied with. The fact of a tube being full, however, does not necessarily indicate true continuity of flow, since, as will be explained in describing Fliegner's experiments, a part of the water round the sides of the

tube may be in a relatively nearly stationary eddying condition, the true form of the discharging jet not being defined by the walls of the tube.

A very interesting series of experiments, made by Professor Fliegner of Zürich, on the outflow of water through submerged inverted conical mouthpieces, shows that in every case investigated by him a loss of energy took place over and above that due to friction. The experiments were carried out with mouthpieces of various proportions and under different pressures. The results proved that with increasing pressure (or velocity) the loss of energy

Fig. 93.



decreased up to a certain point and then became greater, so that there appeared to be in every instance a minimum resistance at some particular velocity of flow.

Professor Fliegner is of opinion, that when the natural angle of the conical jet of water, issuing from the smallest section, is less than that of the mouthpiece, the conditions of continuity of flow are not completely fulfilled; the jet follows, so to speak, an independent course, its surface not being in contact with the walls of the mouthpiece, *vide* Fig. 93. The effect is then the same as if the tube were abruptly enlarged in sectional area, and the loss is as great or even greater. ·

True continuity of flow is only possible when the natural angle of the fluid jet is greater than that of the mouth-

piece, so that the surface of the former comes into contact with the walls of the latter.

Between the assumed theoretical conditions with no loss and abrupt change of flow, various proportions of loss occur, but in no case does the proportion appear to be less than ⅛th of the theoretical loss due to a sudden change from the maximum to the minimum velocity of flow.

If $A_3$ and $A_4$ denote respectively the least and greatest sectional areas of a conically divergent or bell-mouthed tube, $p_3$ and $p_4$ the corresponding pressures, and $c_3$ and $c_4$ the velocities of flow, then when there is no loss—

$$\frac{c_3^2}{2g} + p_3 = \frac{c_4^2}{2g} + p_4;$$

or since

$$c_3 = \frac{A_4 \, c_4}{A_3}$$

$$[\left(\frac{A_4}{A_3}\right)^2 - 1]\frac{c_4^2}{2g} = p_4 - p_3.$$

$p_4$ the pressure in the orifice is generally given, $c_4$ is determined by the effective head $h$—

$$c_4 = \sqrt{2gh}.$$

The smallest possible value of $p_3$ is $O$, and the maximum possible velocity of flow $c_3$ in the smallest section of the tube

$$max \; c_3 = \sqrt{2g \, (h + a)}$$

(where $a = 33\cdot91$ feet, the column of water corresponding to atmospheric pressure).

Assume for example $h = 30\cdot09$ feet, then

$$max \; c_3 = 8\cdot025 \sqrt{64} = 64\cdot200 \text{ feet per second,}$$

and

$$c_4 = 8\cdot025 \sqrt{30\cdot09} = 43\cdot973.$$

In this case the greatest ratio of $A_4$ to $A_3$ with which continuity of flow is theoretically possible is

$$max \; \frac{A_4}{A_3} = \frac{max \; c_3}{c_4} = \frac{64\cdot20}{43\cdot97} = 1\cdot46;$$

with a smaller value of $h$ it would be greater; for instance, with $h = 1$,

$$c_3 = 47 \cdot 476 \text{ and } c_4 = 8 \cdot 025, \text{ whence } max \frac{A_4}{A_3} = 5 \cdot 916.$$

When, as is actually the case, a loss of energy occurs in the conically divergent tube or mouthpiece, the hydro-dynamic equation takes the form

$$\frac{c_3^2}{2g} + p_3 = \frac{c_4^2}{2g}(1 + \zeta_4) + p_4,$$

or

$$\left[\left(\frac{A_4}{A_3}\right)^2 - (1 + \zeta_4)\right]\frac{c_4^2}{2g} = p_4 - p_3,$$

where $\zeta_4$ represents the proportion of the energy lost to that due to the velocity $c_4$, the total loss of head in question being $\zeta_4 \dfrac{c_4^2}{2g}$.

In calculating the efficiency of a turbine using an enlarged suction-tube or Boyden diffusor, in the formula previously employed, it would be necessary to introduce, instead of merely the ratio $\left(\dfrac{A_2}{A_4}\right)^2$, the expression $(1 + \zeta_4)$ $\left(\dfrac{A_2}{A_4}\right)^2$.

## *Practical Limits for Position of Wheel in Suction-Tube.*

Theoretically the limit of height $h_2$ at which a turbine may be placed above the tail-water level in the suction-tube is

$$h_2 = \overset{\text{Feet.}}{33 \cdot 91} - \frac{c_4^2}{2g},$$

where $c_4$ is the velocity with which the water leaves the tube.

P

As $c_4$ bears some proportion, varying generally from $\frac{1}{4}$ to $\frac{1}{6}$, to the theoretical velocity of flow $\sqrt{2gh}$, the theoretically admissible height of suspension or suction $h_2$ will vary with the head $h$.

In practice, however, with the usual cylindrical suction-tube, the height of suspension for various diameters of tube should not exceed the values given in the following table, which is taken from Meissner's work.

| Diameter of suction-tube. Feet. | Admissible height of suspension. Feet. | Diameter of suction-tube. Feet. | Admissible height of suspension. Feet. |
|---|---|---|---|
| 13 | 9·84 | 3·3 | 26·24 |
| 11·5 | 11·15 | 2·3 | 27·88 |
| 9·8 | 12·46 | 1·6 | 27·88 |
| 8·2 | 13·77 | 0·98 | 29·52 |
| 6·5 | 14·76 | 0·49 | 31·16 |
| 4·9 | 19·68 | | |

From the above heights the value of $\dfrac{u^2}{2g}$ must be deducted when the head is greater than 33 feet. If the suction-tube, instead of being cylindrical, is gradually enlarged towards the outflow orifice, as previously explained, the suction height $h_2$ may be greater than indicated in the table, chiefly because with that form the column of water is less liable to pulsations than when the tube is cylindrical.

# CHAPTER VII.

## IMPULSE TURBINES.

THE distinction between reaction and impulse turbines has already been more than once referred to. The essential point in connection with impulse turbines is the principle of *free deviation*, that is, freedom for the water after leaving the guide passages to follow in one direction its own course, and during its transit through the wheel to be at all times under atmospheric pressure only. To insure this result, the buckets of an impulse turbine should never be *filled* by the water, but must be so constructed as to allow free access to the atmosphere. To attain this end with greater certainty, Girard introduced the ventilated buckets shown

in Fig. 94; with these, apertures *c c* are made in the sides of the wheel communicating with each bucket *b* to admit the air, the apertures being so placed that they can never be obstructed by the water.

It has been shown that to act to the best advantage a reaction turbine must work with every portion full of water, and consequently the dimensions of the wheel are to a great extent dependent on the quantity of water the motor is intended to utilize, and when the quantity is very small the area of the guide and wheel passages assume inconveniently or even impossibly minute dimensions, and to obtain the necessary speed at the circumference a very high number of revolutions per minute is required. An impulse turbine is subject to no such restrictions; the water may be admitted to all the buckets round the circumference of the wheel or to only one or two, and the efficiency is not materially affected thereby. The dimensions of an impulse turbine may therefore be chosen to suit practical convenience within very wide limits, more especially as regards the diameter, so that for very high falls and small quantities of water comparatively large wheels running at a moderate number of revolutions can be employed. As for a given fall the quantity of water does not influence the efficiency, impulse turbines can be regulated to any desired extent by closing the guide passages.



Fig. 94.

Owing to these advantages, *Girard* turbines, as impulse wheels are now very generally called, have to a great extent superseded reaction turbines on the continent of Europe, where, in a large proportion of cases in which

water power is available, the *quantity* is variable, and a motor is required which will better adapt itself to changeable conditions than the reaction turbine.

Girard turbines are now to be found of almost all possible varieties, axial, inward-flow, outward-flow, with vertical, horizontal, and inclined axes, and utilizing falls ranging from 4 to 1,800 feet, and quantities of water from 140 to 0·04 cubic feet per second.

## *The Theory of Impulse Turbines.*

The fundamental difference between the theory of reaction and impulse turbines lies in the fact that the condition of continuity of flow in all parts does not apply to the latter, and therefore there is not a constant ratio between the velocities of flow in any two parts of a given impulse turbine.

The general equation (1) applies—as previously stated—equally to reaction and impulse turbines:

Available energy = useful work + work lost on friction, &c., + energy remaining in water due to residual velocity, but owing to the fundamental difference above mentioned, it leads when applied in detail to different results in the two cases.

## *Velocity of Flow.*

The velocity of flow of the water from the guide passages is for impulse turbines always that due to the head less the equivalent of losses arising from friction and other hydraulic resistances; these latter in any given case are a fixed quantity, and hence the velocity of flow

$$c = K \sqrt{2gh_1},$$

where $K$ is a constant co-efficient for any particular conditions. Hence for a certain fall the velocity of flow remains the same whatever the speed of the wheel may be, and is independent of the angles of the wheel vanes or the area of the passages between them.

The head $h_1$ to be used in calculating $c$ is of course the effective head measured from the orifices of the guide passages, which is always somewhat less than the total head, as the impulse wheel is placed above the water. As the water issues from the guides and leaves the wheel at atmospheric pressure, the available energy on entering the wheel is represented by the velocity of flow $c$, and is all in the *kinetic form,* while in a reaction turbine, in most instances, a considerable part of the energy available before the water enters the wheel buckets exists in the *potential* form as pressure; hence for an impulse wheel

(I) $$E_w = \frac{c^2}{2g}$$

and

(II) $2gh_1 = (1 + \zeta_1)\, c^2$, whence $E_w = h_1 - \zeta_1 \dfrac{c^2}{2g}$ (I$a$)

where $\zeta_1$ is a co-efficient giving the proportion of energy lost by friction, &c., before the water leaves the guide passages.

Between $K$ and $\zeta_1$ the following relation exists—

$$K^2 = \frac{1}{1 + \zeta_1}.$$

The independence of $c$ of the proportions of the motors very much simplifies the theory of impulse as compared with reaction wheels.
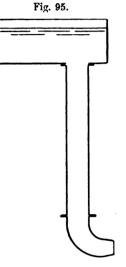
In calculating $c$ of course all the special circumstances of the case must be taken into account in estimating the losses which reduce the effective head for producing

velocity. If, for instance—as very frequently happens with high falls and small quanti-

Fig. 95.

ties of water—the water has to be conveyed through a long pipe from the collecting tank or reservoir to the wheel, *vide* Fig. 95, the loss from friction in this pipe must be estimated by the well-known formula. If, again, the fluid passes through a bend on its way to the wheel, the resulting loss should be calculated. Where the fall is low or moderate, so that the water flows directly from a large tank or reservoir into the guide buckets, the only appreciable loss is that occurring in the guide passages, to which the co-efficient $\zeta_1$ is applicable. As a result of long experience for properly formed guide vanes, the value of $\zeta_1$ is found to range from 0·234 to 0·018, or

$$K_1 = \sqrt{\frac{1}{1 + \zeta_1}} = 0\text{·}9 \text{ to } 0\text{·}95 \text{ ; whence}$$

$$c = 0\text{·}9 \text{ to } 0\text{·}95\sqrt{2gh_1}.$$

## *Useful Work.*

The work done on the vanes of an impulse turbine is expressed in precisely the same manner, and is due to the same causes, as in the case of a reaction turbine; the formula applicable is—

(8) $\qquad W = M \left[ w_2 \left( c_2 \sin a_2 - w_2 \right) + w_1 c \sin a \right]$

*p.* 5%

for radial turbines, which for axial turbines becomes

$$W = M \left[ w \left( c_2 \sin a_2 - w + c \sin a \right) \right].$$

When the entrance of the water takes place without impact, it has been proved that

$$W = w \left( c_2 \sin a_2 - c_1 \sin a_1 \right) M$$

for axial (or parallel-flow) turbines, and

(8a)    $W = \left( w_2 c_2 \sin a_2 - w_1 c_1 \sin a_1 + w_1^2 - w_2^2 \right) M$

for radial turbines.

In equation (8) the quantity $c$ can be calculated at once when the effective head is given, but $c_2$ the relative velocity of outflow cannot be directly determined from the proportions of the motor, and so long as the entry of the water is accompanied by shock it is not possible to ascertain it with any accuracy. For constructive purposes, however, the water must be assumed to enter *without* shock, and in that case the formula (8a) can be applied for the useful work done.

The available energy of the water on entering the wheel is

$$E_w = \frac{c^2}{2g} \text{ (per unit of weight)},$$

during its passage through a parallel-flow wheel with vertical axis or a radial wheel with horizontal axis, as shown in Figs. 96 and 97, a further accession of energy takes place, due to the fall $h_o$ corresponding to the depth of the buckets, which results in an acceleration of the relative velocity of the water during its passage through the wheel; the total available energy for the latter is therefore

$$\frac{c^2}{2g} + h_o.$$

This is expended in doing useful work, overcoming frictional resistance in the buckets, and producing the residual absolute velocity of outflow $u$, or—otherwise

expressed—the useful work done is equal to the available energy $E_w$ less the loss from friction (and other hydraulic resistances) and the energy due to the residual velocity or unutilized energy.
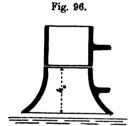
The loss in the buckets may be expressed by
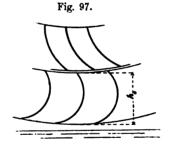
$$\zeta_2 \frac{c_2^2}{2g};$$

and hence

(III) $$E_w - \zeta_2 \frac{c_2^2}{2g} - \frac{u^2}{2g} = W_u.$$

From this equation it is possible, by substituting the detail values of the various quantities, to ascertain the relation

Fig. 96.                                    Fig. 97.



between $c_2$ the relative velocity of outflow and other known values. It may be noted that the method is exactly similar in principle to that employed to calculate the velocity of flow in the case of reaction turbines, only the quantity now required is not the absolute velocity of flow $c$, but the relative velocity $c_2$. As will be subsequently explained, the determination of the latter is of importance for the design of impulse wheels.

When the water enters the buckets without shock (by $C$)

$$c^2 = c_1^2 + w_1^2 - 2 c_1 w_1 \sin a_1,$$

and

$$u^2 = c_2^2 + w_2^2 - 2 c_2 w_2 \sin a_2.$$

If, for the present, it be assumed that $h_o = 0$ (as, for instance, with a radial wheel with vertical axis), then

$$E_{ir} = \frac{c^2}{2g} = \frac{1}{2g}\,(c_1{}^2 + w_1{}^2 - 2\,c_1\,w_1\,sin\,a_1).$$

## *Relative Velocities of Flow.*

By substituting in (III) the values above given, and simplifying and transposing the equation, the following relation between the relative velocities $c_1$ and $c_2$ and the velocities of rotation $w_1$ and $w_2$ is arrived at—

(IV)        $(1 + \zeta_2)\,c_2{}^2 - c_1{}^2 = w_2{}^2 - w_1{}^2.$

The same result can be arrived at by making $p_1 = p_2$ in formula (iii), p. 127, for reaction turbines, that is, when the assumption is made that the pressure in the buckets is constant. In this equation every quantity except $c_2$ is known. The relative velocity with which the water enters the buckets can be ascertained graphically in the now familiar way, or algebraically, precisely in the same manner as for a reaction turbine, on the assumption that the fluid enters without shock.

$w_2$ and $w_1$ the speeds at the circumferences are given in any particular case.

Neglecting, for the moment, the friction in the buckets, equation (IV) becomes

$$c_2{}^2 - c_1{}^2 = w_2{}^2 - w_1{}^2.$$

For an *axial turbine* $w_2 = w_1$, and consequently $c_2 = c_1$, that is, the relative velocity of flow remains constant during the passage of the water through the buckets.

This result it was easy to foresee without the use of mathematics. In an axial wheel every part of a vane curve moves with the same speed, and as, after entering the bucket, no accelerating or retarding force (apart from

friction and the depth of the wheel $h_o$) acts on the water, the relative velocity must necessarily remain the same throughout.

The effect of a varying speed of the vane on the relative velocity is not so easy to follow, but it will be clear on consideration that in this case the relative velocity must change as the fluid traverses a vane and approaches or recedes from the centre, as the case may be. This change is not the result of an accelerating or retarding force acting on the water, but merely of the fact that the latter passes over portions of the wheel moving with increasing or decreasing velocity in which the fluid does not participate.

The following extreme case will best illustrate this. Assume that the relative motion of the water in a radial outward-flow wheel is such that the absolute path is a radial straight line $A$ $B$, Fig. 98, and the absolute velocity—under these conditions constant—represented by $A$ $C$. In this case no work is done, and the velocity with which the water leaves the wheel is the same as that with which it enters. The *relative* velocity of entrance $c_1$ is found by the well-known parallelogram $A$ $E$ $C$ $C_1$, in which $A$ $E = w_1$ the velocity of the circumference where the water enters, and $A$ $C_1 = c_1$.

Similarly the relative velocity of outflow $c_2$ is given by $B$ $D_1$ in the parallelogram $B$ $F$ $D$ $D_1$, in which $B$ $F = w_2$ the velocity of the circumference where the water leaves
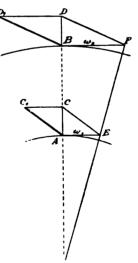
Fig. 98.
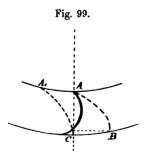
the wheel. It is obvious that in this instance $B D_1$ is greater than $A C_1$, since $B F$ is greater than $A E$, that is, an increase in the relative velocity has taken place.

From equation (IV) it may be seen that when $w_2$ is greater than $w_1$, that is, for an outward-flow wheel, the relative velocity is increased during the passage of the water through the buckets; on the other hand, for an inward-flow wheel, where $w_2$ is less than $w_1$, the relative velocity is reduced.

When in traversing the vanes the water has an additional fall $h_o$ (as before explained), the relative velocity is increased by the amount due to this height reduced to some extent by friction. The complete formula for calculating $c_2$ under these conditions is—

$$(\text{IV}a) \qquad c_2^2 = (c_1^2 + w_2^2 - w_1^2 + 2gh_o)\,\frac{1}{1 + \zeta_2}.$$

It must be noted, that in the case of a radial wheel with horizontal axis, $h_o$ is not necessarily equal to the full depth of the buckets, but is the vertical distance through which the water falls from the moment it enters to that at which it leaves the wheel.

This will be clear from Fig. 99, in which $A B$ represents the absolute path of the water in traversing an outward-flow radial wheel; the point $B$ at which the water leaves the vanes is considerably above the lowest point $C$. It is easy, however, to avoid this by placing the inlet so that the outflow takes place at the lowest point, for instance at $A_1$, $A_1 C$ being the absolute path.

Fig. 99.

This construction is generally only adopted where the quantity of water is so small that it can all be admitted

through two or three guide passages at the lowest part of the turbine.

For a parallel-flow turbine, where $w_2 = w_1$, equation (IV) becomes simply

$$(\text{IV}b) \qquad c_2^2 = (c_1^2 + 2gh_o) \frac{1}{1 + \zeta_2}.$$

## *Efficiency.*

The hydraulic efficiency of an impulse turbine is expressed in the first instance in precisely the same way as that of a reaction wheel. The useful work done is by (8), p. 58:

$$W = M \left[ w_2 \left( c_2 \sin a_2 - w_2 \right) + w_1 c \sin a \right] ;$$

the total available energy

$$E = Mgh,$$

whence the efficiency

$$\epsilon = \frac{W}{E} = \frac{w_2 \left( c_2 \sin a_2 - w_2 \right) + w_1 c \sin a}{gh},$$

or since

$$w_1 = \frac{r_1}{r_2} w_2,$$

$$(\text{V}) \qquad \epsilon = \frac{w_2 \left( c_2 \sin a_2 - w_2 + c \frac{r_1}{r_2} \sin a \right)}{gh}.$$

On the assumption that the velocities $c_2$ and $c$ are constant—independent of the velocity of rotation—the best speed is the same as for a reaction wheel—

$$w_m = \tfrac{1}{2} \left( c_2 \sin a_2 + \frac{r_1}{r_2} c \sin a \right).$$

Here again, however, the formula is incorrect, although somewhat nearer the truth than in the former case, as at any rate $c$ is constant.

The relative velocity of outflow $c_2$ varies with the speed

as a result of the impact or shock which occurs at all speeds except the best, and it is impossible in general to calculate it with any certainty.

Assuming, as previously, entry without shock and radial or axial outflow, there follows for the efficiency

$$\text{(VI)} \quad \epsilon = \frac{w_1 \, c \, \sin a}{gh} = \frac{w_2 \, c \, \dfrac{r_1}{r_2} \sin a}{gh} = \frac{c \, c_2 \, \dfrac{r_1}{r_2} \sin a \, \sin a_2}{gh}.$$

If, instead of comparing the work done with the *whole available energy*, it is compared only with the *energy available for the wheel*, represented by the velocity of flow $c$, and the efficiency thus measured be denoted by $\epsilon_1$, then

$$\text{(VII)} \qquad \epsilon_1 = \frac{w_1 \, c \, \sin a}{\dfrac{c^2}{2}} = \frac{2 \, w_1 \, \sin a}{c}.$$

By (*A*) and (*B*)—for entrance without shock—

$$\frac{w_1}{c} = \frac{\sin (a - a_1)}{\cos a_1};$$

and this substituted in (VII) gives

$$\text{(VIII)} \qquad \epsilon_1 = \frac{2 \, \sin a \, \sin (a - a_1)}{\cos a_1}.$$

As the losses incurred *previous* to the water entering the wheel are entirely independent of what takes place *afterwards*, the expression $\epsilon_1$ really represents the efficiency of the wheel as distinguished from that of the supply-pipe or channel and guide apparatus. From this it will be seen that the maximum efficiency can be expressed entirely in terms of the two angles $a$ and $a_1$, provided always that the conditions of entry without shock and radial exit are fulfilled. The equations (VII) and (VIII) apply only when after entering the wheel the water is subject to no further fall $h_o$ during its passage through the buckets. Denoting by $h_2$ Fig. 100, the mean height of suspension of

a radial wheel with vertical axis, or the height of the lower edge of the vanes of an axial wheel above the tail-water level, then the losses incurred which are independent of the proportions of the wheel are (1) those due to friction and other hydraulic resistances in the supply-pipe and guide apparatus, already considered, and (2) the loss arising from the unutilized portion of the head $h_2$ necessitated by the condition that the turbine must run free of the water. In the latter, particular reaction turbines have an advantage, as owing to their working drowned the whole head is utilized.

Fig. 100.



The first loss is expressed by

$$(IX) \qquad L_1 = \zeta_1 \frac{c^2}{2g} \, ;$$

the second by
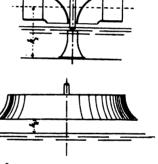
$$(X) \qquad L_2 = h_2.$$

For an axial wheel with vertical axis or a radial outward-flow wheel with horizontal axis of the kind previously referred to on page 216—

$$h - h_2 = h_1 + h_o \, ;$$

by (II) the available energy on entering the wheel

$$E_w = h_1 - \zeta_1 \frac{c^2}{2g},$$

and to this must be added $h_o$ the fall in the buckets; hence the total energy available for the wheel,

$$(XI) \quad E_w^{\ 1} = h_1 + h_o - \zeta_1 \frac{c^2}{2g} = h - h_2 - \zeta_1 \frac{c^2}{2g},$$

and the efficiency of the apparatus *apart from the wheel*

$$\text{(XII)} \qquad \epsilon_2 = \frac{h - h_2 - \zeta_1 \dfrac{c^2}{2g}}{h},$$

or where $h_o = 0$,

$$\epsilon_2 = \frac{h_1 - \zeta_1 \dfrac{c^2}{2g}}{h}.$$

The product of $\epsilon_1$ and $\epsilon_2$ equals the total efficiency $\epsilon$—
$$\text{(XIII)} \qquad \epsilon_1 \epsilon_2 = \epsilon.$$

In dealing with the design of impulse turbines the same assumptions are made with respect to the maximum efficiency of the wheel as for reaction turbines, namely, that the highest efficiency will result from a combination of inflow without impact and radial outflow. In order that these conditions may be complied with, certain relations must exist between the speed of the wheel, velocity of flow, and angles of the vanes.

The formulas for impulse turbines have already been stated in the simplified form resulting from the assumptions in question, and can be applied directly to the calculation of the dimensions.

It will be convenient to consider first the conditions governing the design of parallel-flow or axial impulse wheels, and subsequently how these are modified in the case of a radial turbine.

## Axial Impulse Turbines.

In the first instance it will be assumed, that the turbine the dimensions of which are required is to be so arranged that the water enters over the whole circumference.

The guide passages of an impulse turbine are in all respects like those of a reaction turbine, and must of course be entirely filled with water, but as soon as the streams issuing from the guide passages enter the wheel-buckets there should be an excess of sectional area in the latter, so that from the very commencement of its course in the wheel the water may be freely accessible to the atmosphere.

Since the guide passages are quite full, their sectional area $A$, where the water leaves them, is determined as soon as the quantity of water $Q$ and velocity of flow $c$ are known.

The velocity of flow can be calculated from formula (II) when the various resistances are known, and these can be determined with sufficient accuracy from the dimensions of the conducting pipes and guide apparatus with the help of the various empirical co-efficients applicable to each case.

The orifices of the guide passages are obstructed by the edges of the wheel-vanes moving in front of them, and to compensate for this the area of the passages, as obtained by dividing the quantity of water $Q$ by the velocity $c$, must be increased by multiplying with a certain experimental co-efficient, or (what comes to the same thing and is the usual practice) using a value of $c$ smaller than the true one to calculate $A$. If, after deducting all losses previous to entering the guide passages, $H$ represents the effective head, the actual velocity of flow for well-made guide vanes will be

$$c = 0.95\sqrt{2gH}; \qquad (A)$$

to determine the area $A$, however, a value of $c$ represented by

$$c^1 = 0.85\sqrt{2gH}$$

must be employed, equivalent to multiplying the true value by 0·894, so that

Q

$$A = \frac{Q}{0\cdot894\ c} = \frac{Q}{0\cdot85\ \sqrt{2gH}}. \qquad (B)$$

Retaining the same notation as for reaction turbines

$$A = (2\ \Pi\ r\ \cos a\ -\ z\ t)\ e, \qquad (C)$$

and from this, when $a$, $r$, $z$ and $t$ have been fixed upon, $e$ the width of the guide passages can be determined.

$r$, $z$, and $t$ are, as before, empirically chosen quantities, for which the usual rules will be subsequently given; $a$ the angle at which the water leaves the guide passages must be chosen, and it is easy to see from formula (VIII) that the efficiency increases with the angle $a$. In practice $a$ ranges from 60° to 70°.

Transposing the above equation for $A$,

$$e = \frac{A}{2\ \Pi\ r\ \cos a\ -\ z\ t}, \qquad (D)$$

and the leading proportions of the guide apparatus are settled.

The dimension $e$ should in practice not exceed a certain proportion of the radius $r$, and this proportion ranges from $\frac{1}{4\cdot5}$ to $\frac{1}{6}$ ; for the sake of brevity the ratio $\frac{e}{r}$ may be denoted by $y$. Neglecting the influence of the thickness of the vanes, the area $A$ may be expressed by

$$A = e\ 2\ r\ \Pi\ \cos a,$$

or introducing the ratio $y$,

$$A = 2\ y\ r^2\ \Pi\ \cos a,$$

from which $r$ may be calculated by

$$r = \sqrt{\frac{A}{2\ y\ \Pi\ \cos a}} \qquad (F)$$

It is in some circumstances preferable to *assume* the

value of $y$, and determine the mean radius $r$ by the above formula, instead of from the empirical rule elsewhere given; $r$ being known, $c$ follows approximately from

$$c = y\,r,$$

but should be calculated accurately as before.

Where the graphical method can be applied, it is exactly the same as for reaction turbines; the denominator in formula $(D)$ is simply the sum of all the clear distances $x$ between the guide-vanes at the outflow, of course measured at right angles to the direction of flow,

and $$c = \frac{A}{z\,x}.\qquad\qquad (E)$$

The next step is to determine the proportions of the wheel itself, and the first value required is that of the relative angle of inflow $a_1$.

Upon that angle depends the relative velocity $c_1$ with which the water enters the wheel; this determines the speed of rotation $w$ and the relative velocity of outflow $c_2$ (by formula IV$b$); the latter, again, bears a definite relation to the speed, owing to the condition that the water must leave the buckets parallel with the axis. By a combination of the various formulas expressing these relations, the following approximate connection between the angles $a$ and $a_1$ is arrived at
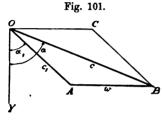
$$a_1 = 2\,a - 90°;\quad (a_1 \text{ negative}).\qquad (G)$$

Geometrically interpreted, this means that the absolute direction in which the water enters bisects the angle formed by the relative direction of inflow with the direction of rotation.

In Fig. 101 let $O\,B$ be the absolute direction of inflow forming the angle $a = B\,O\,Y$ with a vertical $O\,Y$; $O\,A$ the relative direction of inflow forming the angle $a_1 = A\,O\,Y$

with $O\,Y$; then $O\,B$ should bisect the angle $A\,O\,C$, so that $< A\,O\,B\,=\, < B\,O\,C$; then $A\,O\,=\,A\,B$ or $c_1\,=\,w$.

As above intimated, this rule is based only on an approximation, and when, as is sometimes the case, the depth of the wheel $h_o$ forms a considerable portion of the total fall, it is not sufficiently accurate as a basis for the subsequent calculation of $c_x$. The better plan under

Fig. 101.



any circumstances is to temporarily fix $a_1$ as given by the approximate rule in question, then by the graphical method determine $c_1$ and simultaneously $w$.

The relative velocity of outflow can next be calculated by (IV$b$)

$$c_2{}^2\,=\,(c_1{}^2\,+\,2gh_o)\,\frac{1}{1\,+\,\zeta_2},$$

in which $h_o$ the depth of the wheel must be empirically chosen and subsequently corrected if necessary; it may be taken as

$$h\,=\,\frac{r}{4}\;\text{to}\;\frac{r}{5\cdot5},\qquad\qquad (H)$$

$r$ having been already determined.

The co-efficient $\zeta_2$ ranges from 0·05 to 0·1.

When $c_2$ is known the angle $a_2$ is easily constructed, or determined by the relation

$$\sin\,a_2\,=\,\frac{w}{c_2}.$$

The construction is shown in Fig. 102.    $O\,B\,=\,w$; from $O$, with $O\,A\,=\,c_2$ as radius, describe an arc $A\,C$, and at $B$

draw $BA$ at right angles to $OB$, intersecting the arc in $A$; join $O$ and $A$, then $OAB$ is the required angle $a_2$.

It is desirable that $a_2$ should be as great as practicable, in order to reduce the absolute velocity of outflow, and if by the above construction $a_2$ should turn out too small, the assumed value of $a_1$ can be modified—reduced—so as to increase the speed of the wheel, and this will have the desired effect on $a_2$.

Fig. 102.
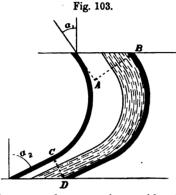
If $a_2$ is made greater without altering $a_1$ the result will be that the absolute direction of outflow is no longer parallel with the axis; provided the deviation is not very considerable, this is not of any practical importance as long as the velocity of outflow is sufficiently low.

It may also happen that the value of $a_2$ obtained is too great—in practice as a rule it does not exceed 72°—and then a modification in the opposite sense to that previously considered is necessary.
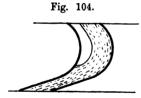
Before all the points involved in finally determining the angle $a_2$ can be thoroughly appreciated, it is essential to follow more closely the course of the water in the wheel-buckets. To insure free deviation and avoid the possibility of the water coming into contact with the convex surface of the vane next to that which it is traversing, no bucket should be filled beyond a certain proportion of its depth, about 0·7 to 1·0 of the latter at the inlet, and 0·5 to 0·75 at the outlet. Assuming, in the first instance, that the width of a bucket is constant throughout, the depth of the stream passing through it must vary inversely as the rela-

tive velocity of flow ; if the latter remains unchanged, so
will the depth. In fact,
a slight increase in the
relative velocity takes
place as the water tra-
verses a parallel-flow
wheel, and therefore un-
der the assumed condi-
tions the depth will some-
what decrease towards the
outflow. As $a_1$ is less than
$a_2$ it is very obvious that
the depth of the passage
between the vanes is considerably greater at the inlet $AB$,
Fig. 103, than at the outlet $CD$; it may, therefore,
easily happen, that although
there is sufficient clearance
between the stream and the
convex vane-surface where the
water enters, the outlet area of
the bucket is much too small,
and the wheel becomes choked
and cannot work as it is intended. This is illustrated in
Fig. 104. The greater the angle $a_2$ the smaller will be the
outflow area, and a little consideration shows that for a
certain depth of stream the inclination of the vanes is
restricted within certain limits. To extend these limits
the depth of the stream must be reduced, and the only
means for effecting this is to widen the buckets towards
the lower (outflow) end. This allows the water to spread
out, and the same sectional area is obtained with less
depth by increased width. Girard turbines are now
almost always constructed in this manner.
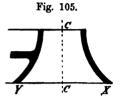
When $a_2$ has been fixed on, it is therefore essential to



Fig. 103.



Fig. 104.

make the width of the buckets, $c_2 = YX$, Fig. 105, at the outflow end sufficiently great to allow a certain proportion of clearance between the water and the convex surface of the adjacent vane. If $\delta_2$ Fig. 106, denote the depth of the stream in each bucket where it leaves the wheel, then the sectional area of the stream is $e_2 \times \delta_2$; this multiplied

Fig. 105.



by the relative velocity of flow $c_2$ must equal the *quantity* of water flowing through each bucket, and if $z_1$ denotes the total number of buckets, then the *total* quantity of water per second $Q = e_2 \, \delta_2 \, z_1 \, c_2$, and from this

$$c_2 = \frac{Q}{\delta_2 \, z_1 \, c_2} \, . \qquad (J)$$

The depth of the stream $\delta_2$ must bear a certain ratio $x_2$ to the depth $d_2$ of the passage between the buckets at the corresponding point, so that

$$\delta_2 = x_2 \, d_2. \qquad (K)$$

Substituting this in $(J)$ there follows—
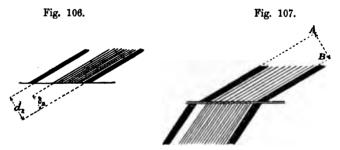
$$e_2 = \frac{Q}{x_2 \, d_2 \, z_1 \, c_2} \, . \qquad (L)$$

$d_2$ can be measured from a drawing when the pitch of the vanes is known, but to determine this the number of vanes $z_1$ must be first settled. As regards the relative number of vanes in the guide apparatus and wheel two different views may be adopted, both of which when put into practice appear to give good results.

According to one of these, the number of guide-vanes should be greater than the number of wheel-vanes, since only a certain proportion $x$ of the inflow area is to be filled by the water, and, to ensure this, the outflow area of the

guides must be less than the corresponding relative inflow area of the buckets; this is illustrated in Fig. 107.

According to the other view of the subject, the pitch of the guide-vanes may be greater than that of the wheel-vanes. The buckets are then, in certain relative positions of the guide and wheel, *filled* at the inflow, but owing to the widening of the wheel casing are only partially filled at other parts. It is assumed in this case, that the actual outflow area from the guide passages is reduced by the edges of the wheel-vanes passing before the orifices, and that consequently no more water can leave the guide passages than the wheel is capable of passing. Besides

Fig. 106.

Fig. 107.



this, a certain clearance is always allowed in the width of the buckets which gives additional area.

If the view first stated is adopted, the number of guide-vanes $z$ is purely empirical, but the number of wheel-vanes $z_1$ must be so determined that only the desired proportion $x_1$ of the buckets at the inflow is filled.

It is easy to arrange this by trial graphically, and when $z_1$ has been thus fixed, its value can be substituted in formula $(L)$ and $c_2$ calculated.

The width $e_1$ of the wheel-buckets at the inlet is nearly the same as that $e$ of the guide passages, there being a slight addition for clearance; the width at any point is

inversely proportional to the velocity of flow, the cosine of the vane angle and the proportion of the sectional area of the bucket filled,[1] hence

$$\frac{e_2}{e_1} = \frac{x_1\, c_1\, cos\, a_1}{x_2\, c_2\, cos\, a_2}. \qquad (M)$$

By substituting in this expression the values of $c_1$ and $c_2$ resulting from the conditions of inflow without shock and vertical outflow respectively, and making use of the relations arising from the equation $(G)$

$$2\, a\, -\, a_1 = 90°,$$

the ratio of the widths of the buckets can be written as follows—

$$\frac{c_2}{e_1} = \frac{x_1}{x_2}\, sin\, 2\, a\, tang\, a_2; \qquad (N)$$

and if $x_1$ be made the same as $x_2$,

$$\frac{e_2}{c_1} = sin\, 2\, a\, tang\, a_2.$$

From this formula the effect of the angles $a$ and $a_2$ on the proportions of the widths of the wheel-buckets can be easily calculated; the following are the results for a few values—

$$a\ =\ 30°,\ 50°,\ 70°,\ 70°.$$
$$a_2\ =\ 70°,\ 70°,\ 70°,\ 60°.$$
$$\frac{e_2}{e_1}\ =\ 2\text{·}38,\ 2\text{·}65,\ 1\text{·}75,\ 1\text{·}10.$$

If $x_2$ is less than $x_1$, these ratios for the same angles are greater in the inverse proportion.

The accurate formula for the value of $\frac{c_2}{e_1}$ is

$$\frac{e_2}{e_1}\ =\ \frac{x_1}{x_2}\frac{c_1}{c_2}\left[\frac{2\, r\, \Pi\, cos\, a_1\, -\, z_1\, t_1}{2\, r\, \Pi\, cos\, a_2\, -\, z_1\, t_2}\right], \qquad (O)$$

where $t_1$ and $t_2$ are the thicknesses of the wheel vanes at inflow and outflow.

[1] When the influence of the thickness of the vanes is neglected.

In practice $\dfrac{c_2}{c_1}$ ranges from 2 to 3.

If the radius $r$ as determined by the preceding method turns out impractically small, the idea of admitting the water over the whole circumference of the wheel must be abandoned and partial admission resorted to.

In this case the radius has no necessary relation whatever to the other parts of the wheel, but is purely empiric.

It may sometimes occur that the number of revolutions of the wheel is prescribed, and in that case $r$ must be calculated from the relation
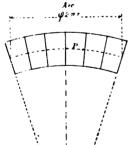
$$w = \frac{\Pi \, r \, n}{30},$$

where $n$ is the number of revolutions to be made per minute; thence

$$r = \frac{30 \, w}{n \, \Pi}.$$

Let the proportion to the whole circumference of that portion over which the water is admitted be denoted by $\phi$, Fig. 108, then the total sectional area of the stream



Fig. 108.

$$A = (\phi \, 2 \, \Pi \, r \, cos \, a - z \, t) \, e, \quad (P)$$

where $z$ denotes the number of guide *passages* and $t$ the thickness of the vanes.

The ratio $y$ of $c$ to $r$ must be assumed—it ranges from $0.1$ to $0.2$—and then since $r$ is known $c$ follows.

From the above equation for $A$ the proportion of the circumference $\phi$ over which water is admitted can be ascertained; in the first in-

stance, neglecting the quantity $z\,t$,

$$\phi = \frac{A}{2\,\Pi\,r\,e\,\cos a}; \qquad (Q)$$

then, choosing an appropriate pitch $P$, the number of passages $z$ can be calculated from

$$z\,P = \phi\,2\,\Pi\,r,$$

$$z = \frac{\phi\,2\,\Pi\,r}{P}; \qquad (R)$$

and a corrected value of $\phi$ determined by again using formula ($P$) in its complete form, whence

$$\phi = \frac{A + z\,t\,e}{e\,2\,\Pi\,r\,\cos a}. \qquad (S)$$

It is not necessary to make use of trigonometrical quantities when the guide passages can be drawn out; the quantity $2\,\Pi\,r\,\cos a$ is simply the aggregate depth of all the guide passages, *including* thickness of vanes ($A\,B$, Fig. 107, being the depth of one passage), and this can be measured direct from the drawing.

A considerable margin should always be allowed, over and above the area $A$ absolutely necessary, in turbines with partial admission, as they are always regulated, and the size of the wheel is not increased thereby.

## *Summary of Formulas for Girard Turbines.*

$$(A) \qquad c = 0.95\,\sqrt{2gH}.$$

$$(B) \qquad A = \frac{Q}{0.85\,\sqrt{2gH}}.$$

$$(C) \qquad A = e\,(2\,\Pi\,r\,\cos a - z\,t).$$

$$(D) \qquad e = \frac{A}{2\,\Pi\,r\,\cos a - z\,t}.$$

$$(E) \qquad e = \frac{A}{z\,x}.$$

$(F)$ $$r = \sqrt{\frac{A}{2\,y\,\Pi\,\cos a}}, \text{ where } y = \frac{c}{r}.$$

$(G)$ $$a_1 = 2\,a - 90° \ (a_1 \text{ negative}).$$

$(H)$ $$h_o = \frac{r}{4} \text{ to } \frac{r}{5\cdot 5}.$$

$(J)$ $$c_2 = \frac{Q}{\delta_2\,z_1\,c_2}.$$

$(K)$ $$\delta_2 = x_2\,d_2.$$

$(L)$ $$c_2 = \frac{Q}{x_2\,d_2\,z_1\,c_2}.$$

$(M)$   Approximately $$\frac{e_2}{e_1} = \frac{x_1\,c_1\,\cos a_1}{x_2\,c_2\,\cos a_2}.$$

$(N)$   Approximately $$\frac{e_2}{e_1} = \frac{x_1}{x_2}\,\sin 2\,a\,\tan a_2.$$

$(O)$   Accurately $$\frac{e_2}{e_1} = \frac{x_1\,c_1}{x_2\,c_2}\,\frac{2\,r\,\Pi\,\cos a_1 - z_1\,t_1}{2\,r\,\Pi\,\cos a_2 - z_1\,t_2}.$$

### For Partial Admission.

$(P)$ $$A = (\phi\,2\,\Pi\,r\,\cos a - z\,t)\,c.$$

$(Q)$   Approximately $$\phi = \frac{A}{2\,\Pi\,r\,e\,\cos a}.$$

$(R)$ $$z = \frac{\phi\,2\,\Pi\,r}{P}.$$

$(S)$   Accurately $$\phi = \frac{A + z\,t\,c}{e\,2\,\Pi\,r\,\cos a}.$$

### Effect of increasing Width of Buckets on the Performance of Axial Impulse Turbines.

By giving the buckets of a Girard wheel a greater width where the outflow takes place, it is possible to make the angle $a_2$ greater than would otherwise be

practicable, and thus increase the efficiency by reducing the absolute residual velocity. At the same time, however, the deviations from the best speed at the outer and inner circumference also increase, and the question naturally occurs whether the advantages obtained in one direction are not neutralized by the drawbacks in another. This is easily tested graphically, and it will be found that under ordinary circumstances there is a decided net gain in efficiency. Strictly viewed, the water on either side of the mean circumference has a radial component of motion, and in consequence during its passage over the vane comes into contact with successive portions of the latter moving at different speeds, so that to some extent the same action takes place as in a radial turbine. On one side of the mean circumference this action is that of an inward-flow, on the other that of an outward-flow wheel; the relative velocity of outflow $c_2$ will consequently be retarded and accelerated respectively, and the residual velocity correspondingly modified. The phenomena are, however, so complicated that it is not possible to follow them with any certainty, especially as only the outer parts of the stream are rigidly constrained in a radial sense.
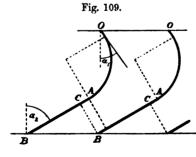
It is for practical purposes sufficiently accurate to take the relative velocity of outflow as unaffected by its radial tendency when measured in projection on a plane parallel with the axis and with the direction of rotation at the point in question.

## On the Form of Vanes for Impulse Turbines.

Similar remarks to those made with regard to reaction wheels apply in this case, except that, so long as there is sufficient room for free deviation, the question of the

sectional area of the passages does not assume quite the same form. The most essential point is, that there shall be no possibility of the buckets becoming choked, especially towards the outlet where there is the greatest risk of this occurring. Impulse turbines have in some instances proved failures owing to defective design in this respect.

The angle $a_1$ in impulse wheels is nearly always *negative;* this is due to the high velocity with which the water leaves the guide passages compared with reaction turbines as usually constructed. If $a_1$ were positive or $0°$, it would generally be impossible to reconcile the two fundamental conditions of inflow without shock and outflow parallel with the axis; for the latter it is essential that the relative velocity of outflow $c_2$ should be nearly equal to (rather greater than) the speed of the wheel $w$, but with a positive value of $a_1$, $w$ would be much greater than the relative velocity of inflow $c_1$, which is not very different from $c_2$.

The vane curves of impulse turbines—like those of reaction wheels—are best drawn by trial. Those for the guide passages are essentially the same as in reaction turbines, while the curves of the wheel-vanes have generally a form similar to that shown in Fig. 109. The

Fig. 109.



lower (outflow) end of the curves should be straight for some portion $AB$ of their length, so that adjacent vane surfaces may run parallel for a certain distance $A C$ to insure the water leaving at the desired angle.

A simple method of construction is to make the upper

part of the vane curve, $O\,A$, an arc of a circle. Very great care should be taken in designing the vanes, and the section should be drawn out not only at the mean but also at the inner and outer circumferences.

In modern wheels the angle $a$ of the guide-vanes is the same for the whole width of the passages, and similarly the relative angle of outflow $a_2$ is constant at all diameters. The surface of the vanes is therefore *not* helical; the form given to them in order to counteract centrifugal action is elsewhere described, but whether this is adopted or not, both $a$ and $a_2$ can be made constant.

## Correction of Vane Angles for different Diameters.

To partially neutralize the evils resulting from the deviation from the best speed at the outer and inner circumference of axial impulse turbines a method of construction similar to that used in the case of reaction wheels may be adopted.

The absolute velocity of outflow from the guide passages
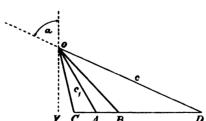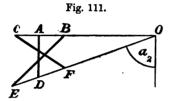
Fig. 110.



is here also to be taken as constant for the whole width, and from the speeds at the outer and inner circumferences

the appropriate values of $a_1$ the relative direction of inflow, can be constructed as shown in Fig. 110, where $D\ A$, $D\ B$, and $D\ C$ represent the speeds at the mean, inner, and outer diameters respectively, and $O\ A$, $O\ B$, and $O\ C$ the corresponding relative velocities of inflow. So far the process is exactly the same as for reaction turbines, but owing to free deviation

Fig. 111.



being possible, subsequently differs. As the relative velocity of inflow $c_1$ varies for different diameters, so also must the relative velocity of outflow $c_2$, which —neglecting the effects of friction—is greater than the former by an amount due to the fall $h_o$ through the wheel.
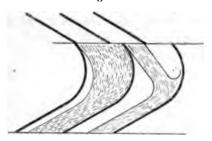
The greatest and least values of $c_2$ correspond to the greatest and least values of $c_1$, and therefore the maximum relative velocity of outflow takes place at the inner and the minimum at the outer diameter. The angle $a_2$ being constant for the whole width of the buckets, it is easy to find the absolute velocity of outflow $u$ at the inner, mean, and outer circumferences, as illustrated by the diagram Fig. 111; in the latter $O\ A$, $O\ B$, and $O\ C$ represent the speeds at the mean, inner, and outer diameters respectively, $O\ D$, $O\ E$, and $O\ F$, the corresponding relative velocities of outflow, all having the same direction, and consequently $A\ D$, $A\ E$, and $A\ F$ the absolute residual velocities.

It is evident that the directions of the latter cannot all be parallel to the axis, but by the construction shown they are made to diverge approximately to an equal extent on either side from the desired course, so that the mean direction is parallel with the axis.

When, as in the ordinary type of wheel, the edges of the wheel-vanes are radial, it is clear that the length of the

curve traversed by the water at the outer circumference
is *greater* than the corresponding length at the inner
circumference ; on the other hand, it has been shown that
the relative velocity of inflow $c_1$ is less for the outer than
for the inner diameter. The *longer* curve is therefore
passed over with the *lower* velocity, and as a consequence
particles entering simultaneously at different points in
the width of a bucket do not leave together, and there is
a relative movement between the various parts of the
stream flowing over a vane which causes internal friction
and loss of energy.

Fig. 112.



The correction of $a_1$ is unnecessary when the width of
the buckets does not exceed $\frac{1}{6}$ to $\frac{1}{12}$ of the mean diameter.
In any given case it is easy to ascertain by graphic
methods the extent of the deviations at the inner and
outer diameters from the assumed conditions, and if these
appear considerable introduce the requisite modifications.

In impulse turbines with partial admission only, a loss
of energy occurs in the wheel-buckets owing to the fact
that a portion of the stream on entering a bucket does
not come into contact with the upper end of the vane, but
impinges on the latter much lower down, the result being
a sudden instead of a gradual change of direction, and

R

consequent shock. This will be best understood by reference to Fig. 112. It is obvious that the greater the pitch of the vanes the lower down will the impact take place, and the more obtuse will be the angle between the stream and the vane. If the distance between the vanes exceeds a certain dimension a portion of the stream will pass right through the bucket without touching the vane. Even in wheels where the water is admitted over the whole circumference something similar may occur if the pitch is too great, since only those portions of the stream near to the vane will be effectively guided by the latter, while those further removed will be to a certain extent free, and cause a disturbance in the flow.

To avoid the evils just referred to, the pitch of the vanes of impulse turbines should be small, especially in those with partial admission.

## Absolute Path of Water.

The method of determining the absolute path of the water in impulse turbines is exactly similar to that described for reaction wheels. The relative velocity of the fluid as it passes through the buckets increases in parallel and outward-flow wheels, but under ordinary conditions for the former, it is often sufficiently accurate to assume it as constant in constructing the curve of the absolute path.

### Turbines to act both as Impulse and Reaction Wheels.

Circumstances often occur in which a variation in the quantity of water available for producing power is accompanied by a considerable alteration in the tail-water

level. In order for the turbine to run clear of the water
at all times, it would under these conditions be necessary
to place it at a greater height above the lowest level
than would be requisite if no alteration took place, the
result being a loss of otherwise available fall at the very
time when the best possible utilization of the power is
most essential, since the tail-race level is lowest in dry
seasons, when the available quantity of water is small.
During floods, the increase in the quantity is generally
proportionately greater than the loss of head from the
rise of the lower level, so that there is a surplus of power
which renders a high efficiency less essential than in the
former case.

In these instances, to avoid the loss of head in question,
a turbine may be so designed that in times of flood it can
work partially drowned, the outflow taking place under
water. It then works as a reaction wheel, since all the
passages are full of water, but the proportions must be
such that there is no excess of pressure over that of the
atmosphere at the guide passage orifices, and conse-
quently the velocity of flow from the latter is that due to
the whole head, and as nearly as possible the same as
when free deviation takes place. This can be accomplished
by choosing a suitable ratio between the area of the
guide passages and the area of the outflow orifices of the
wheel.

For a velocity of flow equal to that occurring with free
deviation, this ratio $\dfrac{A}{A_2}$ is somewhere about $\frac{1}{2}$, and with
the usual values of $a$ and $a_2$ is obtained by widening the
buckets towards the outflow; this, of course, is exactly
what is required for impulse turbines, and the dimensions
must be adjusted in such a manner that the increased
width at the outflow end is sufficient to allow free

deviation if the wheel is acting as an impulse turbine, and at the same time give the necessary ratio $\dfrac{A}{A_2}$ when it works full.

In such cases each passage between two adjacent wheel-vanes should be so designed that the hydrostatic pressure at every cross-section of the stream is the same and equal to the atmospheric pressure. Under these conditions the stream flowing between two vanes should just *touch* the convex vane surface, and, apart from the acceleration of the relative velocity due to the fall in the wheel, the cross-sectional area of the passage should be *constant*. Allowing, however, for the acceleration of the relative velocity, the area must slightly decrease towards the outflow.

The necessary dimensions of the passages are attained generally by the use of back vanes; but some makers of turbines effect the same object by varying the *width* of the buckets, so that the wheel-casing in cross-section appears with a kind of waist. If $c_x$ denote the relative velocity at any section, $h_x$ the mean depth of the section below the inflow, $p_x$ the hydrostatic pressure in that section, then

$$\frac{c_1^2}{2g} + p_1 + h_x = (1 + \zeta_x)\frac{c_x^2}{2g} + p_x.$$

Since $p_1$ must be equal to $p_x$, there follows—

$$\frac{c_x^2}{2g} = \left(\frac{c_1^2}{2g} + h_x\right)\frac{1}{1 + \zeta_x}$$

where $\zeta_x$ is the co-efficient for loss by friction in the passage as far as the section in question.

When $c_x$ is determined, the corresponding area $A_x$ of the passage can be calculated from

$$c_x A_x = c_1 A_1.$$

The friction may safely be neglected, and then

$$\frac{c_x}{2g} = \frac{c_1^2}{2g} + h_x.$$

$A_x$ can be ascertained for a number of points sufficient to enable the form of the passage to be drawn.

The depth $x_2$ of the passage at the outflow is of course fixed by the angle $a_2$, the radius $r$, and the thickness and number of the vanes—

$$A_2 = e_2\, x_2$$

whence
$$e_2 = \frac{A_2}{x_2}.$$

Assuming that the pressure both at inflow and outflow is atmospheric, the proper ratio $\dfrac{A}{A_2}$ is settled by the condition that the co-efficient of flow must be the same as for an impulse turbine working with the same head, hence

$$K_1 = 0\cdot95$$

or
$$\sqrt{\frac{\epsilon}{2\dfrac{A}{A_2}\, sin\, a\, sin\, a_2}} = 0\cdot95.$$

Taking $\epsilon = 0\cdot81$

$$\frac{A}{A_2} = \frac{0\cdot81}{0\cdot9025 \times 2\, sin\, a\, sin\, a_2}.$$

If $a = a_2 = 75^\circ$, then

$$sin\, a\, sin\, a_2 = 0\cdot9331$$

and
$$\frac{A}{A_2} = \frac{0\cdot81}{1\cdot684} = 0\cdot48.$$

When the difference between the heads in times of flood and under the usual conditions is considerable, then, in order to satisfy the conditions for inflow without shock,

and at the same time maintain the same speed of the wheel—

$$0.95 \sqrt{2gH} = K_1 \sqrt{2gh}$$

where $H$ is the effective head when the wheel works as an impulse turbine, while $h$ is the actual head when the wheel is to run full. It is obvious that $K_1$ only equals 0·95 when $H = h$.

As $h$ is usually less than $H$, $K_1$ is generally somewhat greater than 0·95, and the pressure at the inflow becomes slightly less than atmospheric pressure.

With the usual variations of head occurring in practice, it is sufficiently accurate to assume a constant (atmospheric) pressure throughout the buckets.

Turbines of this class, in which the cross-section of the wheel-casing is contracted, in order to vary the width, are very liable to leakage between the guides and wheel, and this fault it is very difficult to avoid.

# CHAPTER VIII.

## IMPULSE TURBINES (*continued*).

Centrifugal action in parallel-flow turbines.—In reaction turbines.—Factors determining centrifugal action.—Centrifugal action in impulse turbines.—Explanation.—Constructions for correcting centrifugal action in impulse turbines.—Outwardly inclined buckets. —Inclined vanes.—Radial impulse turbines.—Distinction between action of water in axial and radial wheels.—Inward and outward flow.—The guide apparatus.—The wheel.—The number and thickness of vanes in impulse turbines.—Influence of dimensions on the efficiency of impulse turbines.—Influence of angles of vanes.—On the difference between impulse and reaction turbines.—Comparison of reaction with impulse turbines.—The Poncelet wheel.—Compound turbines.—Bearings for turbine shafts.—Friction of turbine shafts.— Hydropneumatization.—Governors.—Summary.
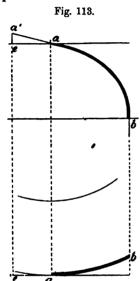
## *The Action of Centrifugal Force in Parallel-flow Turbines.*

THE *so-called* effect of centrifugal force on the water passing through an axial turbine is a point on which misconceptions may easily occur and actually have arisen.

The case of reaction and impulse turbines in relation to this subject must be treated separately, and that of reaction wheels taken first.

Let *a b*, Fig. 113, in elevation and plan represent the absolute path of a filament of water during its passage through the buckets, following the outer surface of the

wheel-casing. In plan, it will be seen, the water is compelled to follow a circular path $a\ b$, and in consequence will exert a certain normal force against the inner side of the outer wheel-casing, due to the deflection from its original course $e\ a$ at $a$. The amount of this force for any particle of water is calculated by the usual formula for centrifugal action, and depends upon the velocity in the circular path and the radius of the latter. The circumferential velocity at any point of the path is the component of the absolute velocity in the direction of the tangent to the circle at that point; for instance, the absolute velocity at $a$, where the water enters the wheel, is represented by $a\ a^1$ (in elevation), and the horizontal projection $a\ e$ of this is the circular velocity. As each particle of water advances from $a$ to $b$ along the absolute path, the absolute velocity decreases, and the direction of its motion becomes more nearly parallel to the axis of the wheel, until at $b$ it is or should be actually parallel with the axis. It is easy to perceive that under these circumstances the horizontal projection, and therefore the circumferential velocity of the water, decreases in passing from $a$ to $b$, until at the latter point it is *nil*.
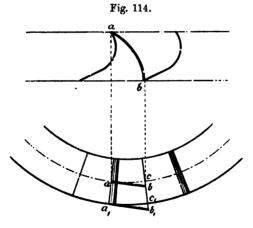
Fig. 113.



The centrifugal action is dependent on the *absolute velocity* and *direction* of the water traversing the buckets, on the distance from the axis and on the length of the arc passed through. If, for example, the absolute path

were a straight line parallel with the axis, it is clear there would be *no* centrifugal action, while if the water were simply carried round in the buckets at the velocity of the latter, without leaving the wheel, it would be a *maximum.* As the outer filaments of water are forced by the surface of the wheel-casing to follow a circular path, so successive inner filaments are compelled by the surrounding contiguous zones of water to take a concentric course. The resultant pressure exerted against the outer wheel-casing is the sum of the action of all the zones of different radii. Under ordinary conditions the centrifugal force exerted against the outer circumference of the wheel is only connected in a secondary manner with the velocity of rotation, in so far as this affects the absolute velocity of flow. The question next arises as to the effect of the centrifugal pressure on the performance of the wheel. It might be supposed that as a pressure between the particles of water, increasing towards the outer circumference, is set up, this pressure would modify in some way the velocity of flow, and consequently the power exerted by the motor. It is easy to show that on general grounds this cannot be the case. In the first place, since all the energy developed comes from that contained in the water, it is evident that the existence of centrifugal pressure— itself a result of the motion of the fluid—cannot produce any *increase* of absolute velocity, which would mean a creation of energy. In the second place, as the centrifugal pressure is in a radial direction in which no motion occurs, no energy is expended on work done by that pressure, and therefore no diminution of velocity (or energy) can result,

*Centrifugal Action in Parallel-flow Impulse Turbines.*

In impulse wheels where free deviation is allowed, only the outer zone of water next the circumference is compelled to take a circular path; nearer the centre the various filaments are free to follow their own course, which will naturally be in a plane, or seen in plan—the axis of the wheel being vertical to the picture plane—a straight line.

Fig. 114.



If $a\ b$, Fig. 114, represent the absolute course of a particle of fluid entering the wheel at the mean diameter at $a$, then—seen in plan—it will try to follow the straight line $a\ b$, instead of the circular path $a\ c$, at the mean radius; consequently, if quite unrestricted, on leaving the vane it will be further from the centre by the distance $b\ c$ than when it entered the wheel. The result of this action is that the water tends to heap itself up at the outer circumference. The extent to which this occurs
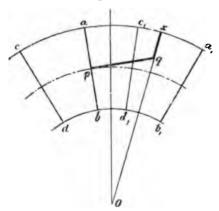
depends on the absolute velocity, the form of the absolute
path, and the diameter of the wheel. A particle of water
entering at $a_1$ on the outer circumference would, if free,
arrive at $b_1$ by the time it left the vane; it is, however,
compelled to follow the cylindrical surface of the wheel-
casing, and so in reality leaves at $c_1$. The tendency of the
water to heap itself up is therefore evidently proportional
to $b_1 c_1$, and increases as the diameter diminishes. In im-
pulse turbines the absolute velocity $c$ with which the water
enters is quite independent of the speed of rotation, but
the latter influences subsequent values through its effect
on the horizontal component and affects the length of the
absolute path; on this length the centrifugal action in
question is entirely dependent for any given diameter and
absolute velocity of inflow.

With vanes of which the surfaces are helical and the
edges radial, the whole centrifugal tendency is not limited
to that which has just been considered, but is augmented
from another cause.

Each particle of water enters the buckets in a direction
which lies in a plane at right angles to the surface and
upper or inflow edge of the vane (seen in plan, for a
vertical axis, the direction itself is at right angles to the
vane edge), but a little consideration will show, that as
the particle proceeds the surface of the vane is no longer
perpendicular to the plane in which that particle moves,
or—what comes to the same thing—to a radial line
through the point with which the particle is in contact.
In consequence of this angular position of the vane-surface
relatively to the course of the water as seen in plan, a
radial *outward* deflection of the fluid is caused, in addition
to that parallel with the circumference and similar in
character. The phenomenon will be best understood by
reference to Fig. 115. $a b$ is the upper edge of a vane,

*HYDRAULIC MOTORS.*

*c d* the lower edge; a particle of water entering at *p* moves in the direction *p q* at right angles to *a b*; by the time it arrives at *q*, however, *a b* will have travelled to $a_1 b_1$, and a radial line *O x* through *q*—which touches the vane-surface throughout the whole width of the latter— will form the obtuse angle *p q x* with the direction of flow *p q*, so that the water traversing that part of the vane will tend to move outwards towards *x*. In any perfectly con-
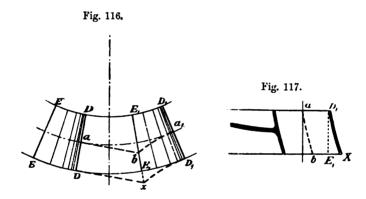
Fig. 115.



structed wheel the surfaces of the vanes should be so formed that there is no *radial* component of the pressure between them and the water during any part of its course.

## Construction for correcting the Centrifugal Action.

When, as illustrated in Fig. 116, the upper edges of the wheel-vanes are radial, and the absolute direction of the water on entering is at right angles to them, or, seen in plan, tangential to the circumference of the wheel, a

particle of fluid entering at *a* will, if unrestricted, leave
the vane at a point *b* further from the axis than *a*. In
Fig. 116, *D D E E* represents in plan the position of a vane
when the particle of water enters at *a*, and *D₁ D₁ E₁ E₁*
that of the same vane at the moment when the particle
leaves it at *b*, by which time *a* has arrived at $a_1$. The
course followed by the particle *relative* to the vane will
be $a_1$ *b*, and the general direction of the water at all
distances from the axis approximately parallel to this.

Fig. 116.



Fig. 117.

In order that the water next the outer circumference˜may
be able to take this course, it would be necessary to so
construct the vane and casing that a particle entering at
$D_1$ might leave at *x* instead of at $E_1$, and it is easy to see
that this requirement would be met by giving the outer
casing a larger diameter at the bottom than at the top,
somewhat as shown in section in Fig. 117.

Modern axial impulse turbines are constructed in such
a manner that in flowing over a vane the stream of water
can spread itself out so as to increase its width—generally
to about two and a half times its original dimension—and

when no regard is paid to the centrifugal action now in question, the casing has a bell-mouthed section symmetrical with respect to a vertical line (parallel to the axis) through the mean circumference, *vide* Fig. 118.

To avoid the centrifugal action when the width of the stream is increased, the lower edge of each vane should be so placed that it is bisected at the point $b$ instead of at $c$ by the centre line $C\,C$; this gives the unsymmetrical form of casing illustrated in Fig. 119.
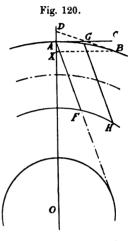
The line $a_1\,b$ on the vane can be very simply constructed from a drawing of the absolute path.

Fig. 118.                    Fig. 119.



Another method of neutralizing the centrifugal action of the water is given by Meissner, which has been successfully tried in practice. This consists in giving the vane-surfaces an inclination inwards (towards the centre of the wheel) such that the relative outward tendency is counteracted by a deflection in the opposite sense, resulting from the angular position of the vane with respect to radial lines from the wheel axis. Let $A\,B$, Fig. 120, be the plan of the absolute path at the outer circumference which the water must be compelled to follow; if free, its course would be $A\,C$, and the extent to which each particle must be, so to speak, deflected inwards is represented by $A\,X$, $B\,X$ being parallel with $A\,C$ (the tangent to the circumference at $A$); the angle through which the deflection takes place is $X\,B\,D$, $B\,D$ being the tangent at $B$. The
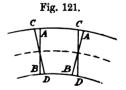
required effect is approximately obtained by placing the lower edge *A F* of each vane at an angle *O A F* equal to *X B D* with respect to the radial line *O A*. The upper edge *G H* of the vane must be parallel with the lower, so that the vane-surface is cylindrical.[1] A result of this is, that the length of the path traversed by particles entering at various points in the width of the vanes is the same (measured parallel with the mean circumference), and the losses arising from internal friction between portions of a stream moving with different velocities are to a great extent avoided.

Fig. 120.



When this arrangement of the vanes of the wheel is adopted, the guide-vanes must also be made with both edges parallel; it is not, however, necessary that the lower edges of the guide-vanes should be parallel with the upper edges of the wheel-vanes, as the direction, as seen in plan, taken by the outflowing stream is determined by the sides of the casing. In order to distribute more uniformly the shocks resulting from the motion of the wheel-vanes in front of the guide-passage orifices, some engineers place the guide-vane edges at an angle with those of the wheel-vanes, as shown in the diagram, Fig. 121, where *A B* and *A B* represent the former and *C D* and *C D* the latter.

Fig. 121.



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[1] Not necessarily circular.

## *Radial Impulse Turbines.*

For impulse turbines the radial arrangement of vanes is generally only adopted with partial admission, and chiefly for outward-flow.

Zuppinger's tangent wheel was of the inward-flow type, designed for use with high falls and small quantities of water, but it is not now often employed, the outward-flow type being found more convenient. As a rule, turbines of this kind are made with horizontal axes, an arrangement which has some practical advantages over that with vertical axes, especially where the driving is effected by belts. Zuppinger's wheel generally has a vertical axis.

The chief distinction between the theory of axial and radial impulse turbines is that in the latter an acceleration or retardation of the relative velocity of flow occurs during the passage of the water through the buckets independently of the action of gravity. By this the relative velocity of outflow as compared with axial wheels is modified, and therefore also to some extent the relations between the vane-angles.

In other respects the construction of radial wheels is simpler, owing to the fact that the speed of rotation is constant for all points in the width of a bucket where the water enters and leaves it.

The water is admitted to the buckets of outward-flow impulse wheels with horizontal shafts at the lowest possible point of the circumference, to avoid loss of head, and enters through only a small number of guide passages, sometimes only one or two.

For Zuppinger's tangent wheels the admission is equally distributed at opposite sides, to avoid lateral pressure on the shaft and bearings.

*The Guide Apparatus.* The total area of the guide passages is determined exactly as for a parallel-flow wheel:

$$c = 0.95 \sqrt{2gH} \text{ to } 0.92 \sqrt{2gH}$$

$$A = \frac{Q}{0.894\, c}.$$

If $\phi$ denotes the proportion of the circumference over which the water enters the wheel, then approximately

$$A = \phi\, 2\, \Pi\, r_1\, e \cos a,$$

and from this

$$\phi = \frac{A}{2\, \Pi\, r_1\, e \cos a}.$$

The angle $a$, as before, is chosen, the value ranging between the limits previously given—for tangent wheels it is as much as 80°.

A certain ratio $y$ between $e$ and $r_1$ is selected, which in practice lies between $\frac{1}{8}$ and $\frac{1}{20}$, and then $\phi$ can be expressed as follows—

$$\phi = \frac{A}{2\, \Pi\, y\, r_1{}^2 \cos a}.$$

Very often it is convenient to determine $r_1$ independently, with reference to the number of revolutions the shaft is required to make, and in that case $\phi$ must be calculated from the above formula, in the first instance approximately.

When the number of revolutions is not fixed, the rule $r = 1.5 \sqrt{Q}$ can be employed for tangent wheels, for outward-flow wheels $r = k \sqrt{A}$, where $k$ is a co-efficient the value of which is given in the summary of rules and formulas.

s

The pitch $P$ of the guide-vanes must then be selected, and the number of passages $z$ calculated—

$$z = \frac{\phi \, 2 \, \Pi \, r_1}{P};$$

the corrected value of $\phi$ can finally be obtained from the accurate formula—

$$\phi = \frac{A + z \, t \, e}{e \, 2 \, \Pi \, r_1 \cos a},$$

where the thickness $t$ of the vanes is empirically fixed, and the width of the passages from

$$e = y \, r_1.$$

The above formulas are the same as those used for parallel-flow wheels with partial admission, except that instead of the mean radius $r$ appears the radius $r_1$ at which the water enters.

The value $\phi$ should never exceed $\frac{1}{4}$ of the circumference.

*The Wheel.* The angles $a$ and $a_2$ are chosen as in the case of axial turbines. Approximately, too, the same relation between the angles obtains, namely

$$a_1 = 2 \, a - 90°.$$

This must be temporarily adopted, and $c_1$ and $w_1$ graphically determined in the now familiar way.

Having ascertained the relative velocity of inflow $c_1$, the relative velocity of outflow $c_2$ is given by formula (IV$a$). In this appears the depth of the buckets $h_o$, which, assuming that the water leaves at the lowest possible point, is for outward-flow wheels with horizontal axis about equal to the difference between the inner and outer radius, $r_2 - r_1$; this dimension is purely empiric, and varies from $\frac{1}{4}$ to $\frac{1}{6}$ of the mean radius $\dfrac{r_1 + r_2}{2}$.

$h_o$ is, as previously indicated, by no means always exactly equal to the depth of the buckets, but, according to the points where the water enters and leaves the wheel, may be either greater or less. By formula (IV$a$)

$$c_2^2 = (c_1^2 + w_2^2 - w_1^2 + 2gh_o)\frac{1}{1 + \zeta_2},$$

where $\qquad \dfrac{1}{1 + \zeta_2} = 0{\cdot}9 \text{ to } 0{\cdot}95;$

for tangent wheels with vertical axis $h_o = O$ with respect to the middle of the buckets.

From this point on the calculation is similar to that for a parallel-flow wheel.

The ratio of the width at the outlet of the buckets to that at the inlet is

$$\frac{e_2}{e_1} = \frac{x_1}{x_2}\frac{c_1}{c_2}\left[\frac{2\,r_1\,\Pi\,\cos a_1 - z_1\,t_1}{2\,r_2\,\Pi\,\cos a_2 - z_1\,t_2}\right].$$

The values of this ratio adopted in practice are the same as for parallel-flow wheels.

The relation

$$\sin a_2 = \frac{w_2}{c_2} = \frac{r_2}{r_1}\frac{w_1}{c_2}$$

gives the correct angle $a_2$; if this does not agree with that at first assumed, $a_1$ must be modified.

It should be noted that with *inward-flow* a *retardation* of the relative velocity is associated, $c_2$ being less than $c_1$.
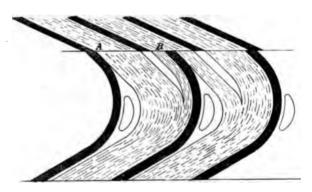
In radial impulse wheels no correction of the angles or curves of the vanes is required; the form of the latter is therefore simpler than for axial wheels. The relative velocities of inflow and outflow are the same for all points of the width, as projected on a plane at right angles to the axis of rotation, and the particles of fluid which enter simultaneously also leave the buckets together.

For these reasons, where it is possible a radial outward-flow wheel should be adopted in preference to one with parallel-flow; as already stated, however, this type of motor is only applicable for partial admission, and for large quantities of water and low falls would assume unwieldy dimensions.

## Notes on Vanes.

The guide-vanes of impulse turbines are generally of wrought-iron, or steel plate cast in with the casing, and

Fig. 122.



the wheel-vanes of cast-iron. It is desirable to make the vanes as thin as practicable, especially the guide-vanes, in order that they may interfere as little as possible with the flow and distribution of the water in the buckets.

Fig. 122 illustrates the nature of this interference; in practice, however, the upper edges of the wheel-vanes would be tapered off almost to a point, instead of being flat as shown. It is obvious that, as regards obstruction of the flow of water, it is advantageous to make the number of

guide-vanes relatively small, but in practice there is never very much difference between the number of guide- and wheel-vanes, whichever of the views previously explained in regard to this point is adopted.

## *Influence of Dimensions on the Efficiency of Impulse Turbines.*

For the efficiency of the wheel as distinguished from the accessory arrangements the expression was found—

$$\epsilon_1 = \frac{2 \sin a \sin (a - a_1)}{\cos a_1}.$$

It was further shown that approximately

$$a_1 = 2 a - 90°,$$

or $\quad\quad a - a_1 = 90° - a$ ($a_1$ being negative);

whence $\quad\quad \sin (a - a_1) = \cos a,$

and $\quad\quad \cos a_1 = \sin 2 a.$

Substituting these equivalents in the above formula, there follows—

$$\epsilon_1 = \frac{2 \sin a \cos a}{\sin 2 a}.$$

The efficiency is here expressed exclusively in terms of *one* angle, and the question is—For what value of this angle is it a maximum? $\sin 2 a$ is identical in value with $2 \sin a \cos a$, so that the above formula gives an efficiency of 100 per cent. whatever $a$ may be. This of course is impossible, and merely a result of the approximations used in developing the expression, but it shows that, provided the conditions of inflow without shock and radial or axial outflow are observed, the value of $a$ has very little influence on the efficiency.

The greater $a_2$, as in the case of reaction wheels, the higher must of course be the efficiency, as an increase of $a_2$ is accompanied by a reduced absolute residual velocity $u$.

## *On the Difference between Impulse and Reaction Turbines.*

A clear understanding of the essential characteristics of a reaction as distinguished from an impulse turbine is so important for the student of the subject, that a somewhat more detailed consideration of this point is required.

It has been already explained that in reaction turbines continuity of flow [1] is a necessity, while in impulse turbines it must be avoided.

As a result of the condition of continuity the water on leaving the guide passages has a pressure $p_1$, which is different to the atmospheric pressure; under this pressure —neglecting the effects of leakage and other interference —it enters the wheel-buckets. When, as is now assumed, a suction-tube is employed, the height $h_1$ of water above the wheel is less than the total head $h$, and if the water issued from the guide passages at atmospheric pressure (as in an impulse turbine), the whole energy available would be represented by the velocity of outflow only, due to the head $h_1$, and the difference $h - h_1$ the height of the outflow orifices above the tail-water would be all *lost.* In a reaction turbine, the water generally passes through the wheel-buckets under a decreasing pressure, and in that

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[1] It must be noted that the author uses the term "continuity of flow" to express the fact that the flowing water is always in contact with all the internal surfaces of the passages. In a general sense there must of course be continuity of flow in *all* turbines.

case the energy available is represented, not merely by the velocity, but in addition by the difference between the pressure of the water leaving the guide passages and that leaving the wheel-buckets. The available energy is therefore at the moment of leaving the guide passages made up—so to speak—partly of velocity and partly of pressure, and during the flow of the water through the wheel-buckets the energy of pressure becomes converted into energy of velocity owing to the acceleration, relative to the buckets, produced by the excess of pressure $p_1 - p_2$.

It must be distinctly understood that the hydrostatic pressure *as such* produces no effect in driving round the wheel, but only when converted into *velocity relative to the wheel*. The language of some authors on this point appears misleading, and an unnecessary distinction is drawn between the action of the fluid on the vanes in reaction and in impulse turbines.

As far as the force exerted on the vanes is concerned, the *mode* of action is precisely the same in both systems, but while in impulse turbines—apart from friction and any difference of level of the entrance and exit orifices of the buckets—there is no accelerating (or retarding) force affecting the water during its passage between the vanes, the contrary is true of reaction wheels.

The acceleration produced in the relative velocity intensifies the effort exerted by the fluid on the vanes, and, under otherwise equal conditions, increases the work done in turning the wheel.

The pressure $p_1$ at which the water leaves the guide passages may be either greater or less than atmospheric pressure; when it is *less* the velocity of outflow is of course greater than in the former case, and may represent an amount of energy *in excess* of that due to the total head, but under these conditions the pressure *increases* as

the fluid passes through the buckets, consequently $p_1 - p_2$ is negative, and the available energy is *less* than that due to the velocity of outflow from the guide passages.

Where no suction-tube is used and the off-flow into the tail-water takes place direct from the wheel, $p_2$ equals atmospheric pressure, and the energy available for the wheel is $\frac{c^2}{2g} + p_1 - a$. The *absolute* velocity $u$ with which the water leaves the wheel then represents the lost energy not absorbed in the turbine. When, however, there is a suction-tube, this loss is expressed by $\frac{u^2}{2g} + p_2 - a$, and in many cases $p_2$ is less than $a$, and hence the energy wasted is not so great as that due to the velocity $u$; this occurs when the final velocity of off-flow $c_4$ under the sluice is smaller than $u$, as the former must be the measure of the loss from residual energy, and therefore

$$\frac{u^2}{2g} + p_2 - a = \frac{c_4^2}{2g}.$$

This explains the action of an enlarged suction-tube or a Boyden diffusor in improving the efficiency of a turbine; the principle involved may be stated by saying *that what is lost in velocity is gained in pressure.* The excess of the velocity $u$ over $c_4$ is compensated by $p_2$ being lower than $a$, and consequently a greater available energy. It has been throughout assumed for the sake of simplicity that there are no frictional or other resistances apart from the work done on the wheel.

The velocities and pressures, it is scarcely necessary to repeat, depend, under given conditions as to head of water and position of wheel, entirely on the proportions of the different parts of the apparatus; it is therefore not necessary in practice to determine the pressures $p_1$ and $p_2$ as such.

## Comparison of Reaction with Impulse Turbines.

In comparing one system of turbine with another it is not merely the maximum possible efficiency under the most favourable circumstances which should, for practical purposes, be the standard according to which their relative merits are estimated, but rather the average performance under the varying conditions occurring in practice, and the capacity of any motor for adapting itself with the least possible disadvantage to such variations.

Reaction turbines give a somewhat higher efficiency than impulse turbines when running at the best speed and with the full quantity of water for which they are designed, and have the further advantage that they can be used with a suction-tube and placed within certain limits at any convenient height above the tail-water.

On the other hand, all the practicable methods of regulating reaction wheels to suit varying quantities of water involve with the decrease of the latter a reduction of the efficiency, and consequently these motors are only suited for cases in which the quantity of water is tolerably constant, or at any rate does not fall below that required by the wheel when working at its best speed.

For high falls the dimensions of reaction turbines become very diminutive, and this necessitates a very large number of revolutions in order to obtain the requisite velocity at the circumference, while, when in addition to a high fall the quantity of water is small, it becomes impossible to use a reaction wheel.

The maximum efficiency obtainable with impulse turbines is not so high as in the case of reaction turbines, but—as has been previously fully explained—they can be advantageously regulated to almost any extent for varying

quantities of water, and the dimensions are almost inde-
pendent of the latter, so that this system of motor can be
applied under .nearly any conditions, either for low falls
and great quantities of water, or for high falls and small
quantities of water.

Owing to their great practical advantages in these
respects, impulse turbines are very largely used on the
continent of Europe, and have to a great extent super-
seded reaction turbines, since, in so very ·many instances
where hydraulic power is available, the quantity of water
is changeable with the seasons;

In America and Canada reaction turbines appear to be
chiefly employed, as the practically unlimited supplies of
water in the great rivers in those countries are favourable
to their use.

Whether, in any given case, a reaction or an impulse
turbine is appropriate, must depend on the particular
circumstances—each system has its own sphere of useful-
ness ; the actual results obtained may be studied from the
particulars of experiments given in the sequel.

Where the available *head* of water varies, a Jonval
wheel may be advantageously used by dividing it into
several concentric parts, and, according as the head is
greater or less, using the inner or outer rings so as to
obtain the appropriate speed.

A radial reaction turbine is obviously not adapted to
this method of regulation for varying falls, since the
velocity of all points at which the water enters is the
same.
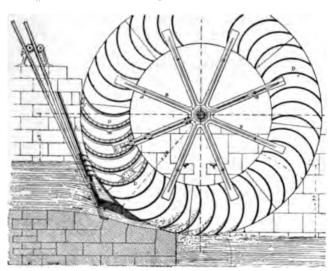
There is, of course, no hard-and-fast line at which the
employment of a reaction turbine must cease and the
application of an impulse wheel begin.  The first Four-
neyron turbine ever made, of which particulars are given
elsewhere (Chap. XI.), was constructed for a high fall,

with small diameter, and to run at an enormous number
of revolutions per minute, but this is quite contrary to
present practice.

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## The " Poncelet" Water-wheel.

Although generally classed as a water-wheel in the
narrower sense of the term, the " Poncelet " wheel is,

Fig. 123.
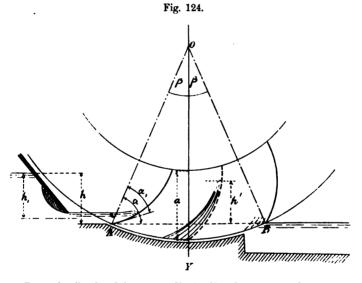


according to the definition given, in reality a turbine.
The water, *vide* Fig. 123, enters at the bottom as in an
undershot wheel, but the vanes, instead of being straight,
are curved in such a manner that the inflow takes place
without shock, and the water in consequence of its velocity
flows up a certain height along the vanes, until, brought
relatively to rest by the action of gravity, it falls again,

still following the vane surface.  Both on its upward and downward course the fluid exerts pressure on the vanes as a result of the deflection caused by their curvature; to a slight extent, too, the water acts by its weight on the buckets, but this action is very insignificant in comparison with that due to the relative velocity.

The sluice under which the water is admitted to a

Fig. 124.



" Poncelet" wheel is generally inclined at an angle of from 40° to 60° to the horizon.

The axis, as shown by the illustration, is horizontal, the diameter is comparatively great, and the velocity of rotation slow, as in ordinary water-wheels.

It is clear that, if none of the available head is to be lost, the water after attaining the highest point in the buckets should fall through exactly the same height to which it has risen before reaching again the tail-race

level. This end is obtained by so choosing the point $A$, Fig. 124, at which the water enters, with reference to the speed of the wheel and velocity of the water relative to the vanes, that the fluid leaves the buckets at a point $B$, equi-distant with $A$ from a vertical line $O\ Y$ through the centre of the wheel, whence $A\ Y = B\ Y$, and $A\ O\ Y = \angle B\ O\ Y = \beta$.

The water enters the wheel from under the sluice in a horizontal direction, forming the angle $a = O\ A\ B$, with the radius $O\ A$, while $\beta = 90° - a$.

The velocity of flow from under the sluice is

$$c = 0\cdot922 \sqrt{2gh_1},$$

where $h_1$ is the effective head measured to the centre of the sluice orifice.

The conditions for inflow without shock are the same as for a turbine.

If $h_1$ denote the height above the tail-race level to which the water rises in the buckets, then

$$\frac{c_1^2}{2g} = h_1;$$

and, neglecting friction,

$$c_2 = c_1,$$

since the water falls through the same height to which it has risen. This equation, combined with the well-known conditions for inflow without shock and radial outflow, leads to the relation

$$a_1 = \tfrac{1}{2}\ a.$$

It is easy to see that the depth of the buckets $a$, the difference between the outer and inner radii $r_1 - r_2$, must be at least equal to the height $h^1$ *plus* the depth to which the lowest point of the wheel is submerged below the tail-water.

According to Zeuner

$$r_1 \beta = 1.6 \, w \sqrt{\frac{a}{g}} \, ;$$

generally $\beta = 15°$, and consequently

$$a = 90° - \beta = 75° ; \text{ limits } 72° \text{ to } 78°.$$

If $b$ denote the clear breadth of the wheel-buckets, and $x$ the proportion filled by the water, then

$$Q = x \, a \, b \, w,$$
$$x = \tfrac{1}{2} \text{ to } \tfrac{2}{3}.$$

The following data may be taken as a guide in the construction of "Poncelet" wheels:

Maximum value of $b = 13$ feet.
     „        „     $r = 10$ feet.
$P = 8$ to 10 inches; $z = 36$;
Vertical sluice opening 4 to 12 inches.

The greatest head suitable for this type of motor is about $5\tfrac{1}{2}$ feet.

It should be unnecessary to remark that the "Poncelet" wheel is an impulse turbine with free deviation; with it an efficiency of about 70 per cent. has been obtained.

## *Compound Turbines.*

If a turbine is allowed to run at a much lower speed than the best, the water leaves the buckets with a very considerable absolute velocity, and there is a corresponding loss from unutilized energy. This energy might, however, be usefully employed in driving a second turbine, the water after leaving the first being deflected by a set of stationary guide-vanes to cause it to enter the second

wheel at the proper angle. Both turbines could be keyed to the same shaft, and their speed would be much lower than that of a single turbine driven by the same head of water and utilizing it to the same extent. This arrangement would constitute a *compound* turbine, and it is clear that, instead of *two* wheels only, *three* or more might be employed in the same way, the speed being lower the greater the number of wheels. The only object in using a compound turbine in preference to a single one would be to reduce the speed in cases where the head was very great and a high velocity of rotation inconvenient or impracticable. The principle of the compound turbine has, in so far as the author is aware, not hitherto been applied in practice with hydraulic power, but is embodied in Mr. Parson's steam turbine, in which a very large number of wheels keyed to the same shaft are used. The velocity of rotation necessary to utilize steam efficiently in a single turbine would be impossibly high, owing to the very great velocity of flow.

With water, the losses in a compound turbine would certainly be much greater than in a single wheel, in consequence of there being a double set of guide-vanes and buckets, in *each* of which the frictional losses would be at least as great as in the single motor; notwithstanding this, it might in certain cases be desirable to adopt the compound system, in order to avoid the use of gearing and countershaft for obtaining a relatively low speed from a quick-running motor.

## Bearings for Turbine Shafts.

The forms of bearings generally used for turbine shafts are the following :

(1) Subaqueous step bearing.

(2) Collar bearing above water.

(3) Overhead (Fontaine) suspension bearing.

In every case, in addition to the supporting bearing which takes the weight of the motor, there are one or more bearings or stuffing-boxes which steady the shaft laterally.

## Subaqueous Step Bearings.

These require no special description, being of the usual construction, and either resting on the floor of the tail-race or supported by brackets secured to the walls of the suction-tube where one is employed. For lubrication an oil pump is necessary.

This class of bearing has the drawback that it is not easily accessible. Lignum vitæ has also been employed for step bearings, in which case no lubrication is required.

In at any rate one American turbine the shaft runs on the conical end of a cylindrical block of lignum vitæ held in a socket; the end of the shaft is of course recessed to fit the taper end of the block. (*Vide* illustration of "Risdon" turbine.)
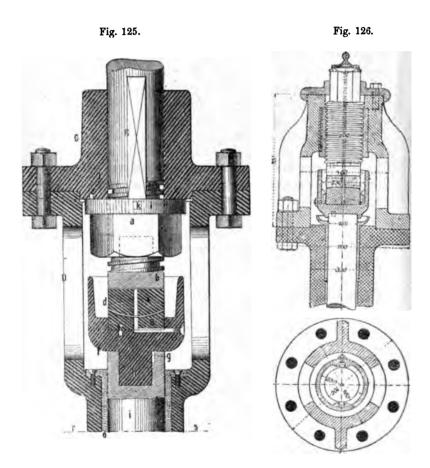
## Collar Bearings.

With these a turbine is suspended from the upper end of the shaft, which is guided lower down in a stuffing-box to insure lateral stability. In some cases the bearing rests in a spherical socket to allow greater freedom of adjustment. The same rules are applicable to collar bearings whether used for turbines or other purposes, the bearing surface being proportional to the load on the shaft.

## Overhead (*Fontaine*) Suspension Bearings.

With these the load is supported on the upper end of a spindle of steel or iron rigidly fixed in a socket of cast-iron on the tail-race floor  The turbine wheel itself is keyed to a hollow cast-iron shaft which at its upper end carries a kind of lantern.  Through the top of this lantern, which is formed as a boss, passes a short spindle, often continued upwards to form the driving shaft, the lower end of which rests on a loose steel washer, sometimes with spherical base, running on a bearing rigidly attached to the top of the fixed spindle previously referred to.  The position of the short spindle, which is screwed on the outside, is adjustable by means of a large nut; a feather key prevents it from turning relatively to the top of the lantern.  The top of the lantern is attached to the body by bolts passing through flanges.  The bearing is easily accessible through the openings in the sides of the lantern. The construction will be readily understood from the illustrations, Figs. 125 and 126.

In Fig. 125, $D$ is the cast-iron lantern, $G$ the flanged top in the boss $I$ of which is secured the spindle or shaft $b$ by means of the feather key $R$; $a$ the nut with flange $k$ fitted on the screwed portion of the spindle $b$ by which the position of the wheel is vertically adjusted; $d$ a loose washer of gun-metal between the bearing surface of the spindle and the steel bearing-plate $e$; $f$ a cup of cast-iron containing oil for lubrication, having at its lower end a cylindrical neck $h$ fitting into a socket in the upper end of the fixed spindle $i$.  The hollow cast-iron shaft just below the lantern is lined with a long gun-metal bush $o$ fitting the spindle $i$.  The plate $e$ is prevented from turning by a pin $h$, while the cup $f$ is held by a cotter $g$. In the under side of the washer $d$ are radial grooves to

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Fig. 125.

Fig. 126.

admit the oil. The lower end of the bearing spindle *b* is loose and fitted into a cylindrical recess in the end of the screwed portion; this is in order to allow the nut to be fitted on. The spindle *b* should be of steel. It is unnecessary to make the under surface of the washer *d* spherical, as shown, and this is now not generally done.

The pressure on the bearing surfaces may range from 1,100 to 1,400 pounds per square inch.

For the diameter of bearing spindles for turbines Meissner gives the following formulas:

$$\text{For } wrought\text{-}iron \ d = \sqrt{\frac{P}{2,750}} + 0\text{·}2 \text{ inch};$$

$$\text{For } steel \ d = \sqrt{\frac{P}{7,640}} + 0\text{·}12 \text{ inch};$$

where *d* is the diameter of the spindle in inches, and *P* the load on it in pounds.

Fig. 126 illustrates the construction of the overhead bearings employed for two double Jonval turbines at Schaffhausen, recently erected by Messrs. J. J. Rieter and Co. of Winterthur. A description of these will be found in Chap. XI.

The bearings in question differ somewhat in detail from that shown in Fig. 125, but the design is essentially the same.

## *Shaft Friction of Turbines.*

Mr. J. C. Bernhard-Lehmann published in 1879[1] the results of experiments carried out by him on the friction

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[1] *Zeitschrift des Vereins Deutscher Ingenieure*, 1879, vol. xxiii. p. 121.

of turbine shafts, including the resistance of all the bearings and stuffing-boxes, &c.

To determine this frictional resistance a cord was passed round the driving-wheel or pulley on the turbine shaft, and to the free end weights were attached until the turbine was set slowly in motion. In this way the co-efficient of friction when the wheel was nearly at rest could be determined, and the same co-efficient was employed to calculate the power absorbed by friction at the usual speed of the motor. As the co-efficient of friction for shafting at rest is in nearly all cases greater than when it is in motion, the results thus obtained by Mr. Lehmann err if anything on the safe side.

The following is a summary of the conclusions arrived at :

(1) For Axial Turbines with vertical solid shaft, ordinary footstep, and leather collar in the guide-wheel cover, with bearings from 1 to 2·8 inches diameter—

Loss by friction, with full admission, **1·4** to **2·4** per cent.
    „ half     „     **2·3** „ **3·6**   „

(2) For Axial Turbines with vertical hollow shaft, Fontaine's overhead bearings, and no leather collar, with bearings from 3·2 to 4·8 inches diameter—

Loss by friction, with full admission, **1·5** to **3·2** per cent.
    „ half     „     **2·3** „ **5·4**   „

(3) For Axial Turbines with vertical solid shaft, collar bearings, and leather collar in the guide-wheel cover, with bearings from 1·6 to 2·8 inches diameter—

Loss by friction, with full admission, **2·1** to **3·4** per cent.
    „ half     „     **2·7** „ **4·7**   „

(4) For Radial Outward-flow Turbines with vertical solid shaft, and step bearings without stuffing-boxes, with bearings from 1·8 to 3·6 inches diameter—

Loss by friction, with full admission, **0·8** to **1** per cent. ; with bearings from 1·8 to 2·4 inches diameter—

Loss by friction, with half admission, **0·5** to **0·9** per cent.

(5) For Radial Outward-flow Turbines with vertical hollow shaft and Fontaine's overhead bearings, with bearings from 2 to 3·2 inches diameter—

Loss by friction, with full admission, **0·9** to **1·2** per cent.

(6) For Radial Inward-flow Turbines with vertical solid shaft, step bearing with stuffing-box and leather collar in guide-wheel cover, for bearings from 2 to 2·8 inches diameter—

Loss by friction, with full admission, **0·9** to **1·1** per cent.

(7) For Radial Inward-flow Turbines with vertical solid shaft, collar bearings, and without leather collar, for bearings from 2·4 to 3·2 inches diameter—

Loss by friction, with full admission, **1·3** to **1·7** per cent.

(8) For Radial Outward-flow Impulse Turbines with horizontal axis and ordinary bearings, with bearings from 1·6 to 3·6 inches diameter—

Loss by friction, 1 to **1·6** per cent.

A few experiments with bearings of lignum vitæ showed that with this material the friction was considerably higher than with metal.

All the experiments were undertaken with the motors in their ordinary condition and with the usual lubrication, while the weight of the water in the wheel, and, for axial turbines, its downward pressure on the vanes, was taken into account in calculating the total friction.

The pressure on a parallel-flow turbine tending to increase the shaft friction is made up of two factors, (1) the excess of pressure $(p_1 - p_2)$ at the inflow acting on the horizontally measured area of the buckets, and (2) the force resulting from the deflection of the stream $\left(Y = \dfrac{1}{g} \ (c \ cos \ a - c_2 \ cos \ a_2) \ \text{for the unit of weight}\right).$

In radial-flow turbines this source of increased friction is absent.

## *Hydropneumatization.*

To enable impulse turbines to be used with very low heads, which would otherwise be impracticable on account of the very small depth of water left above the guide-apparatus, Girard devised a process to which he gave the above somewhat cumbrous title.

This process consists in enclosing the turbine in a casing dipping below the surface of the tail-water and pumping in air until the level is sufficiently depressed to allow the necessary clearance for the wheel, while keeping the lower edge of the latter at or below the original tail-race level. In this way it is possible to maintain a depth of water over the turbine which would otherwise be reduced by the required clearance. There is no actual gain of head by this process, as the water issues from the guide orifices against a pressure which exceeds that of the atmosphere in proportion to the clearance obtained; the advantage achieved consists solely in the avoidance of the disturbances arising when the depth of fluid over the wheel is very small, which cause air to be drawn in with the water and interfere with its proper action. When once the tail-water level has been depressed, only sufficient air has to be further supplied to compensate for leakage.

Hydropneumatization has not been adopted in practice to any considerable extent, as it is complicated and expensive, and the power necessary to replace the air lost by leakage is very appreciable.

## *Girard Turbine with Draft Tube.*

In Vol. III. of the *Annales du Conservatoire des Arts et Métiers* (1891, p. 291), a description is given of an interesting installation, designed with a view to combining the advantages of the draft tube with those of the impulse turbine.

It was necessary, owing to the special circumstances of the case, to place the turbine at a considerable height above the tail-water level, and this entailed the use of the draft tube. In order to employ this in connection with an impulse wheel, the column of water in the tube is allowed to sink a few inches below the lower edge of the wheel, and is maintained at that level by means of a regulating device consisting of a float in connection with a small air-valve. When the water rises too far in the tube, the valve is opened and admits air above the column, slightly increasing the pressure there, so that the column sinks again.

By this means the wheel works as an *impulse wheel in a partial vacuum*, and of course the velocity of flow from the guide passages corresponds to the increase of head due to the vacuum, and compensates for the height at which the turbine is placed above the tail-water.

The principle involved is really the same as that of hydropneumatization, only the air pressure is *less* than that of the atmosphere, and the tail-water level is raised instead of being depressed.

## *Governors.*

Centrifugal governors when used with turbines have to be of the *indirect acting* class, that is, the governor does not itself work the regulating mechanism, but merely

put it into gear with a shaft driven by the motor itself, so that the latter exerts the force required for regulation. This indirect action is rendered necessary by the very considerable power which has generally to be exerted to move the regulating mechanism of turbines. In some of these the governor acts on reversing clutches which it throws in and out of gear; in others a balanced valve is employed which admits water under pressure to either side of a piston actuating the regulating mechanism of the turbine.
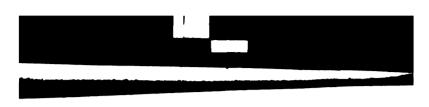
The residual velocity of the water leaving the wheel has also been utilized for governing turbines by acting on an auxiliary small wheel.

When turbines are used for driving dynamos, a good governing mechanism is of great importance, while a reduction of the efficiency at part gate is of less moment than the maintenance of a constant speed.

A balanced cylindrical sluice-gate is in such instances probably the best form of regulator, used in conjunction with an inward- or outward-flow radial turbine, subdivided by partitions perpendicular to the axis.

As previously explained, a sluice on the draft tube of a Jonval turbine does not act very rapidly, and when employed in connection with a governor the construction must be such that a comparatively small movement of the sluice causes a considerable alteration of the outflow orifice.

| Medium quantity of water and medium fall $\text{where } \dfrac{Q}{c}$ greater than 2 and less than 16 sq. ft. | Small quantity of water and high fall $\text{where } \dfrac{Q}{c}$ less than 2 sq. ft. |
|---|---|
| ............... 75° to 70° ................. | .........75° to 73°. |
| ............... 74½° to 73° ............... | .........77° to 74°. |
| ... 1·5 to 0·5, usually about 1 ... | 1·5 to 0·5, usually about 1 |
| .......... $K_1$ from Table I. ......... | ...$K_1$ from Table I. |

$a_1$
$+ a_1) \; sin \; a,$
may be taken    For ordinary turbines $K_1 = 0·67.$

| | 53 to 176 cubic feet | 35 to 53 cubic feet. |
|---|---|---|
| ( feet ... | ... 53 to 176 cubic feet | ... 35 to 53 cubic feet. |
| ... | .......5 to 26 feet | ...26 to 40 feet. |
| ... | ... 66° to 72° | 72° to 75°. |
| a ... | ... 68° to 74° | 74° to 77°. |
| j | | |
| h | | |

p, area-ratios, &c.

| | | |
|---|---|---|
| ... | ... 1·5 to 1·75 $\sqrt{A}$ | ... 1·75 to 2 $\sqrt{A}$. |
| ... | $\frac{r}{9}$ to $\frac{r}{10}$ | $\frac{r}{7\cdot5}$ to $\frac{r}{9}$. |
| ... | 54 to 60 | 45 to 54. |
| ... | $\frac{3}{16}''$ to $\frac{1}{4}''$ | $\frac{1}{8}''$ to $\frac{3}{16}''$. |
| ... | $\frac{r}{4\cdot5}$ to $\frac{r}{5\cdot5}$ | $\frac{r}{5\cdot5}$ to $\frac{r}{6}$. |
| ... | $\frac{1}{4\cdot5}$ to $\frac{1}{4\cdot5}$ | $\frac{1}{5\cdot5}$ to $\frac{1}{6}$. |
| ... | ... 42° to 54° | 54° to 60°. |
| ... | $\frac{r}{5}$ to $\frac{r}{5\cdot5}$ | $\frac{r}{4}$ to $\frac{r}{5}$. |

sin

| ( $\frac{l}{\ell}$ ... | 2·5 to 3 | 2·7 to 3·5 |
|---|---|---|

| ... | ' 50 to 56 | 40 to 50. |
| ... | $\frac{5}{16}''$ to $\frac{3}{8}''$ | $\frac{1}{4}''$ to $\frac{3}{8}''$. |

( $\frac{a}{a}$

| ... | $\frac{r}{5}$ to $\frac{r}{5\cdot5}$ | $\frac{r}{4}$ to $\frac{r}{5}$. |
|---|---|---|
| ... | $\frac{1}{4}$ to $\frac{1}{2}$ $h_0$ | $\frac{1}{4}$ to $\frac{1}{2}$ $h_0$. |

# CHAPTER IX.

SUMMARY OF RULES AND FORMULAS AND NUMERICAL EXAMPLES.

Rules and formulas :—Reaction turbines : parallel-flow ; radial flow.—Impulse turbines : parallel-flow ; radial-flow.—Examples : Jonval wheel No. 1 ; Jonval wheel No. 2 ; inward-flow wheel ; axial Girard turbine No. 1 ; turbine to work both as impulse and reaction wheel, axial No. 2 ; radial outward-flow impulse wheel No. 3.

## NUMERICAL EXAMPLES.

### *Reaction Turbines.*

#### (1) *Axial Turbines No.* 1.

AN axial turbine is required to utilize a fall of 13·5 feet and a quantity of water 210 cubic feet per second.

Data:—$h = 13·5$ feet.  $Q = 210$ cubic feet.

Weight of water per second $G = Q\lambda = 13,109·25$ lbs. Available horse-power

$$E_{hp} = \frac{13,109·25 \times 13·5 \times 60}{33,000} = \text{say } 322.$$

In Jonval turbines the width of the buckets is the same throughout. In the variety known as the Henschel wheel the width where the water leaves the buckets is greater than where it enters.

It is assumed in the present instance that the width is to be constant, or nearly so; in that case, as has been already shown, the ratio $\dfrac{A}{A_2}$ is fixed as soon as the angles $a$ and $a_2$ are chosen.

Assume $a = 75°$ and $a_2 = 80°$

$$\frac{A}{A_2} = \frac{\cos a}{\cos a_2} = \frac{0·259}{0·174} = 1·488, \text{ say } 1·5.$$

From (viii), p. 128, $\dfrac{c_1}{c_2} = \dfrac{A}{A_2}\dfrac{r_2}{r_1}\dfrac{\cos a_2}{\cos a}$; for axial turbines

$\dfrac{r_2}{r_1} = 1$, hence

$\dfrac{c_1}{c_2} = 1\cdot5\,\dfrac{1}{1\cdot488} = \mathbf{1\cdot008}$, practically 1, as assumed.

The velocity of flow $c$ must next be determined :—

$$c = K_1 \sqrt{2gh} = K_1 \sqrt{2 \times 32\cdot2 \times 13\cdot5} = K_1\,29\cdot483.$$

On referring to Table I. it will be found that for $\dfrac{r_1}{r_2} = 1$, $a_2 = 80°$, $a = 75°$, and $\dfrac{A}{A_2} = 1\cdot5$, the corresponding value of $K_1 = 0\cdot535$, hence

$c = 0\cdot535 \times 29\cdot483 = \mathbf{15\cdot773}$ feet per second.

The mean velocity of rotation $w$ can now be calculated by (v)

$$w = c\,\frac{A}{A_2}\,sin\,a_2;$$

therefore $w = c\,1\cdot5 \times sin\,80° = 1\cdot5 \times 0\cdot985 \times 15\cdot946,$
$$w = 1\cdot477 \times 15\cdot773 = \mathbf{23\cdot297} \text{ feet per second,}$$

or $\qquad K_2 = \dfrac{w}{\sqrt{2gh}} = \dfrac{23\cdot297}{29\cdot483} = \mathbf{0\cdot8}$ nearly.

$w$ may also be graphically determined, by the construction so frequently referred to, as one side of a right-angled triangle of which the hypothenuse $c_2$ and one of the angles $a_2 = 80°$ is given, *vide* Fig. 60 (p. 134).

$c_2 = c\,\dfrac{A}{A_2} = 15\cdot773 \times 1\cdot5 = \mathbf{23\cdot659}$ feet per second.

It will be instructive to compare the values so far

obtained with those resulting from other preliminary assumptions, for instance,

$$a_2 = 75° \text{ and } a = 75°.$$

In this case $\dfrac{A}{A_2} = 1$ (since $cos\ a_2 = cos\ a$); from Table I.,

$$K_1 = \mathbf{0\cdot665}, \text{ hence}$$

$$c = 0\cdot665 \times 29\cdot483 = \mathbf{19\cdot606},$$

and $K_2 = 0\cdot665 \times .1 \times sin\ 75° = 0\cdot665 \times 0\cdot966 = \mathbf{0\cdot642}$; therefore $w = 0\cdot642 \times 29\cdot483 = \mathbf{18\cdot928}$ feet per second.

From this it is evident that a very much lower speed is necessary in the second case than in the first. The head and quantity of water assumed are the same as for the "Goeggingen" turbine in Table A, for which the best experimental results are there given; on reference to these it will be seen that the experimental value of $K_2$ agrees very well with that last calculated, for $a = 75°$ and $a_2 = 75°$. The angles of the vanes for the turbine in question are not given, but from the correspondence in the values of $K_2$ it is probable that $a$ and $a_2$ are somewhere near $75°$; for the sake of further comparison the calculation will be continued with the figures obtained on this assumption instead of those first proposed.

The next step is to ascertain the angle $a_1$; this when designing can be done most conveniently by the graphic method, *vide* Figs. 61 and 62 (p. 135).

By formula (ix) *tang* $a_1 = tang\ a_2 - tang\ a$, since in this instance $a_2$ and $a$ are equal; *tang* $a_1 = 0$, and therefore $a_1 = 0$, that is to say, the relative direction of entrance is parallel with the axis of the turbine.

The effective area of the guide passages

$$A = \frac{Q}{c} = \frac{210}{19\cdot606} = \text{ say } \mathbf{10\cdot7} \text{ square feet.}$$

Had the first assumptions been adhered to for which $c = 15\cdot773$, it is clear that the area $A$ must have been greater and the wheel altogether larger.

For medium falls and quantities of water the empirical formula may be used—

$$r = 1\cdot25 \text{ to } 1\cdot5\sqrt{A},$$

say　　$r = 1\cdot25 \sqrt{10\cdot7} = 1\cdot25 \times 3\cdot3 = 4\cdot125$ feet,

or　　$d = 8\cdot25$ feet, the mean diameter.

The Jonval turbine in the table referred to has a mean diameter of $8\cdot036$ feet; for the turbine under consideration, $d = 8$ feet may be assumed, or $r = 4$ feet.

The number of guide- and wheel-vanes is respectively 36 and 38 for the wheel actually constructed, and the same numbers may be adopted for the subsequent calculations—

$$z = 36; \; z_1 = 38.$$

Assuming the vanes to be of cast-iron, they may be taken as having a thickness of $\frac{1}{2}$ inch at the extremities, $t = \frac{1}{2}$ in., $t_1 = \frac{1}{2}$ in. From the preceding data the width $c$ of the guide passages can now be found from

$$c = \frac{A}{2\,r\,\Pi\,\cos a - z\,t - z_1\,t_1\,\dfrac{\cos a}{\cos a_1}}.$$

$t$ and $t_1$ must of course be expressed as fractions of a foot, as the foot is the unit employed for other measurements;

$$t = t_1 = \tfrac{1}{24} \text{ foot.}$$

$$c = \frac{10\cdot7}{25\cdot13 \times 0\cdot259 - \dfrac{36}{24} - \dfrac{38}{24} \times \dfrac{0\cdot259}{1}} = \frac{10\cdot7}{4\cdot6}.$$

$$c = 2\cdot326 \text{ feet} = \text{say } 2 \text{ feet } 4 \text{ inches.}$$

The turbine of which the particulars are given in the table has a width of only 1 foot $5\frac{3}{4}$ inches; it is evident from this that the angles $a$ and $a_2$ must be less than has been assumed; it will be noted on reference to Table I. that for equal values of $a$ and $a_2$ with a given ratio $\dfrac{A}{A_2}$, the co-efficient $K_1$ is very nearly the same in all cases, so that provided $a = a_2$ the velocity of flow would remain very nearly unchanged; for $a = a_2 = 70°$, for instance, $K = 0\cdot672$ and $K_2 = 0\cdot632$.

On going through the calculations again with these values it will be found that

$$e = 1\cdot65 \text{ feet} = 1 \text{ foot } 7\cdot8 \text{ inches.}$$

This is not very different from the width of the Jonval turbine in question, and if the vanes were of wrought-iron, and therefore thinner, the agreement would be closer.

The calculated efficiency with $a = a_2 = 70°$ is **80** per cent., that actually obtained 83 per cent. including friction.

This proves that the co-efficients of resistance $\zeta_1$ and $\zeta_2$ for this wheel were in reality lower than has been assumed, and accounts for the best speed as proved by experiment being somewhat greater than that determined by calculation, in the ratio of $0\cdot642$ to $0\cdot632$ approximately.

It has been assumed throughout that $e_1 = e_2'$. To allow for inaccuracies in erection and vibration it is usual in practice to make the width $e_1$ of the wheel slightly greater than that $e$ of the guide passages; this clearance ranges from $\frac{1}{8}$ to $\frac{3}{8}$ inch.

To insure agreement with the preliminary assumptions as far as possible, $e_2$ may be calculated from

$$e_2 = \frac{A_2}{2\, r\, \Pi\, \cos a_2 - z_1\, t_2}$$

(where in the present case $A_2 = A$), and its value will be found slightly less than that of $e$, if the thickness of the ends of the vanes where the water leaves the wheel is the same as where it enters; substituting numerical values

$$e_2 = \frac{10\cdot7}{25\cdot13 \times 0\cdot259 - \frac{38}{24}} = \textbf{2·172 feet,}$$

$$c_2 = \textbf{2 feet 2} \tfrac{1}{16} \textbf{ inches.}$$

If, however, $c_2$ is made exactly the same as $e$, then the assumed condition $A = A_2$ may be complied with by altering the thickness $t_2$ of the vanes where the water leaves them.

## *Reaction Turbines.*

### (1) *Axial Turbines No. 2.*

A Jonval turbine is to be designed for a fall of **15** feet and to utilize **136** cubic feet of water per second.

Assumed value of $a$　=　**73° 30′.**

　　　　,,　　　　,,　　$a_2$ =　**74° 30′.**

$$\frac{A}{A_2} = \frac{cos\ a}{cos\ a_2} = \frac{0\cdot284}{0\cdot267} = \textbf{1·064.}$$

Velocity of flow $c = \sqrt{\dfrac{2gh}{2 \times 1\cdot064 \times 0\cdot959 \times 0\cdot964 +}}$

$$\overline{0\cdot125 + 1\cdot132 \times 0\cdot271}\ \cdot$$

$K_1 = \textbf{0·646.}$

$c = 0\cdot646 \sqrt{2g15} = 0\cdot646 \times 31 = \textbf{20·026 feet per} second, say **20** feet per second.

Velocity of rotation $w = K_2 \sqrt{2gh}.$

$K_2 = 0\cdot646 \times 1\cdot064 \times 0\cdot964 = \textbf{0·662.}$

$w = 0.662 \times 31 = \mathbf{20\cdot52}$ feet per second.

$tang\ a_1 = 3.606 - 3.376 = 0.230.$

$a_1 = \mathbf{13°}.$

$A = \dfrac{136}{20} = \mathbf{6\cdot8}$ square feet.

$r = 1.25\ \sqrt{A}$ (since $\dfrac{Q}{c}$ less than 16 sq. ft.).

$r = 1.25\ \sqrt{6\cdot8} = 1.25 \times 2.6 = \mathbf{3\cdot25} = \mathbf{3}$ ft. $\mathbf{3}$ in., or diameter = $\mathbf{6}$ feet $\mathbf{6}$ inches.

$P = \dfrac{r}{4\cdot5} = \dfrac{39}{4\cdot5} =$ say $\mathbf{9}$ inches.

$z = \dfrac{3.1416 \times 6\ \text{ft.}\ 6\ \text{in.}}{9\ \text{in.}} = \dfrac{245''}{9''} = \mathbf{27}.$

$z_1 = z + 1 = \mathbf{28}.$

$t = \tfrac{3}{8}'' = \mathbf{0\cdot03}$ ft.

$t_1 = \tfrac{3}{8}'' = \mathbf{0\cdot03}$ ft.

$c = \dfrac{6\cdot8}{20.42 \times 0.284 - 27 \times 0.03 - 28 \times 0.03 \times \dfrac{0.284}{0.974}}.$

$c = \dfrac{6\cdot8}{5.8 - 0.81 - 0.24} = \dfrac{6\cdot8}{4.695} = \mathbf{1\cdot235}$ ft.

$c = \mathbf{1}$ ft. $\mathbf{2\tfrac{13}{16}}$ in.

$A_2 = \dfrac{1}{1.064} \times 6.8 = \mathbf{6\cdot39}$ sq. ft.

$c_2 = \dfrac{6\cdot39}{5.45 - 0.84} = \mathbf{1\cdot386}$ ft. $= \mathbf{1}$ ft. $\mathbf{4\tfrac{5}{8}}$ in.

$c_1 = c + \tfrac{3}{16} = \mathbf{1}$ ft. $\mathbf{3}$ in.

The depth of wheel $h_o = \dfrac{r}{5} = \dfrac{39''}{5} =$ say $\mathbf{8}''$. It is very usual to make the depth $h_o$ of the guides the same as that of the wheel, and in this case $h^1 = 8''$. Some engineers make $h^1\ \tfrac{2}{3}$ to $\tfrac{3}{4}\ h_o$. To some extent the depth of the guides and wheel must depend on the pitch of the vanes;

U

with an exceptionally large pitch in proportion to the diameter, a greater depth than usual would be necessary in order to insure the water being sufficiently guided between the vanes and given the required direction of flow.

### *Reaction Turbines.*

#### *Inward Mixed-Flow Turbine.*

A fall of 30 feet, with a quantity of water equal to 120 cubic feet per second, is to be used for driving a turbine of the inward mixed-flow class.

The water—it is assumed—enters radially and leaves axially.

Owing to the nature of the ground and surroundings the water has to be conducted to the wheel through a wrought-iron pipe having a length $l$ of about 50 feet, consequently a certain amount of loss by friction is incurred before the water enters the turbine.

This loss may be approximately calculated by the well-known formula for pipe friction when the diameter of the pipe has been determined. The velocity $c_0$ in the pipe may be taken as 5 feet per second, and the area of the pipe must, in that case, be

$$A = \frac{120}{5} = 24 \text{ square feet,}$$

whence the diameter

$$d = \text{say } 5\cdot5 \text{ feet.}$$

The loss of head from friction is given by the formula

$$L = 0\cdot025 \, \frac{l}{d} \, \frac{c_0{}^2}{2g},$$

and when the numerical values above given are sub-
stituted—

$$L = 0 \cdot 025 \frac{50}{5 \cdot 5} \frac{25}{64 \cdot 4} = \mathbf{0 \cdot 088} \text{ foot.}$$

This is so small a proportion of the total head—only $0 \cdot 29$
per cent.—that in calculating the velocity of flow it may
safely be neglected. If there are no sharp bends or other
sources of loss in the pipe (and this in the present instance
is assumed to be the case), the co-efficient of velocity $K_1$
may be taken from Table No. I., or calculated from one of
the formulas given—that in which the hydraulic efficiency
occurs is the best for practical purposes, a value of $\epsilon$ being
employed which has been proved by experience to be safe
for a well-designed wheel.

Should the losses before entering the wheel be con-
siderable, say 5 per cent., then it would be necessary to
take into account their effect on the velocity of flow $c$.
This can most easily be accomplished by using a value
of the efficiency less by 5 per cent. than that otherwise
employed in determining the velocity of flow; instead
of 81 per cent., for instance, only 76 per cent.

Returning now to the design of a mixed-flow turbine
for the given fall and quantity of water, the angles $a$ and
$a_2$ and the ratios $\dfrac{A}{A_2}$ and $\dfrac{r_1}{r_2}$ may be assumed as follows:

$$a = \mathbf{75°}; \; a_2 = \mathbf{70°};$$

$$\frac{A}{A_2} = \mathbf{0 \cdot 75}; \; \frac{r_1}{r_2} = \tfrac{3}{2}.$$

The angle $a_2$ of outflow from the buckets is taken as
greater than $a$, the angle of outflow from the guides, in
order to avoid a very great width of bucket.

It must be understood that the assumed values apply
to the mean angle of each stream of water issuing from a
passage.

Referring to Table No. I., the value of $K_1$ corresponding to the preceding assumptions will be found to be

$$K_1 = 0.653.$$

The theoretical velocity due to the head is

$$V = \sqrt{2gh} = 43.953,$$

whence          $c = K_1 V = 28.70.$

The effective area of the guide passages,

$$A = \frac{Q}{c} = \frac{120}{28.70} = \text{say } 4.2 \text{ square feet.}$$

Approximately $\dfrac{c_1}{c_2} = \dfrac{r_1}{r_2} \dfrac{A}{A_2} \dfrac{\cos a_2}{\cos a} = 0.66$;

this is a perfectly practicable ratio, so that the calculations may be proceeded with on the above basis.

The outer radius of the wheel

$$r_1 = 1.25 \sqrt{A} = 1.25 \times 2.049,$$

or say          $r_1 = 2.5$ feet.

$r_2 = \frac{2}{3} r_1 = 1\frac{2}{3}$ foot.

The pitch of the guide-vanes at the radius $r_1$ may be taken as

$$P = \frac{r_1}{4} = \frac{2.5}{4} = 0.62,$$

say          $P = 0.625$ foot $= 7\frac{1}{2}$ inches.

Number of guide-vanes

$$z = \frac{2\,\Pi\,r_1}{P} = 25.$$

True pitch $P = 0.6284$ foot $= 7.540$ inches.

Number of wheel-vanes

$$z_1 = 0.7\,z = \text{say } 18;$$

this is perhaps rather a small value of $z_1$, resulting in a

somewhat large pitch for the wheel-vanes, and it will be better to make
$$z_1 = 20.$$
The relative angle of inflow $a_1$ can be determined graphically or by the formula
$$tang\ a_1 = \frac{A}{A_2}\frac{r_1}{r_2}\frac{sin\ a_2}{cos\ a} - tang\ a,$$
whence $tang\ a_1 = 4{\cdot}0826 - 3{\cdot}7320 = 0{\cdot}3506,$
and $a_1 = 19° 20'.$

It will be found on drawing out the vanes that, for the same *mean* angles $a$ and $a_2$, the depth available for the passage of the stream flowing between two guide-vanes is least when it is taken as the shortest distance $x$ between them (measured as explained in Chapter V., p. 122), and as this method of gauging the outflow areas agrees best with practical results, it will be here adopted.

Taking the thickness of the guide-vanes as about $\frac{3}{8}$ inch, it will be found on drawing out the curves that the depth of each stream is
$$x = 1\tfrac{3}{8}\ \text{inch} = 0{\cdot}1145\ \text{foot},$$
and the depth $s_1$ obstructed by each passing wheel-vane
$$s_1 = \tfrac{3}{32}\ \text{inch} = 0{\cdot}0078\ \text{foot}.$$
The width of the guide passages must therefore be
$$e = \frac{A}{0{\cdot}9\ (25 \times 0{\cdot}1145 - 18 \times 0{\cdot}0078)},$$
$$e = \frac{4{\cdot}2}{0{\cdot}9 \times 2{\cdot}724} = 1{\cdot}713\ \text{foot},$$
a co-efficient $0{\cdot}9$ being allowed for contraction.

As the water leaves axially, the wheel-vanes can be made parallel at the outlet, so that no contraction need be allowed for. The distance $x_2$ between the vanes is, as measured from a drawing,
$$x_2 = 1{\cdot}91\ \text{inch} = 0{\cdot}1591\ \text{foot}.$$

The area    $A_2 = \frac{4}{5} A = 5\cdot6$ square feet,

whence    $e_2 = \dfrac{5\cdot6}{20 \times 0\cdot1591} = 1\cdot757$ foot.

If the outflow is entirely axial, this width of buckets would make the greatest diameter at the outflow $3\cdot333 + 1\cdot757 = 5\cdot087$ feet, that is, larger than the diameter at the inflow, and although there are some American turbines constructed in this way, it is in the author's opinion very undesirable, as either that portion of the water entering at the lower edge of the buckets has to make a very sharp turn to fill the outflow orifices, or, what is more probable, the outflow orifices are only partially filled, and then an element of uncertainty is introduced into the design.

Under these circumstances it will be preferable to make the outflow inclined, with both a radial and axial component, but more nearly radial than axial. The same amount of contraction may then be allowed for as at the inflow. On setting out the vanes again to suit this modification in the design, the depth $x_2$ will be found as

$$x_2 = 1\tfrac{13}{16} \text{ inch} = 0\cdot1514 \text{ foot,}$$

and

$$e_2 = \frac{5\cdot6}{0\cdot9 \times 20 \times 0\cdot1514} = 2\cdot06 \text{ feet.}$$

The co-efficient of speed

$$K_2 = K_1 \frac{r_1}{r_2} \frac{A}{A_2} \, sin\, a_2 = K_1 \frac{sin\,(a + a_1)}{cos\, a_1},$$
$$K_2 = 0\cdot690.$$

The two methods of calculation afford a check on the accuracy of the calculation or construction of the angles.

It is not necessary to calculate $K_2$, as $w_1$ the velocity of rotation can be obtained by graphical methods, and if the construction is correct the inflow should be without shock

and the outflow radial. The velocity of rotation (by whichever method determined) is

$$w_1 = 0.690 \ V = \mathbf{30\cdot327} \text{ feet per second};$$

hence the number of revolutions per minute,

$$n = \frac{30\cdot327 \times 60}{15\cdot71} = \mathbf{115\cdot8},$$

or say $n = \mathbf{115}$ revolutions.

The available power,

$$E_{hp} = 30 \times 120 \times 0.1135 = \mathbf{408\cdot6} \text{ horse-power,}$$

and assuming the efficiency to be 75 per cent., the work done,

$$W_{hp} = \mathbf{306\cdot45}.$$

The total work done by the motor per second may be theoretically calculated by the formula

$$W = \frac{G}{g} \ c^2 \frac{r_1}{r_2} \frac{A}{A_2} \ sin \ a \ sin \ a_2 \text{ (foot lbs.),}$$

whence $W = 195,742\cdot3$ foot lbs. = $\mathbf{355\cdot9}$ horse-power, and the hydraulic efficiency

$$\epsilon = \frac{355\cdot9}{408\cdot6} = 0.87 \text{ or } \mathbf{87} \text{ per cent.;}$$

allowing 3 per cent. off this for friction, the actual efficiency of the wheel would be

$$\epsilon_e = \mathbf{84} \text{ per cent.}$$

In a wheel of the radial mixed-flow type, it must not be forgotten that the guide- and wheel-*vane angles* are much greater than the mean angles of outflow from the guides and buckets respectively; in the present case the vane angle for the guides, as measured from a drawing, is

$$a = \mathbf{82°},$$

while that for the buckets is

$$a_2 = \mathbf{83°}.$$

For the quantity of water and head given it would be better to give the turbine a larger diameter than that assumed, and thus reduce the width of the guide passages and buckets. It would then be possible to make the outflow axial, or nearly so, without any abrupt change in the direction of flow, and at the same time avoid the necessity of allowing for contraction in the bucket orifices.

By making the angles $a$ and $a_2$ *less* than has been assumed the dimensions of the motor could be still further reduced, and although this would involve some increase in the residual velocity and of the unutilized energy due to it, there would probably be no loss of efficiency, as the form of the guide and bucket passages would be more favourable, the changes in sectional area being less abrupt than with the greater angles.

## *Impulse Turbine.*

### (1) *Axial No.* 1.

A Girard turbine is required to utilize a quantity of water of 176 cubic feet per second, with a fall of 10 feet; the wheel must be of the parallel-flow type with vertical shaft.

$$Q = 176; \; h = 10.$$
$$a \text{ assumed} = 64°.$$
$$a_2 \quad „ \quad = 66°.$$
$$h_2 \quad „ \quad = 1 \text{ foot.}$$

As the fall is moderate the water may be conducted to the wheel through a casing or tube of sufficiently large area to render the loss in it by friction, before the water enters the guide apparatus, practically negligible, so that $L = 0$.

The effective head $H$ is therefore $h - h_2 = 10 - 1 = 9$, and the velocity of flow from the guide passages

$$c = 0\text{·}95\sqrt{2g\ 9} = \mathbf{22\text{·}871}\ \text{feet per second.}$$

The area of outflow of the guide passages

$$A = \frac{Q}{0\text{·}89\ c} = \frac{176}{20\text{·}355} = \mathbf{8\text{·}646}\ \text{square feet.}$$

The mean radius of the wheel may be taken as

$$* \ r = 1\text{·}4\ \sqrt{A} = 1\text{·}4 \times 2\text{·}94 = \mathbf{4\text{·}116}\ \text{feet;}$$

consequently the diameter = 8·236, say **8 feet 3** inches.
The pitch of the guide-vanes

$$P = \frac{r}{10} = \text{say } \mathbf{5} \text{ inches;}$$

hence $$z = \frac{2\ \Pi\ r}{P} = \mathbf{62.}$$

The turbine No. 7 in Table B, actually constructed for the assumed conditions, has 64 guide-vanes, and as within reasonable limits $z$ is a matter of choice, to facilitate further comparison it may be given the same value.

The thickness of the guide-vanes, which should be of wrought-iron or steel, may be assumed as

$$t = \tfrac{5}{16}\ \text{inch} = \mathbf{0\text{·}026}\ \text{foot.}$$

The clear width of the guide passages

$$c = \frac{A}{2\ \Pi\ r\ cos\ a\ -\ z\ t} = \frac{8\text{·}646}{11\text{·}353\ -\ 64\ \times\ 0\text{·}026}.$$

$$c = \mathbf{0\text{·}892}\ \text{feet} = \mathbf{10\text{·}7}\ \text{inches.}$$

$$a_1 = \text{(approximately) } 2\ a - 90° = 128° - 90° = \mathbf{38°}\ (-)$$

* As the rule for determining $r$ is empirical, the factor—ranging from 1·25 to 1·5—has been chosen so as to give a result agreeing with turbine No. 7 in Table B.

$$w = c\,\frac{\sin\,(a + a_1)}{\cos a_1} = 22\text{·}871 \times 0\text{·}556 = \mathbf{12\text{·}716} \text{ feet}$$

per second, should in designing be determined graphically.

$$c_1 = w = \mathbf{12\text{·}716}.$$

$$h_o = \frac{r}{5} = \text{say } 10'' = 0\text{·}833 \text{ feet}.$$

$$c_2 = 0\text{·}96\,\sqrt{(12\text{·}716)^2 + 64\text{·}4 \times 0\text{·}833}$$

$$= 0\text{·}96\,\sqrt{215\text{·}33}.$$

$$c^2 = \mathbf{14\text{·}086} \text{ feet per second.}$$

True value of $a_2$ for vertical outflow calculated by

$$\sin a_2 = \frac{w}{c_2} = \frac{12\text{·}716}{14\text{·}086} = \mathbf{0\text{·}903}\,;$$

$a_2$ can be conveniently ascertained graphically

$$a_2 = \mathbf{64°\ 30'}.$$

The assumed value of $a_2$ was 66°, and if vertical outflow is to be rigidly adhered to, $a_1$ must be revised. For practical purposes, however, it is sufficiently accurate. The outflow merely deviates very slightly, almost inappreciably, from the vertical direction; the originally assumed values may therefore be retained. For the turbine No. 7 in the table, $a_1 = 40°$ instead of 38°. The velocity $w$ of this wheel has the ratio 0·500 to the theoretical velocity of flow $\sqrt{2gh}$; it will be interesting to compare this with the result obtained above. It was found that

$$w = 0\text{·}556\ c = \mathbf{12\text{·}716}.$$

The theoretical velocity $V = \sqrt{64\text{·}4 \times 10} = \mathbf{25\text{·}375}$,

hence          $$\frac{w}{V} = \frac{12\text{·}716}{25\text{·}375} = \mathbf{0\text{·}501}\,;$$

from this it will be seen that the agreement is very close.

For $z_1$, as for $z$, the number given for turbine No. 7 in the table may be assumed, $z_1 = \mathbf{60}$.

$t_1$ and $t_2$ can be taken as equal to $t$,

$$t_1 = t_2 = t = \tfrac{5}{16}'' = 0\cdot026 \text{ foot.}$$

The ratio of the width at outflow to that at inflow, should be calculated by the accurate formula

$$\frac{e_2}{e_1} = \frac{x_1}{x_2}\,\frac{c_1}{c_2}\left(\frac{2\,\Pi\,r\,cos\,a_1 - z_1\,t_1}{2\,\Pi\,r\,cos\,a_2 - z_1\,t_2}\right);$$

$x_1 = 0\cdot9$ and $x_2 = 0\cdot7$, both assumed.

It should be noted that $\frac{e_2}{e_1}$ depends only on the ratio of $x_1$ : $x_2$. The absolute value of $x_1$ is fixed by the number and thickness of the vanes, and must be settled graphically.

$$\frac{e_2}{e_1} = \frac{0\cdot9}{0\cdot7}\,\frac{12\cdot716}{14\cdot086}\,\frac{25\cdot92 \times 0\cdot788 - 60 \times 0\cdot026}{25\cdot92 \times 0\cdot407 - 60 \times 0\cdot026}.$$

$$\frac{e_2}{e_1} = 1\cdot3 \times 0\cdot903 \times 2\cdot146 = 2\cdot5.$$

This is the same ratio as that given in the table for turbine No. 7.

Depth of guides $h^1 = \tfrac{3}{4}\,h_o = \tfrac{3}{4}\,10'' = 7\tfrac{1}{2}''$.

## *Impulse Turbine.*

### (1) *Axial No. 2.*

A turbine has to be constructed to utilize a fall of water subject to the following conditions :—

During nine months of the year, on the average, the available head is 11 feet and the quantity of water 106 cubic feet per second; for the rest of the year (in times of flood) the head is 9 feet and the quantity of water 114 cubic feet.

It will be seen by comparing the products of the head and quantity of water in the two cases, that the power available is *less* during times of flood with the smaller

head and greater quantity of water; as, however, this state of affairs only lasts for three months out of the twelve, it is desirable that the turbine should be designed chiefly with a view to suit the normal conditions, but at the same time to give a reasonably good efficiency in times of flood. The diminution of the head in the latter period is almost entirely due to the rise of the water-level in the tail-race, amounting to 2 feet. Unless, therefore, the wheel is suspended at least 2 feet above the normal tail-water level, it must work *drowned* in times of flood, but an impulse turbine would give poor results under such conditions. On the other hand, to place the wheel 2 feet above the lower surface of the water means the loss of over 18 per cent. of the available power during three-quarters of the year. The best solution of the difficulty consists in so designing the turbine that at ordinary times it acts as an impulse wheel, with the usual clearance above the water surface, while in times of flood it works as a reaction turbine, and may, without detriment to its efficiency, be submerged.

In the first place, it is evident, the guide passages and wheel-buckets must be of sufficient area to allow for the flow of the maximum quantity of water under the minimum fall, and therefore with the minimum velocity; in other respects the proportions must be determined as for an impulse turbine with free deviation.

As the wheel has to run at all times at a constant speed, matters must, if possible, be so arranged that the *best speed* is very nearly the same whether the motor works with free deviation or with reaction. As no long pipes are necessary for leading the water to the turbine, and the latter can be placed in a chamber or reservoir of relatively large sectional area, the velocity of flow from the guide passages, when free deviation is permitted, may be taken

as $0.95\sqrt{2gh}$, and in order that the conditions as regards speed may be the same with reaction as with free deviation, the co-efficient of flow $K_1$ must be so chosen that the following equation is satisfied:—

$$0.95\sqrt{(h-h_2)\,2g} = K_1\sqrt{2gh_1},$$

where $h$ is the maximum and $h_1$ the minimum head, while $h_2$ denotes the height above the lower water-level of the outflow orifices of the guide passages.

Taking as usual $h_2 = 1$ foot, there follows

$$0.95\sqrt{2g\,10} = K_1\sqrt{2g\,9}, \qquad \text{and since } 2g$$

is a factor common to both sides of the equation,

$$0.95\sqrt{10} = K_1\sqrt{9},$$

whence

$$K_1 = 0.95\sqrt{\frac{10}{9}} = 0.95 \times 1.054,$$

$$K_1 = 1.001, \text{ or say } 1\,;$$

that is, the velocity of flow when reaction is used must equal the velocity due to the head.

To attain this the turbine must be constructed with a low value of the ratio $\dfrac{A}{A_2}$, which can easily be determined as follows.

By the well-known formula—

$$K_1 = \sqrt{\frac{\epsilon}{2\dfrac{A}{A_2}\sin a \sin a_2}}\,;$$

$a$ and $a_2$ may be chosen, say $a = 64°$ and $a_2 = 68°$. The efficiency $\epsilon$, in order to be on the safe side, should not, in a turbine of this class, be taken as more than 75 per

cent., which, when friction is deducted, is equivalent to a mechanical efficiency of 70 to 71 per cent. From the above equation, when the assumed values are substituted, the ratio $\dfrac{A}{A_2}$ can be calculated.

$$\frac{A}{A_2} = \frac{\epsilon}{K_1^2\, 2\, \sin a \sin a_2}$$

$$\text{or } \frac{A}{A_2} = \frac{0\cdot75}{2 \times 0\cdot899 \times 0\cdot927} = \frac{0\cdot75}{1\cdot667},$$

$$\frac{A}{A_2} = 0\cdot45.$$

In a turbine with these proportions, it may be noticed, the pressure at the guide-passage orifices will be slightly *less* than that of the atmosphere, as otherwise the velocity of flow could not equal that due to the head.

The velocity of flow

$$c = 0\cdot95 \sqrt{64\cdot4 \times 10} = 1\cdot001 \sqrt{64\cdot4 \times 9},$$

$$c = 24\cdot099, \text{ say } 24 \text{ feet per second.}$$

The area of the guide passages $A$ must be calculated for the largest quantity of water likely to pass through the wheel, 114 cubic feet, hence

$$A = \frac{114}{0\cdot89 \times 24} = 5\cdot337 \text{ square feet.}$$

In determining $A$, the formula has been used which is applicable to Girard turbines, in which the obstruction of the wheel-vanes passing in front of the guide passages is allowed for by the co-efficient $0\cdot89$ instead of the method given for reaction wheels; the former is somewhat more simple than the latter method, and gives results which err on the safe side.

The mean radius of the wheel

$$r = 1\cdot5 \sqrt{A} = 1\cdot5 \times 2\cdot31 = 3\cdot46,$$

say $r = 3.5$ feet; $d = 7$ feet.

The pitch of the guide-vanes may be taken as—

$$P = \frac{r}{9} = \text{say } 4.5 \text{ inches,}$$

and $z = \dfrac{2\, r\, \Pi}{P} = \dfrac{263.9 \text{ inches}}{4.5} = \text{say } 58.$

The thickness of the guide-vanes of wrought-iron

$$t = \tfrac{5}{16} \text{ inch} = 0.0259 \text{ foot.}$$

The width of the guide passages

$$c = \frac{5.34}{21.90 \times 0.438 - 58 \times 0.0259} = \frac{5.34}{8.133},$$

$$c = 0.656 \text{ foot} = 7\tfrac{13}{16} \text{ inches.}$$

$$a_1 = 2 \times 64° - 90° = 38° \;(-).$$

The mean velocity of rotation—

$$w = 24 \times \frac{\sin(64° - 38°)}{\cos 38°} = 24 \times \frac{\sin 26°}{\cos 38°} = 24 \times \frac{0.438}{0.788}$$

$$w = 24 \times 0.556 = 13.34 \text{ feet per second.}$$

$c_1$ the relative velocity of inflow $= w$.

The depth of the wheel $h_o$ for a turbine intended to work, as in this case, both as an impulse and as a reaction turbine, should be somewhat greater than for an ordinary impulse turbine, since back vanes are used, and it will be found in drawing out the form of these that a greater length than usual is required in order to avoid too sudden curvature, and to make the lower ends parallel both on the back and front side.

It may therefore be assumed that

$$h_o = \frac{r}{4} = \text{say } 10 \text{ inches} = 0.833 \text{ foot,}$$

instead of $\dfrac{r}{5}$, as would be the case for an impulse wheel under ordinary circumstances.

The relative velocity of outflow from the buckets $c_2$ can now be calculated.

$$c_2 = 0.96 \sqrt{177.9 + 64.4 \times 0.833} = 0.96 \sqrt{231.54},$$
$$c_2 = \textbf{14·60 feet per second.}$$

The exact value of $a_2$ for vertical outflow would be given by—

$$\sin a_2 = \frac{13.34}{14.60} = \textbf{0·914},$$

whence $\qquad\qquad a_2 = \textbf{66}°;$

this differs by only 2° from the assumed value, so that the latter may be allowed to remain unaltered.

The ratio of the width of the buckets at the outflow to that at the inflow $\dfrac{c_2}{c_1}$ is determined for the wheel when working submerged by the ratio $\dfrac{A}{A_2}$ in conjunction with the other dimensions. On the other hand, when the action is that of an impulse turbine and the wheel runs clear of the tail-water, the width $c_2$ of the buckets at the outflow must be sufficiently great to prevent any possibility of the buckets being choked, that is, the proportion $x_2$ of the outflow orifice filled must be less than 1; it remains to be ascertained whether these two conditions are compatible with each other.

The relative velocity of outflow when the wheel works submerged is

$$c_2 = c \frac{A}{A_2} = 24 \times 0.45 = \textbf{10·80 feet;}$$

with free deviation it was found to be

$$c_2 = \textbf{14·60 feet.}$$

It is evident from this that if in the first case the water just *fills* the orifice, in the second, where the velocity is

greater, it will *not* quite fill it, so that clearance between the convex surface of the vane and the water is insured. It is consequently only necessary to proportion the widths of the buckets according to the requirements of the turbine when working submerged.

When the section through a guide passage is drawn out, the depth $x$ can be measured, also the depth $s_1$ of the portion of the passage obstructed by one wheel-vane; similarly, the depth $x_2$ of the outflow orifices of the buckets; then, as before demonstrated—

$$A = e \, (z \, x - z_1 \, s_1)$$

and

$$A_2 = e_2 \, z_1 \, x_2;$$

whence

$$\frac{e_2}{e} = \frac{A_2}{A} \frac{z \, x - z_1 \, s_1}{z_1 \, x_2}.$$

Assuming $t_1 = t = \frac{5}{16}''$ and $z_1 = 60$ it will be found that

$$\frac{c_2}{c_1} = \mathbf{2\cdot4}.$$

## *Impulse Turbines.*

### (2) *Radial Outward-Flow No. 3.*

A head of water $h = \mathbf{590}$ feet with a supply $Q = \mathbf{16}$ cubic feet per second is to be utilized. The head is measured to the orifices of the guide passages. In this case—the fall being very great and the quantity of water small—an outward-flow radial impulse wheel with horizontal shaft is the most suitable type of motor. In Table B the particulars of a turbine designed by Messrs. J. J. Rieter and Co. for similar conditions, and working at Terni in Italy, will be found, and may be compared with the results of calculation.

The available power is

$$E = \frac{590 \times 16 \times 62\cdot425}{550} = 1071\cdot4 \text{ horse-power.}$$

x

$$a \text{ assumed} = 70°,$$
$$a_2 \quad \text{,,} \quad = 70°.$$

It may be supposed for the sake of simplicity that the pipe conducting the water to the turbine is of such dimensions that the loss in it by friction is very small and is covered by using a lower value of the co-efficient with which $\sqrt{2gh}$ has to be multiplied to determine $c$. It is very easy to calculate the losses in the pipe when the dimensions are known; with high falls like that in question, the velocity of flow in the pipe may be 5 or 6 feet per second. For a velocity of 5 feet per second the area of the pipe must be $\frac{16}{5} = 3.2$ square feet and the diameter say 2·1 feet.

If the length of the pipe be twice the head, or say 1,600 feet, it will be found on calculation that the total loss from friction is equivalent to a head of about 5·1 feet, or less than 0·9 per cent. of the total fall; for a sharp bend with a mean radius of curvature equal to the diameter of the pipe, the loss of head would be—for the same velocity—about 0·8 of a foot. These figures make it clear that a co-efficient 0·92 will amply cover all losses incurred up to the moment the water leaves the guide orifices.

$$c = 0.92\sqrt{2g\ 590} = 179.33,$$

say
$$c = 179 \text{ feet per second.}$$

Area of guide passages $A = \dfrac{Q}{0.89\ c} = \dfrac{16}{159}.$

$$A = 0.1 \text{ square foot.}$$

For the inner radius $r_1$ a large value may be assumed in order to avoid too great a number of revolutions:

$$r_1 = 13\sqrt{A} = 13 \times 0.316 = 4.125,$$

or
$$r_1 = 4 \text{ feet } 1\tfrac{1}{2} \text{ inch.}$$

The pitch $P$ assumed $= 3$ inches $= 0.25$ foot.

Ratio $\dfrac{e}{r_1} = y = \frac{1}{10}$ assumed.

$$e = 0\cdot4125 \text{ foot,}$$

or say $\qquad e = 5$ inches $= \mathbf{0\cdot417}$ foot.

Proportion of circumference, exclusive of thickness of vanes, over which the water is admitted

$$\phi = \frac{A}{2\,\Pi\,r_1\,e\,\cos a} = \frac{0\cdot1}{25\cdot920 \times 0\cdot417 \times 0\cdot342},$$

$$\phi = \mathbf{0\cdot027}.$$

Number of vanes in guide apparatus

$$z = \frac{2\,\Pi\,r_1\,\phi}{P} = \frac{25\cdot920 \times 0\cdot027}{0\cdot25},$$

$$z = \mathbf{2\cdot8}, \text{ say } \mathbf{3}.$$

On reference to the dimensions of the turbine for similar conditions given in Table B it will be seen that there are only *two* guide passages; this is a matter of minor importance in a turbine in this class. Provided the water is sufficiently guided to leave the orifices in the required direction, the resistance is reduced by diminishing the number of vanes. The pitch in such cases is greater than the pitch of the wheel-vanes, and consequently the proportion of the depth of the buckets occupied by water at *the inflow* must be taken as $x_1 = 1$.

Adhering to the calculated number of guide passages $z = 3$, the *corrected* value of $\phi$ can now be determined after fixing the thickness of the vanes; this latter for steel vanes let into the guide-casing may be taken as $t = \frac{3}{16}$ inch $= \mathbf{0\cdot0156}$ foot.

$$\phi = \frac{A + z\,t\,e}{2\,\Pi\,r_1\,e\,\cos a} = \frac{0\cdot1 + 3 \times 0\cdot417 \times 0\cdot0156}{3\cdot708},$$

$$\phi = \mathbf{0\cdot032}.$$

From this the corrected value of the pitch

$$P = \frac{2\,\Pi\,r_1\,\phi}{z} = \frac{25\cdot920 \times 0\cdot032}{3} = \mathbf{0\cdot276},$$

or $P = 3\frac{5}{16}$ inches.

This of course can be more simply ascertained by graphical construction in a way requiring no description.

$$a_1 = 2a - 90° = 50°,$$

$a_1$ being of course negative, as is always the case in impulse wheels.

The velocity of the inner circumference of the wheel

$$w_1 = c\, \frac{\sin (a - a_1)}{\cos a_1} = c\, \frac{\sin 20°}{\cos 50°},$$

$$w_1 = \frac{0\cdot 342}{0\cdot 643} c = 0\cdot 532\, c,$$

or $0\cdot 532 \times 0\cdot 92 \sqrt{2gh} = 0\cdot 489 \sqrt{2gh};$

$$w_1 = 95\cdot 228 \text{ feet per second,}$$

and also $c_1 = 95\cdot 228$ feet per second.

The depth of the buckets $h_o = r_2 - r_1'$ may be taken as $h_o = \frac{1}{5} r_1 = 0\cdot 825.$

As the flow of the water through the buckets is *radially outward*, an increase of the relative velocity takes place over and above that due to the additional fall $h_o$, and hence

$$c_2 = 0\cdot 95 \sqrt{c_1{}^2 + w_2{}^2 - w_1{}^2 + 2gh_o},$$

or $c_2 = 0\cdot 95 \times 114\cdot 502 = 108\cdot 777,$

the velocity of the outer circumference being

$$w_2 = \frac{r_2}{r_1}\, w_1 = \frac{r_1 + h_o}{r}\, w_1 = \tfrac{6}{5}\, w_1,$$

$$w_2 = 1\cdot 2 \times 95\cdot 228 = 114\cdot 27.$$

The correct value of $a_2$ should be given by the relation $\sin a_2 = \frac{w_2}{c_2}$ (for radial outflow). As $c_2$ is less than $w_2$, it is evident that the outflow cannot be radial with the assumed angles; $a_1$ must therefore be modified.

By way of trial, let it be assumed that

$$a_1 = 54°,$$

then $\quad w_1 = c \dfrac{\sin 16°}{\cos 54°} = c \times \dfrac{0\cdot276}{0\cdot588} = 0\cdot470\, c,$

or $w_1 = 0\cdot432\sqrt{2gh} = 84\cdot13$ feet per second; $(K_2 = 0\cdot432).$

$$c_1 = c \dfrac{\cos 70°}{\cos 54°} = c \dfrac{0\cdot342}{0\cdot588} = 0\cdot581\, c,$$

$$c_1 = 104 \text{ feet per second,}$$

$$w_2 = \tfrac{6}{5}\, w_1 = 100\cdot9 \text{ feet per second,}$$

and $\quad c_2 = 0\cdot95\sqrt{c_1{}^2 + w_2{}^2 - w_1{}^2 + 2gh_o},$

$$c_2 = 112\cdot29.$$

The number of revolutions per minute will be

$$n = \dfrac{w_1 \times 60}{2\,\Pi\ r_1} = \text{say } 195$$

$$\sin a_2 = \dfrac{w_2}{c_2} = \dfrac{100\cdot9}{112\cdot29} = 0\cdot898$$

whence $\qquad a_2 = 64°.$

Referring to the Table B, the angle $a_1$ for the turbine before mentioned will be found to have the value 54° assumed last, and the angle $a_2$ calculated from this differs by 6° from the corresponding angle in the turbine as actually made. The proportion $\dfrac{r_2}{r_1}$ is a little different to that adopted in the preceding calculations, and this modifies slightly the results. The effect of these differences on the performance of the turbine is not great; the calculated velocity of rotation agrees very well with that adopted by the makers, as may be seen by comparing the values of $K_2$.

The ratio of the width of the buckets at the outflow to that at the inflow has now to be determined.

$$\frac{e_2}{c_1} = \frac{x_1}{x_2} \frac{c_1}{c_2} \left( \frac{2 \, \Pi \, r_1 \cos a_1 - z_1 \, t_1}{2 \, \Pi \, r_2 \cos a_2 - z_1 \, t_2} \right).$$

Assuming $P = 3''$, the number of vanes $z_1$ in the wheel is

$$z_1 = \frac{2 \, \Pi \, r_1}{P} = \frac{25 \cdot 920}{0 \cdot 25} = \text{say } \mathbf{100},$$

whence the correct value of $P = \mathbf{0 \cdot 259}$, or

$$P = \mathbf{3 \cdot 111} \text{ inches.}$$

The thickness of the vanes at the inflow and outflow may be taken as the same—

$$t_1 = t_2 = \tfrac{1}{2} \text{ inch} = \mathbf{0 \cdot 0416}.$$

By substituting these values in the preceding formula there follows :

$$\frac{e_2}{c_1} = \frac{x^1}{x_2} \frac{104}{112 \cdot 29} \left( \frac{25 \cdot 920 \times 0 \cdot 588 - 0 \cdot 0416 \times 100}{31 \cdot 102 \times 0 \cdot 438 - 0 \cdot 0416 \times 100} \right).$$

$$x_1 = \mathbf{1}, \text{ and } x_2 \text{ can be assumed as}$$
$$x_2 = \mathbf{0 \cdot 5},$$

whence $\qquad \dfrac{c_2}{c_1} = 2 \times 0 \cdot 926 \times 1 \cdot 083 = \mathbf{2 \cdot 17},$

say $\qquad\qquad \dfrac{e_2}{e_1} = \mathbf{2 \cdot 2}.$

$$c_1 = e + \tfrac{1}{2} = \mathbf{5\tfrac{1}{2}} \text{ inches,}$$
$$c_2 = 2 \cdot 2 \, e_1 = \text{say } \mathbf{12} \text{ inches.}$$

With the greater value of $a_2$ adopted in the turbine of Messrs. Rieter and Co. a smaller residual velocity is obtained, and although the outflow is not exactly radial, it only deviates by a few degrees from the radial direction. The formulas employed, although not mathematically accurate under such conditions, give results which are close enough to the truth for practical purposes, and the gain in efficiency is very appreciable. A strict agreement might be obtained by choosing a somewhat smaller value of $a_1$—between 54° and 50°—but it is hardly worth while to repeat the calculation for this purpose.

As is evident from the formula for the ratio $\frac{e_2}{e_1}$, an increase in the angle $a_2$ results in a greater value of that ratio, and for $a_2 = 70°$—other factors remaining as before— $\frac{e_2}{e_1} = 2\cdot94$. It is possible to carry the increase of $a_2$ so far that the advantages obtained are neutralized by the excessive axial components of the motion of the streams leaving the buckets, but this does not happen with the proportions adopted in practice.

Finally, for the sake of comparison, the work done can be calculated from the formula

$$W_{hp} = \frac{M\ w_1\ c\ sin\ a}{550},$$

$$M = \frac{\text{Weight of water per second}}{32\cdot2} = 31\cdot018,$$

$$W_{hp} = \frac{31\cdot018 \times 84\cdot13 \times 179 \times 0\cdot94}{550},$$

$$W_{hp} = 798 \text{ horse-power.}$$

This gives a hydraulic efficiency

$$\epsilon = \frac{W}{E} = \frac{798}{1071} = 0\cdot745,$$

or say $\epsilon = 74\frac{1}{2}$ per cent.

In consequence of the greater value of the angle $a_2$ in the wheel as actually made, the loss by unutilized energy is about $2\cdot9$ per cent. less than under the conditions assumed in the calculations, and this brings the hydraulic efficiency up to $77\cdot4$ per cent. Allowing 2 per cent. for shaft friction, the actual efficiency would thus be $75\cdot4$ per cent. The makers guarantee 75 per cent.

For this type of wheel the power absorbed in friction does not in practice exceed $1\cdot6$ per cent., so that in reality the allowance of 2 per cent. is more than sufficient, and leaves a certain margin beyond that already included in the co-efficients used in determining the velocities of flow.

# CHAPTER X.

## MEASUREMENT OF THE QUANTITY OF FLOWING WATER.

Enumeration of methods.—Direct measurement.—Measurement by current meters.—Measurement by floats.—Measurement over weirs : theoretical discharge, actual discharge, influences affecting discharge, Braschmann's formula, Donkin and Salter's experiments and table, Weisbach's formula, Francis's formula, Fteley and Stearns's experiments, comparisons.—Comparative table of results of experiments, &c., on discharge over weirs.—Measurement by meters.—Measurement of water levels : hook gauge, positions for measuring levels.—New method of gauging the flow of water ; the "Venturi" meter.

IN determining the available energy of a fall of water the most important—and at the same time most difficult—measurement to be made is that of the quantity of water passing in a given time. Errors amounting to many per cent. may arise from carelessness or a defective method in determining this factor of the power of a falling stream.

The following are the principal methods which have been employed for ascertaining the quantity or volume of water flowing through any given section of a stream or channel :—

(1) Direct measurement of volume in a calibrated collecting tank or reservoir ;

(2) Measurement of velocity of flow in any section of known dimensions by means of current meters ;

(3) Measurement of velocity of flow by floats;
(4) Measurement over weirs or tumbling bays;
(5) Measurement by various forms of meter.

## (1) *Direct Measurement in a calibrated collecting Tank.*

It is very rarely that this method for the measurement of water is practicable for any but the very smallest hydraulic motor; when applicable, it is by far the most reliable, being independent of experimental co-efficients or the errors of current or other meters, and requiring only average care in determining fixed dimensions.

The water as it leaves the motor is all collected in a tank or reservoir of known capacity, which it is most convenient to calibrate beforehand.  In some instances it may be practicable to use several tanks in succession, care being taken that in transferring the inflow from one tank to another no loss of fluid is incurred.

Sometimes with small quantities of water it is possible to use two comparatively small tanks or vessels, which are alternately filled and emptied.

## (2) *Measurement by Current Meters.*

With this method the velocity of flow of the water in a channel of known section is measured by current meters, and the quantity of water calculated from it. To arrive at accurate results it is necessary to subdivide a section of the stream, taken at right angles to the general direction of flow, into a number of parts, preferably of equal area, and to observe the velocity, as indicated by the current meters, in each of these parts for some time.  From the observed velocities in the different

parts the mean velocity for the whole section is deter-
mined.

The current meters used are those in which a screw-
shaped fan is made to revolve by the action of the
moving water; in some cases these are connected with
electrical stopping and starting apparatus.

It is preferable to use several meters simultaneously
as checks on each other, and all of them, previous to the
experiments, should be carefully tested to ascertain their
degrees of accuracy.

The axis of each current meter should be parallel to
the general direction of flow in which the velocity of the
stream is to be measured, as, if free to adjust itself to
the direction of flow at the particular point where it is
placed, the recorded velocity may be too great, since
*locally* the water may have a motion deviating in direction
from that in which the velocity is required, and in that
case only the *component* of the local velocity in the
general direction of flow must be taken into account.

If carefully carried out there is no doubt that this
method gives very reliable results; for very large quanti-
ties of water it is often the only one possible, with the
exception of that in which floats are used, to be subse-
quently described.

In each portion of the section of the stream several
observations should be taken at intervals. During the
trial of the Jonval turbine at Goeggingen, of which an
account is elsewhere given, *eight* separate observations,
each lasting *one minute*, were made with each of the
instruments employed.

As the surface of the water forms the boundary of the
top of the uppermost row of subdivisions of the channel
section, the area of these divisions of course varies some-
what during the experiments in consequence of changes in

the water level, and where accuracy is desired the latter must be observed and allowed for.

## (3) *Measurement by Floats.*

This consists in observing the speed with which a float is carried down the stream to be measured, and inferring the velocity of the latter from the former. It depends for its accuracy on a knowledge of the relation existing between the velocity of the float and the mean velocity of the water in the particular part of the stream traversed by the float.

As in the previous method, the channel should be divided into several parts, in each of which observations must be made, and the velocity should be ascertained at three different depths at least. For this purpose a body of slightly greater specific gravity than the fluid, and presenting a considerable surface to the action of the stream, is suspended at the desired depth from another body floating on the surface, of such a form as to offer much less resistance to the stream than the first, so that without sensible error the velocity with which the instrument is carried along by the current is that of the submerged body and of the stream at the particular depth below the surface at which it is placed. A hollow metallic ball supported from a small ball or cylinder of wood has been employed for this kind of float.

Another form of float is that consisting of two bodies of equal size and shape connected by a thin rod; one of these swims just below the surface, the other at any desired depth, regulated by the connecting rod. The velocity with which the apparatus moves is taken as being the mean of the velocities of the water at the depths at which the two bodies float. In this way the mean between the

surface velocity and that at any depth can be ascertained for several positions of the lower body. It is not necessary that the upper body should be close to the surface ; it may be made to swim at any required depth below, but must then be kept in place by being connected to a light float of small resistance at the surface.

Another form of measuring float is a cylindrical wooden staff weighted so as to swim in a nearly vertical position. Its velocity gives the mean velocity of the water throughout the depth occupied by the staff. For weighting this kind of float a metal cap containing shot, attached to the lower end, is employed.

### (4) *Measurement of Discharge of Water over Weirs or Tumbling Bays.*

This method of water-gauging is perhaps that which has been most frequently employed to measure the quantity of water consumed by hydraulic motors, chiefly because in many instances it was the most convenient method practicable.

The stream to be measured is dammed by a weir or other suitable obstruction, and all the water compelled to flow through a rectangular aperture open at the top, cut out from the upper edge of the weir, and generally known as a Tumbling Bay. Very frequently the water is allowed to fall over the whole length of the weir, in which case the sides of the canal or stream form the ends of the bay, and should be vertical and parallel for some distance above the weir (up-stream).

If $h$ denote the height of the water-level above the lower edge of the bay, measured some distance above the weir where there is no appreciable gradient on the surface

of the water, and $L$ the length of the bay, then the theoretical quantity of water which should be discharged over the bay is

$$Q = L \tfrac{2}{3} \sqrt{2gh^3}.$$

Experiment has proved that in reality the quantity discharged is *less* than this, so that the theoretical result has to be multiplied by some co-efficient $C_1$ less than unity to arrive at the true quantity.

A large number of investigations have been carried out under various conditions to determine the co-efficient, and assuming that all the observations are correct, the accuracy of the results obtained depends upon whether the co-efficient employed is applicable to the particular conditions under which the experiment is made. Very great care should therefore be taken to select a co-efficient determined under circumstances corresponding as closely as possible to those in which the measurement is to be made.

It has been found that the co-efficient $C_1$ varies with the following dimensions and conditions :—

Length of bay ;
Head over bay ;
Width of canal of approach ;
Nature and thickness of edges of bay ;
Presence or absence of thin edges at ends of bay (full width of canal or otherwise) ;
Distance from bottom of bay to bottom of canal.

With a given thickness and shape of the edges of the bay, the co-efficient $C_1$ varies chiefly with the *ratio* of the length of the bay to the width of the canal and with the *head* over the bay ; the distance from the bottom of bay to the bottom of the canal appears to have little influence on the results.

When the length of the bay is less than the width of

the canal of approach, so that the bay has thin edges at
the ends, it is said to have *end contractions*, since the thin
ends cause a contraction of the stream.

Braschmann has constructed a formula for determining
the value of the co-efficient, *including the factor* $\frac{2}{3}$, in which
the ratio of the length $L$ of the bay to the width $B$ of the
canal of approach and the head $h$ over the bay are taken
into account; it is as follows :—

$$Q = \left(0{\cdot}3838 + 0{\cdot}0386 \, \frac{L}{B} + \frac{0{\cdot}00174}{h}\right) L \sqrt{2gh^3},$$

all dimensions being in *feet*.

This formula is very generally used in Germany, and in
many cases in which the author has compared it with
experimental results shows a very fair agreement with the
latter, in others a considerable discrepancy.

A formula of this kind must be used with discretion,
and only applied within the limits for which it has been
constructed.

The following table has been chiefly compiled from that
given by Messrs. Donkin and Salter in their paper " On
the Measurement of Water over Weirs," published in
the *Minutes of Proceedings of the Institution of Civil
Engineers*, vol. lxxxiii., 1885-86 ; the arrangement, how-
ever, is somewhat different, being according to the length
of the bays to facilitate reference for practical purposes.
Some slight additional data have been added, and two
columns containing respectively the ratio of the length of
the bay to the width of the canal of approach and the
ratio of the length of the bay to the head.

The values of the co-efficients given are in most cases
those resulting from the mean curves, also taken from
Messrs. Donkin and Salter's paper.

Weisbach gives the following formula as embodying

the results of the experiments of Poncelet and Lesbros on
the flow of water over tumbling bays :—

$$Q = \mu \, L \, h \, \sqrt{2gh,}$$

where $\qquad\qquad \mu = \mu_o \, (1 + 1 \cdot 718 \, n^4),$

and $\qquad\qquad n = \dfrac{L \, h}{F_o},$

$F_o$ being the sectional area of the canal of approach and
$\mu_o$ the co-efficient of Poncelet and Lesbros for a bay $0 \cdot 2$
mètre ($7 \cdot 874$ inches) long.

Francis, whose data are generally employed in America,
gives the formula

$$Q = 3 \cdot 33 \, L \, \sqrt{h^3,}$$

equal to $\qquad\qquad Q = 0 \cdot 416 \, L \, \sqrt{2gh^3,}$

when there are no end contractions,

and $\qquad\qquad Q = 3 \cdot 33 \, (L - 0 \cdot 1 \, n \, h) \, \sqrt{h^3,}$

when end contraction takes place, where $n$ is the number
of end contractions, generally 2.

Braschmann's formula agrees very well with Francis's
results, but it will be seen on reference to the latter, in
the case in which the weir has a length equal to the width
of the canal of approach without end contractions, that the
co-efficient is practically constant. The same is true of
the experiments of Fteley and Stearns with the longer
weir. With the shorter weir, 5 feet long, the results
obtained by the same experimenters, with heads under
6 inches, show that the discharge is influenced by the
head ; above 6 inches the co-efficient is nearly constant.

According to Braschmann's formula, the co-efficient
varies in all cases with the head.

## COMPARATIVE TABLE OF RESULTS OF EXPE-

| Name of experimenter. | Length of bay of weir $L$ | Depth over weir $h$ | Width of canal of approach $B$ | | Co-efficient in formula $Q = C\,L\sqrt{2gh^3}$ $C$ |
|---|---|---|---|---|---|
| | Inches. | Inches. | Ft. | In. | |
| Lesbros. | 0·7874 | 3 | 12 | 1 | 0·435 |
| ,, | ,, | 6 | ,, | | 0·434 |
| ,, | ,, | 12 | ,, | | 0·433 |
| ,, | ,, | 18 | ,, | | 0·431 |
| ,, | ,, | 23½ | ,, | | 0·425 |
| Castel. | 1·181 | 1½ | 2 | 5⅛ | 0·4198 |
| ,, | ,, | 2 | ,, | | 0·4194 |
| ,, | ,, | 3 | ,, | | 0·4186 |
| ,, | ,, | 4 | ,, | | 0·4182 |
| ,, | ,, | 5 | ,, | | 0·4182 |
| ,, | ,, | 6 | ,, | | 0·4185 |
| ,, | ,, | 7 | ,, | | 0·4186 |
| ,, | ,, | 8 | ,, | | 0·4190 |
| Donkin and Salter. | 1·5 | ½ | 1 | 6 | 0·4256 |
| ,, | ,, | ¾ | ,, | | 0·421 |
| ,, | ,, | 1¼ | ,, | | 0·4174 |
| ,, | ,, | 1½ | ,, | | 0·4156 |
| ,, | ,, | 2 | ,, | | 0·4146 |
| ,, | ,, | 2¾ | ,, | | 0·4122 |
| ,, | ,, | 3 | ,, | | 0·412 |
| Prof. Kennedy. | 1·75 | ½ | 1 | 5 | 0·4666 |
| ,, | ,, | ¾ | ,, | | 0·435 |
| ,, | ,, | 1 | ,, | | 0·4256 |
| ,, | ,, | 1½ | ,, | | 0·414 |
| ,, | ,, | 2 | ,, | | 0·4053 |
| Castel. | 1·968 | 1½ | 2 | 5⅛ | 0·4090 |
| ,, | ,, | 2 | ,, | | 0·4086 |
| ,, | ,, | 3 | ,, | | 0·4082 |
| ,, | ,, | 4 | ,, | | 0·4080 |
| ,, | ,, | 5 | ,, | | 0·4080 |

-RIMENTS ON FLOW OF WATER OVER WEIR.

| Thickness of crest of bay or weir $t$ | Distance from crest of bay to bottom of canal $d$ | | Distance back from weir at which head was measured $l$ | | Ratio of length of bay to width of canal $\frac{L}{B}$ | Ratio of length of bay to head $\frac{L}{h}$ |
|---|---|---|---|---|---|---|
| | Ft. | In. | Ft. | In. | | |
| Very thin metal. | | 2⅛ | 11 | 6 | 0·00543 | 0·2624 |
| ,, | ,, | | | ,, | ,, | 0·1312 |
| ,, | ,, | | | ,, | ,, | 0·0656 |
| ,, | ,, | | | ,, | ,, | 0·0437 |
| ,, | ,, | | | ,, | ,, | 0·0335 |
| $\frac{1}{16}$ | | 6¾ | 1 | 7¼ | 0·0405 | 0·787 |
| ,, | ,, | | | ,, | ,, | 0·590 |
| ,, | ,, | | | ,, | ,, | 0·393 |
| ,, | ,, | | | ,, | ,, | 0·295 |
| ,, | ,, | | | ,, | ,, | 0·236 |
| ,, | ,, | | | ,, | ,, | 0·197 |
| ,, | ,, | | | ,, | ,, | 0·169 |
| ,, | ,, | | | ,, | ,, | 0·148 |
| $\frac{1}{16}$ | | 7 | 1 | 6 | 0·0833 | 3 |
| ,, | ,, | | | ,, | ,, | 2 |
| ,, | ,, | | | ,, | ,, | 1·2 |
| ,, | ,, | | | ,, | ,, | 1 |
| ,, | ,, | | | ,, | ,, | 0·75 |
| ,, | ,, | | | ,, | ,, | 0·545 |
| ,, | ,, | | | ,, | ,, | 0·5 |
| $\frac{1}{16}$ | | 7 | 1 | 9 | 0·1029 | 3·5 |
| ,, | ,, | | | ,, | ,, | 2·333 |
| ,, | ,, | | | ,, | ,, | 1·75 |
| ,, | ,, | | | ,, | ,, | 1·166 |
| ,, | ,, | | | ,, | ,, | 0·875 |
| $\frac{1}{16}$ | | 6¾ | 1 | 7¼ | 0·0679 | 1·312 |
| ,, | ,, | | | ,, | ,, | 0·984 |
| ,, | ,, | | | ,, | ,, | 0·656 |
| ,, | ,, | | | ,, | ,, | 0·492 |
| ,, | ,, | | | ,, | ,, | 0·393 |

Y

| Name of experimenter. | Length of bay or weir $L$ | Depth over weir $h$ | Width of canal of approach $B$ | | Co-efficient in formula $Q = C\,L\,\sqrt{2gh^3}$ $C$ |
|---|---|---|---|---|---|
| | Inches. | Inches. | Ft. | In. | |
| Castel. | 1·968 | 6 | 2 | 5⅛ | 0·4082 |
| ,, | ,, | 7 | ,, | | 0·4086 |
| ,, | ,, | 8 | ,, | | 0·4090 |
| ,, | ,, | 9 | ,, | | 0·410 |
| Smeaton. | 3 | ¼ | Not given. | | 0·442 |
| ,, | ,, | 1 | ,, | | 0·424 |
| ,, | ,, | 2 | ,, | | 0·412 |
| ,, | ,, | 3 | ,, | | 0·388 |
| Bidone. | 3·048 | 3·47 | 2 | 1·31 | 0·3994 |
| ,, | ,, | 6·66 | ,, | | 0·4113 |
| Castel. | 3·937 | 1¼ | 2 | 5⅛ | 0·411 |
| ,, | ,, | 1½ | ,, | | 0·403 |
| ,, | ,, | 2 | ,, | | 0·398 |
| ,, | ,, | 3 | ,, | | 0·395 |
| ,, | ,, | 4 | ,, | | 0·394 |
| ,, | ,, | 6 | ,, | | 0·395 |
| ,, | ,, | 8 | ,, | | 0·396 |
| ,, | ,, | 9½ | ,, | | 0·396 |
| Smeaton. | 6 | ½ | Not given. | | 0·445 |
| ,, | ,, | 1 | ,, | | 0·431 |
| ,, | ,, | 2 | ,, | | 0·425 |
| ,, | ,, | 3 | ,, | | 0·420 |
| ,, | ,, | 4 | ,, | | 0·412 |
| ,, | ,, | 5 | ,, | | 0·402 |
| ,, | ,, | 6 | ,, | | 0·390 |
| Bidone. | 6·721 | 3·97 | 2 | 1·31 | 0·4003 |
| Castel. | 7·874 | 1¼ | 2 | 5⅛ | 0·417 |
| ,, | ,, | 2 | ,, | | 0·407 |
| ,, | ,, | 3 | ,, | | 0·397 |
| ,, | ,, | 4 | ,, | | 0·395 |
| ,, | ,, | 6 | ,, | | 0·395 |
| ,, | ,, | 7 | ,, | | 0·397 |
| ,, | ,, | 8 | ,, | | 0·397 |

| Thickness of crest of bay or weir $t$ | Distance from crest of bay to bottom of canal $d$ | | Distance back from weir at which head was measured $l$ | | Ratio of length of bay to width of canal $\frac{L}{B}$ | Ratio of length of bay to head $\frac{L}{h}$ |
|---|---|---|---|---|---|---|
| | Ft. | In. | Ft. | In. | | |
| $\frac{1}{16}$ | 6¾ | | 1 | 7¼ | 0·0679 | 0·328 |
| ,, | ,, | | ,, | | ,, | 0·281 |
| ,, | ,, | | ,, | | ,, | 0·246 |
| ,, | ,, | | ,, | | ,, | 0·218 |
| Not given. | Not given. | | Not given. | | — | 12 |
| ,, | ,, | | ,, | | — | 3 |
| ,, | ,, | | ,, | | — | 1·5 |
| ,, | ,, | | ,, | | — | 1 |
| 1·33 | 5·76 | | 1 | 7¼ | 0·1204 | 0·878 |
| ,, | ,, | | ,, | | ,, | 0·457 |
| $\frac{1}{16}$ approx. | 6¾ | | 1 | 7¼ | 0·1351 | 3·149 |
| ,, | ,, | | ,, | | ,, | 2·624 |
| ,, | ,, | | ,, | | ,, | 1·968 |
| ,, | ,, | | ,, | | ,, | 1·312 |
| ,, | ,, | | ,, | | ,, | 0·984 |
| ,, | ,, | | ,, | | ,, | 0·656 |
| ,, | ,, | | ,, | | ,, | 0·492 |
| ,, | ,, | | ,, | | ,, | 0·414 |
| Not given. | Not given. | | Not given. | | — | 12 |
| ,, | ,, | | ,, | | — | 6 |
| ,, | ,, | | ,, | | — | 3 |
| ,, | ,, | | ,, | | — | 2 |
| ,, | ,, | | ,, | | — | 1·5 |
| ,, | ,, | | ,, | | — | 1·2 |
| ,, | ,, | | ,, | | — | 1 |
| 1·33 | 5·76 | | 1 | 7¼ | 0·2655 | 1·693 |
| $\frac{1}{16}$ | 6¾ | | 1 | 7¼ | 0·2703 | 6·299 |
| ,, | ,, | | ,, | | ,, | 3·937 |
| ,, | ,, | | ,, | | ,, | 2·625 |
| ,, | ,, | | ,, | | ,, | 1·968 |
| ,, | ,, | | ,, | | ,, | 1·312 |
| ,, | ,, | | ,, | | ,, | 1·125 |
| ,, | ,, | | ,, | | ,, | 0·984 |

| Name of experimenter. | Length of bay or weir $L$ | Depth over weir $h$ | Width of canal of approach $B$ | | Co-efficient in formula $Q = C L \sqrt{2gh^3}$ $C$ |
|---|---|---|---|---|---|
| | Inches. | Inches. | Ft. | In. | |
| Lesbros. | 7·874 | 1¼ | 12 | 1 | 0·410 |
| ,, | ,, | 2 | | ,, | 0·403 |
| ,, | ,, | 3 | | ,, | 0·397 |
| ,, | ,, | 4 | | ,, | 0·395 |
| ,, | ,, | 5 | | ,, | 0·394 |
| ,, | ,, | 6 | | ,, | 0·393 |
| ,, | ,, | 7 | | ,, | 0·392 |
| Castel. | 15·748 | 1⅜ | 2 | 5⅛ | 0·423 |
| ,, | ,, | 2 | | ,, | 0·417 |
| ,, | ,, | 3 | | ,, | 0·414 |
| ,, | ,, | 4 | | ,, | 0·414 |
| ,, | ,, | 5 | | ,, | 0·415 |
| ,, | 23·622 | 1¼ | | ,, | 0·435 |
| ,, | ,, | 1½ | | ,, | 0·430 |
| ,, | ,, | 2 | | ,, | 0·429 |
| ,, | ,, | 3 | | ,, | 0·430 |
| ,, | ,, | 4 | | ,, | 0·429 |
| Boileau. | 34·4 | 2 | 2 | 11·43 | 0·420 |
| ,, | ,, | 4 | | ,, | 0·429 |
| ,, | ,, | 6 | | ,, | 0·437 |
| ,, | ,, | 8 | | ,, | 0·444 |
| Blackwell. | 36 | 1 | 30 | 0 | 0·450 |
| ,, | ,, | 2 | | ,, | 0·446 |
| ,, | ,, | 3 | | ,, | 0·453 |
| ,, | ,, | 4 | | ,, | 0·411 |
| ,, | ,, | 5 | | ,, | 0·401 |
| Fteley and Stearns. | 60 | 1¼ | 5 | 0 | 0·456 |
| ,, | ,, | 1⅓ | | ,, | 0·443 |
| ,, | ,, | 1¾ | | ,, | 0·431 |
| ,, | ,, | 2 | | ,, | 0·428 |
| ,, | ,, | 2½ | | ,, | 0·424 |
| ,, | ,, | 3 | | ,, | 0·422 |

| Thickness of crest of bay or weir $t$ | Distance from crest of bay to bottom of canal $d$ | | Distance back from weir at which head was measured $l$ | | Ratio of length of bay to width of canal $\dfrac{L}{B}$ | Ratio of length of bay to head $\dfrac{L}{h}$ |
|---|---|---|---|---|---|---|
| | Ft. | In. | Ft. | In. | | |
| Knife edge. | 1 | 9¼ | 11 | 6 | 0·0543 | 6·299 |
| ,, | ,, | | ,, | | ,, | 3·937 |
| ,, | ,, | | ,, | | ,, | 2·625 |
| ,, | ,, | | ,, | | ,, | 1·968 |
| ,, | ,, | | ,, | | ,, | 1·575 |
| ,, | ,, | | ,, | | ,, | 1·312 |
| ,, | ,, | | ,, | | ,, | 1·125 |
| $\frac{1}{16}$ | | 6¾ | 1 | 7¼ | 0·5406 | 11·453 |
| ,, | ,, | | ,, | | ,, | 7·874 |
| ,, | ,, | | ,, | | ,, | 5·249 |
| ,, | ,, | | ,, | | ,, | 3·937 |
| ,, | ,, | | ,, | | ,, | 3·149 |
| ,, | ,, | | ,, | | 0·8109 | 18·897 |
| ,, | ,, | | ,, | | ,, | 15·748 |
| ,, | ,, | | ,, | | ,, | 11·811 |
| ,, | ,, | | ,, | | ,, | 7·874 |
| ,, | ,, | | ,, | | ,, | 5·905 |
| Sharp edge. | 1 | 1·381 | Head measured by glass gauge. | | 0·9709 | 17·2 |
| ,, | ,, | | ,, | | ,, | 8·6 |
| ,, | ,, | | ,, | | ,, | 5·733 |
| ,, | ,, | | ,, | | ,, | 4·3 |
| $\frac{1}{16}$ | Not given. | | Not given. | | 0·1000 | 36·0 |
| ,, | ,, | | ,, | | ,, | 18·0 |
| ,, | ,, | | ,, | | ,, | 12·0 |
| ,, | ,, | | ● ,, | | ,, | 9·0 |
| ,, | ,, | | ,, | | ,, | 7·2 |
| 0·0792 | 3·17 feet. | | 6 | 0 | 1·000 | 48·0 |
| ,, | ,, | | ,, | | ,, | 40·0 |
| ,, | ,, | | ,, | | ,, | 34·289 |
| ,, | ,, | | ,, | | ,, | 30 |
| ,, | ,, | | ,, | | ,, | 24 |
| ,, | ,, | | ,, | | ,, | 20 |

| Name of experimenter. | Length of bay or weir $L$ | Depth over weir $h$ | Width of canal of approach $B$ | | Co-efficient in formula $Q = C L \sqrt{2gh^3}$ $C$ |
|---|---|---|---|---|---|
| | Inches. | Inches. | Ft. | In. | |
| Fteley and Stearns. | 60 | 4 | 5 | 0 | 0·419 |
| ,, | ,, | 5 | ,, | | 0·418 |
| ,, | ,, | 6 | ,, | | 0·4174 |
| ,, | ,, | 7 | ,, . | | 0·4170 |
| ,, | ,, | 8 | ,, | | 0·4166 |
| ,, | ,, | 9 | ,, | | 0·4170 |
| ,, | ,, | 10 | ,, | | 0·4174 |
| Deacon. | 96 | 4·248 | — | | 0·4504 |
| ,, | ,, | 6·000 | — | | 0·4343 |
| ,, | ,, | 6·756 | — | | 0·4167 |
| Francis. | 120 | 7 | 14 | 0 | 0·4146 |
| ,, | ,, | 8 | ,, | | 0·4146 |
| ,, | ,, | 9 | ,, | | 0·414 |
| ,, | ,, | 10 | ,, | | 0·414 |
| ,, | ,, | 12 | ,, | | 0·414 |
| ,, | ,, | 14 | ,, | | 0·4146 |
| ,, | ,, | 16 | ,, | | 0·4148 |
| ,, | ,, | 18 | ,, | | 0·4143 |
| Francis. | 168 | 9 | 14 | 0 | 0·4162 |
| ,, | ,, | 10 | ,, | | 0·4163 |
| ,, | ,, | 11 | ,, | | 0·416 |
| ,, | ,, | 12 | ,, | | 0·416 |
| Fteley and Stearns. | 228 | 6 | 19 | 0 | 0·412 |
| ,, | ,, | 8 | ,, | | 0·4114 |
| ,, | ,, | 10 | ,, | | 0·411 |
| ,, | ,, | 12 | ,, | | 0·4106 |
| ,, | ,, | 14 | ,, | | 0·4110 |
| ,, | ,, | 16 | ,, | | 0·4111 |
| ,, | ,, | 18 | ,, | | 0·4110 |
| ,, | ,, | $19\frac{1}{2}$ | ,, | | 0·4103 |

| Thickness of crest of bay or weir $l$ | Distance from crest of bay to bottom of canal $d$ | Distance back from weir at which head was measured $l$ | | Ratio of length of bay to width of canal $\dfrac{L}{B}$ | Ratio of length of bay to head $\dfrac{L}{h}$ |
|---|---|---|---|---|---|
| | | Ft. | In. | | |
| 0·0792 | 3·17 feet. | 6 | 0 | 1·000 | 15 |
| ,, | ,, | ,, | | ,, | 12 |
| ,, | ,, | ,, | | ,, | 10·000 |
| ,, | ,, | ,, | | ,, | 8·571 |
| ,, | ,, | ,, | | ,, | 7·500 |
| ,, | ,, | ,, | | ,, | 6·666 |
| ,, | ,, | ,, | | ,, | 6·000 |
| $\frac{3}{32}$ | — | — | | ,, | 22·599 |
| ,, | — | — | | ,, | 16·000 |
| ,, | — | — | | ,, | 14·209 |
| $\frac{1}{4}$ | 5   0 | 6 | 0(?) | 0·7143 | 17·143 |
| ,, | ,, | ,, | | ,, | 15·000 |
| ,, | ,, | ,, | | ,, | 13·333 |
| ,, | ,, | ,, | | ,, | 12·000 |
| ,, | ,, | ,, | | ,, | 10·000 |
| ,, | ,, | ,, | | ,, | 8·571 |
| ,, | ,, | ,, | | ,, | 7·500 |
| ,, | ,, | ,, | | ,, | 6·666 |
| $\frac{1}{4}$ | 5   0 | 6 | 0(?) | 1·000 | 18·666 |
| ,, | ,, | ,, | | ,, | 16·800 |
| ,, | ,, | ,, | | ,, | 15·273 |
| ,, | ,, | ,, | | ,, | 14·000 |
| $\frac{1}{4}$ | 6·55 | 6 | 0 | 1·000 | 38·000 |
| ,, | ,, | ,, | | ,, | 28·500 |
| ,, | ,, | ,, | | ,, | 22·800 |
| ,, | ,, | ,, | | ,, | 19·000 |
| ,, | ,, | ,, | | ,, | 16·285 |
| ,, | ,, | ,, | | ,, | 14·250 |
| ,, | ,, | ,, | | ,, | 12·666 |
| ,, | ,, | ,, | | ,, | 11·692 |

In order that Francis's formula may give reliable results, its author states that the edges of the weir (or bay) presented to the current must be sharp; if bevelled or rounded off in any perceptible degree a material effect will be produced on the discharge. It is essential also, that the stream should touch the orifice only at these edges, after passing which it should be discharged through the air.

The velocity of the water approaching the weir should be very slow, and to this end the depth of the bottom of the canal below the weir-crest should not (according to Francis) be less than *one-third* of the length of the weir, otherwise the velocity of approach must be taken into account, as it tends to increase the quantity of water discharged.

With a thick weir-crest the co-efficient of discharge is greater than with a thin one. Experiments with weirs formed of planks 1 inch thick and over were made by Blackwell, but as the majority of other experiments of this kind have been carried out with thin bays, it has not been considered necessary to record the former here.

The most recent experiments on this subject, as far as the author is aware, are those carried out by Bazin, and recorded in the *Annales des Ponts et Chaussées* (October 1888, p. 446).

## (5) *Measurement by Meters.*

The gauging of water by meters is only applicable to comparatively small quantities, and is rarely practicable in the case of hydraulic motors. It depends for its accuracy on the construction of the particular form of meter used, which in every case should be carefully tested, and the error, if any, ascertained before it is employed.

## *Measurement of Water Levels.*

For ascertaining the level of the surface of the water, for the purpose of determining either the head acting on a turbine or the depth over a weir, the hook gauge is the best instrument to employ. It consists of a metal rod bent up at one end to form a hook with a sharp point, and adjustable by means of a nut working on a thread cut in the long shank of the hook, the readings being taken by a micrometer. The hook is used point upward, the point being at first submerged, and the position noted on the scale at which it emerges from the surface of the water; this can be done with great accuracy, much exceeding that obtainable by the reverse process, that of observing the position in which a point, at first above the water, comes into contact with it. The hook must, of course, be raised and lowered vertically, and should be enclosed in a box or casing, open at the top, to which the water is admitted only through a comparatively small hole in the bottom in order to protect the surface of the fluid in the immediate neighbourhood of the gauge from disturbances.

In ascertaining the upper level of the water driving a turbine, the measurement should *not* be taken *directly over the motor*, but in the head-race immediately before the water enters the turbine chamber or wheel pit, at a point where the surface is as nearly as possible undisturbed. Over the motor, especially when the entrance to the chamber is not central, there is often a depression of the surface caused by a whirling motion or vortex, and if observed here the head would have too small a value.

## The " Venturi" Water Meter.

A new method of measuring the quantity of water passing through a pipe has recently been proposed and tried by Mr. Clemens Herschel, the engineer of the Holyoke Water-power Company.[1]

This method is based upon the fact, investigated by Venturi, that when water flows through a pipe of which the section is contracted and subsequently gradually increased, the pressure in the smallest section is much less than in the largest on either side of the contraction, and may with suitable proportions sink below the atmospheric pressure, so that it can be measured by a vacuum gauge. The velocity in the smallest section is *theoretically* that due to the effective head corresponding to the difference between the pressure in the largest section before the contraction and that in the smallest section *plus* the influence of the velocity in the largest section, generally very slight. To obtain the *actual* velocity the theoretical quantity has to be multiplied with an experimental co-efficient. It is evident that when the latter is accurately known, together with the sectional area of the smallest part of the pipe, the true velocity of flow can be calculated from the observed difference of pressure above referred to.

Mr. Clemens Herschel's experiments were directed to determining the value of the co-efficient. Two sizes of " Venturi" ajutages were tried of exactly similar geometric form, one inserted in a pipe of 1 foot diameter, the other in a pipe of 9 feet diameter. The ajutage (or " venturi," as it has been christened by Mr. Herschel) consists of a pipe which converges and then diverges, and constitutes

[1] *The Venturi Water Meter*, by Clemens Herschel, M.Am.Soc.C.E., *Transactions of the American Society of Civil Engineers*, vol. xvii., Nov. 1888.

with the apparatus for recording the pressure the actual meter.

The diameter at the smallest section was one-third the diameter of the large ends of the conical portions, the angle of the cones being small. The length of the smaller cone was in each case about 1·68 diameters, that of the larger 7·37 diameters—referred to the greater diameter. The apparatus was placed so that the *shorter* cone was at the up-stream end. In the trials with the 1-foot tube an annular air-chamber surrounded the "venturi" at the smallest diameter, communicating by four accurately drilled holes, ¼ inch in diameter, with the interior of the pipe. The interior of the "venturi" at the small part was carefully polished, leaving the edges of the holes sharp and square.

For the 9-foot pipe the "venturi" had *eight separate* air-chambers, one for each ¼-inch hole communicating with the interior. These several air-chambers had each a suction-pipe attached, while only a single suction-pipe was used with the annular chamber of the 1-foot "venturi."

The trials with the smaller size of "venturi" were made first, and in the subsequent experiments with the 9-foot meter certain defects which had made themselves apparent were remedied, so that the second series of trials may be taken as the more accurate.

Experiments were made with velocities of flow ranging from 2 to 50 feet per second, while the depression at the "venturi" ranged from a very small amount to about 40 feet of water column.

In the case of the 9-foot tube the deviation of any single experiment from the resultant mean was only ½ per cent. for the whole range of velocities, and if both series of experiments are taken, the deviation did not exceed 3 per cent.

By limiting the use of the meter to velocities greater

than 9 feet per second through the "venturi," or about 1 foot per second through the corresponding pipe, the range of variation becomes even less than that above stated.

The discharge during the experiments always took place under water. The quantity of water discharged was measured in a collecting tank in the case of the 1-foot tube, and by a weir for the 9-foot tube.

The value of the co-efficient $C$ to be used in the formula—

$$\text{Velocity} = C \sqrt{2gh},$$

where $h$ is the effective head on the "venturi," was found in most cases to be near $0.99$, but varied slightly with the velocity.

If $p_1$ denote the pressure in the largest section of the meter on the up-stream side of the contraction, $p_2$ the pressure in the smallest section, $A_1$ and $A_2$ respectively the areas of the largest and smallest sections, then by the hydrodynamic equation—

$$p_1 + \left(\frac{A_2}{A_1}\right)^2 \frac{c^2}{2g} = p_2 + \frac{c^2}{2g}.$$

—where $c$ is the velocity of flow through the smallest section—and thence

$$c = \sqrt{\frac{(p_1 - p_2)\, 2g}{1 - \left(\frac{A_2}{A_1}\right)^2}},$$

where $p_1$ and $p_2$ are expressed as *head*, and therefore

$$p_1 - p_2 = h.$$

Since $\quad \dfrac{A_2}{A_1} = \tfrac{1}{9}, \ 1 - \left(\dfrac{A_2}{A_1}\right)^2 = \dfrac{80}{81},$

and theoretically $\quad c = 1.0062 \sqrt{2gh}.$

Figures 127 and 128 show the construction of the "Venturi" meter in the 9-foot pipe.

Fig. 127.

Fig. 128.

Since the publication of the first edition of this work, the "Venturi" meter has come into practical use in the United States. At the Columbian Exposition at Chicago (1893) a 36″ "Venturi" meter was employed for measuring the main water supply to the Exhibition.

The meter is now furnished with an automatic recorder, which shows by ordinary dials the volume of flow.

As described by Mr. Clemens Herschel, "The Recorder is connected with the tube by pressure pipes, which lead to it from the chamber surrounding the up-stream end, and the throat of the tube. It is operated in part by a weight and in part by clockwork. The difference of pressure or head at the entrance and at the throat of the meter is balanced in the Recorder by difference of level in two columns of mercury in cylindrical receivers, one within the other. The inner carries a float, the position of which is indicative of the quantity of water flowing through the tube. By its rise and fall the float varies the time of contact between an integrating drum and the counters by which the successive readings are registered.

"Usually the drum revolves once in every ten minutes, and at each revolution the counter registers, on ordinary dials, the volume flowing for that period of time. The interval of time may be shortened if desired."

The apparatus may be placed in any convenient position within one thousand feet of the tube, and there may also be an electric device by which the record can be made at any distance from the meter.

# CHAPTER XI.

### DESCRIPTIONS OF AND EXPERIMENTS WITH TURBINES.

General remarks.—Rittinger's experiments with Jonval turbines.
—Tables of results.—Haenel's turbine.—Tables of results.—Curves.
—Jonval turbines at Goeggingen.—Turbines at Olching.—Francis's
experiments; Tremont and Boott turbines.—Analysis of results.
—Holyoke testing station.—"Collins" turbine; with and with-
out suction-tube. — "Boyden" turbine. — "Hercules" turbine. —
Francis's experiments with "Humphrey" turbine.—"Little Giant"
turbine.—Girard turbines at Varzin.—Outward-flow impulse tur-
bines at Terni.—Turbine at Immenstadt.—Turbines at Schaff-
hausen.—The "Pelton" wheel.—Small impulse turbines for high
falls.—Fourneyron turbine at St. Blaise.—Niagara Installation.—
Francis's experiments with a "Swain" turbine.—Table of experi-
mental and calculated results, A.—Tables of dimensions of turbines
actually made, B and C.—The best speed of turbines as tested by
experiment.—Causes of differences between calculated and experi-
mental results.—Fliegner's experiments.—General remarks.

ALTHOUGH a large number of experiments and tests of
turbines have been carried out at various times, only a
comparatively small proportion of the published results
obtained are of any use for the purpose of accurate com-
parison and study. In this respect, some of the older
experiments are more valuable than those of more recent
date, owing to their greater thoroughness and the more
general conditions under which they were performed, more

especially as regards the range of speeds. In many instances the dimensions given are incomplete, so that a comparison of observed with calculated results is not possible; generally it is the angles of the vanes which are omitted. In all tests with turbines, the two chief quantities measured are the available power and the power actually developed. The latter can be easily determined with tolerable accuracy by means of the brake when ordinary precautions are taken, but for the former it is necessary to ascertain the quantity of water used by the motor, and this is an operation requiring more than average care and. nicety of observation if reliable results are to be obtained.

In the following, those experiments have been selected for description of which the fullest account of the water- and power-measurements is available, and which appear from the character of the latter to afford the best internal evidence of their trustworthiness. In general the most complete record obtainable of the dimensions of the turbines employed is given, and will be found, together with the relative proportions and various particulars as to speed and velocity of flow for the maximum efficiency, in Table A, as well as in the detailed account of the experiments in question.

In Table A are also stated the results obtained by calculation from observed quantities. From the *observed* quantity of water and *measured effective area* of guide passages the velocity of flow has been determined, and the corresponding speed of rotation calculated for comparison with that experimentally obtained; the theoretical co-efficient $K_1$ for the velocity of flow has been calculated from the observed efficiency by the formula—

$$K_1 = \sqrt{\dfrac{\epsilon}{2\dfrac{A}{A_2} \sin a \sin a_2}} \quad (vide \text{ p. } 109);$$

this affords the means of testing the agreement between theory and practice.

In the case of the older turbines, such as Rittinger's, the efficiencies of which are considerably below those now attained by the best wheels, the values of the co-efficients $\zeta$ and $\zeta_1$ as previously given are not applicable, and if $c$ is calculated by formula (vi A) with those values, the agreement between the observed and calculated results will not be good. For this reason the above expression for $K_1$ containing the efficiency, which can be directly noted, has been chosen as affording an accurate test of the correctness of theoretical assumptions.

The *measured* area $A$ it must be remembered is not the *effective* area, but to arrive at the latter an allowance must be made for the obstruction of the orifices by the passing wheel-vanes.

In some cases this has only been estimated, as the thickness of the wheel-vanes at the edges is not given.

### Rittinger's Experiments with Jonval Turbines.

Results of these experiments have been several times quoted in illustration of various points in the theory of reaction turbines. They are of special interest for several reasons: in the first place, each series was made with turbines of approximately the same power and working with the same head, but differing in some of their proportions; in the second place, they extended over a considerable range of speed, and last, but not least, the quantity of water was actually measured in a collecting

z

reservoir, so that there is no doubt as to the accuracy of this part of the observations, which in ordinary tests is most open to question.

Three different series of experiments at various times were made by Mr. Rittinger with eight turbines in all, and the leading results of these are given in the sequel, to some extent re-arranged, together with the principal dimensions and proportions.

All these experimental turbines of Rittinger's were as compared with present practice somewhat abnormal in their design, chiefly in the dimensions of the vane-angles; in most of them the angle $a_1$ was negative, and several had back-vanes. It is probably partly on this account that the efficiency did not prove higher. In turbines Nos. 1, 4, 5, 6, 7 and 8, the angle $a_1$ was negative, while in turbines 2 and 3 the same angle had a very considerable positive value, in all cases very different to that now usually adopted.

Experiments were made with all the guide passages fully open, and also with part of them closed, but only the former are here given, as the others have not so much interest for the purpose of comparison with theoretical results.

For comparison with the observed speed, the mean velocity of rotation has been calculated for three assumptions, (1) that the inflow takes place without shock, (2) that the outflow is vertical, and (3) that the best speed is represented by $\frac{1}{2}$ ($c_2$ *sin* $a_2$ + $c$ *sin* $a$), an expression, it will be remembered, obtained on the hypothesis that the velocity of flow is constant for all speeds, and therefore from a theoretical point of view inaccurate.

It will be seen that the calculated speeds for inflow without shock and vertical outflow with some exceptions agree very fairly; this proves that the turbines have

been designed in accordance with the generally accepted principles.

The correspondence between the speed thus fixed and that obtained by experiment is, however, not very close, but is generally better than between the latter and the velocity as calculated on assumption (3).

That the agreement is not closer is due to the unusual dimensions of the vane-angles; as long as the latter are within the limits usual in ordinary construction, theory and practice harmonize very well together, but from causes explained in the chapter "On the Best Speed of Turbines," this concord is not so complete under abnormal conditions, as then the relative direction of inflow accompanied by least resistance does not coincide with the vane-angle.

The results in the following tables are given in the original Austrian measures, which do not differ very greatly from English standards.

1 Vienna foot     = 1·03710 English foot.
1 Vienna sq. foot  = 1·07557 English sq. foot.
1 Vienna cubic foot = 1·11546 English cubic foot.

### *Results of Rittinger's Experiments.*

#### *Turbine No. 1.*

$a = 78° 0'$. Angle of guide-vanes at outflow.

$a_1 = -61° 0'$. Angle of wheel-vanes at inflow.

$a_2 = 64° 0'$. Angle of wheel-vanes at outflow.

$r = 10·833$ Vienna inches: Mean radius of wheel.

$z = 12$ : Number of guide-vanes.

$z_1 = 24$ : Number of wheel-vanes.

$c_1 = 4·333$ Vienna inches: Width of wheel-buckets at top.

$c_2 = 4·333$ Vienna inches: Width of wheel-buckets at bottom.

Depth of guide passages at outflow, 15 Vienna lines.
Depth of wheel-buckets at outflow, 18 Vienna lines.

N.B.—1 *Vienna inch* = 12 *lines.*

$$\frac{A}{A_2} = 0\cdot522. \quad A = 0\cdot452 \times 0\cdot9.$$

(0·9 is a co-efficient to allow for obstruction by wheel-vanes.)

| | $h$ | $Q$ | $n$ | $\epsilon_e$ | Remarks. |
|---|---|---|---|---|---|
| | Vienna feet. | Vienna cubic ft. | | | |
| 1 | 6·25 | 4·98 | 160 | 0 | |
| 2 | 6·21 | 5·25 | 147 | 0·326 | |
| 3 | 6·19 | 5·33 | 144 | 0·395 | |
| 4 | 6·18 | 5·40 | 140 | 0·455 | |
| 5 | 6·17 | 5·48 | 136 | 0·509 | |
| 6 | 6·16 | 5·57 | 131 | 0 553 | |
| 7 | 6·15 | 5·66 | 126 | 0·590 | Low degree of reaction. |
| 8 | 6·14 | 5·74 | 122 | 0·626 | |
| 9 | 6·13 | 5·83 | 116 | 0·646 | |
| 10 | 6·12 | 5·92 | 110 | 0·659 | |
| 11 | 6·11 | 6·02 | 105 | 0·671 | |
| 12 | **6·10** | **6·07** | **102** | **0·673** | |
| 13 | **6·09** | **6·12** | **99** | **0·673** | **15·044** = $c$. |
| 14 | 6·08 | 6·16 | 96 | 0·672 | |
| 15 | 6·08 | 6·22 | 93 | 0·667 | |
| 16 | 6·07 | 6·32 | 87 | 0·656 | |
| 17 | 6·06 | 6·43 | 80 | 0·631 | |

Feet per second.

|  |  |
|---|---|
| Best speed as observed . . . = | 9·358 |
| (1) Speed for inflow without shock . . = | 9·056 |
| (2) Speed for vertical outflow . . . = | 7·060 |
| (3) Speed calculated by | |
| $w = \frac{1}{2}(c_2 \sin a_2 + c \sin a)$ . . = | 10·866 |
| Mean of (1) and (2) . . . . = | 8·058 |

### Turbine No. 2.

$a$ = 73° 0′

$a_1$ = 54° 0′.

$a_2$ = 74° 0′.

$r$ = 10·833 Vienna inches.

$z$ = 12.

$z_1$ = 24.

$c_1$ = 4·333 Vienna inches.

$c_2$ = 4·333 Vienna inches.

Depth of guide passages at outflow, 17 Vienna lines.

Depth of wheel-buckets at outflow, 6 Vienna lines.

$\dfrac{A}{A_2}$ = 1·274.   $A$ = 0·511 × 0·9 Vienna square feet.

|    | $h$ | $Q$ | $n$ | $\epsilon_e$ | Remarks. |
|----|-----|-----|-----|-----|----------|
|    | Vienna feet. | Vienna cubic ft. |    |    |    |
| 1  | 6·25 | 5·25 | 242 | 0 |    |
| 2  | 6·25 | 5·12 | 201 | 0·452 |    |
| 3  | 6·25 | 5·13 | 189 | 0·533 |    |
| 4  | 6·25 | 5·11 | 177 | 0·601 |    |
| 5  | 6·25 | 5·08 | 164 | 0·655 | High degree of reaction. |
| 6  | 6·25 | 5·06 | 150 | 0·687 |    |
| 7  | 6·25 | 5·06 | 143 | 0·695 |    |
| 8  | **6·25** | **5·04** | **135** | **0·697** | **10·955** = $c$. |
| 9  | 6·25 | 5·02 | 127 | 0·695 |    |
| 10 | 6·25 | 5·01 | 119 | 0·688 |    |
| 11 | 6·25 | 4·99 | 102 | 0·650 |    |
| 12 | 6·25 | 4·97 | 84 | 0·586 |    |

Feet per second.

Best speed as observed . . . = 12·762

(1) Speed for inflow without shock . . = 14·899

(2) Speed for vertical outflow . . . = 13·400

(3) Speed calculated by

$w = \frac{1}{2} (c_2 \sin a_2 + c \sin a)$ . . = 11·943

Mean of (1) and (2) . . . . = 14·149

## *Turbine No. 3.*

$a$  = 68° 0'.

$a_1$ = 65° 0'.

$a_2$ = 75° 0'.

$r$  = 10·833 Vienna inches.

$z$  = 12.

$z_1$ = 12.

$c_1$ = 4·333 Vienna inches.

$c_2$ = 4·333 Vienna inches.

Depth of guide passages at outflow, 24 Vienna lines.

Depth of wheel-buckets at outflow, 13 Vienna lines.

$\frac{A}{A_2}$ = 1·661.   $A$ = 0·722 × 0·9 Vienna square feet.

|   | $h$ | $Q$ | $n$ |   | Remarks. |
|---|---|---|---|---|---|
|   | Vienna feet. | Vienna cubic ft. |   |   |   |
| 1 | 5·95 | 6·97 | 268 | 0 |   |
| 2 | 6·01 | 6·66 | 224 | 0·405 |   |
| 3 | 6·03 | 6·60 | 211 | 0·479 |   |
| 4 | 6·05 | 6·55 | 198 | 0·543 | Very high degree of |
| 5 | 6·06 | 6·49 | 183 | 0·589 | reaction. |
| 6 | 6·08 | 6·43 | 168 | 0·622 |   |
| 7 | 6·09 | 6·40 | 160 | 0·631 |   |
| 8 | **6·10** | **6·38** | **151** | **0·632** | **9·811** = $c$. |
| 9 | 6·10 | 6·35 | 142 | 0·630 |   |
| 10 | 6·11 | 6·32 | 132 | 0·618 |   |
| 11 | 6·13 | 6·24 | 113 | 0·587 |   |
| 12 | 6·15 | 6·19 | 91 | 0·519 |   |

Feet per second.

Best speed as observed       .     .       . = 14·272

(1) Speed for inflow without shock   .     . = 16·933

(2) Speed for vertical outflow   .     .       . = 15·746

Feet per second.

(3) Speed calculated by

$$w = \tfrac{1}{2} (c_2 \sin a_2 + c \sin a) \quad . = 12\text{·}418$$

Mean of (1) and (2) . . . $= 16\text{·}339$

With respect to Rittinger's turbines Nos. 1, 2, and 3, with which the first series of trials was carried out at Blankso, it should be noticed that they were chiefly designed to test the influence of varying degrees of reaction on the performance of the motors. The results showed that turbine No. 3, with the highest degree of reaction, gave the lowest efficiency, but in the experiments made with part of the guide passages closed, it also proved to be less sensitive in the matter of efficiency to the influence of this method of regulation.

The closing of half the guide passages reduced the efficiency of turbine No. 1 by 13 per cent., of turbine No. 2 by 19 per cent., and of turbine No. 3 by only 10 per cent.

A deviation from the *best speed* of 10 per cent. reduced the efficiency

of turbine No. 1 by 1 to 2 per cent.
    „    No. 2 by $\tfrac{1}{2}$ per cent.
    „    No. 3 by $\tfrac{1}{3}$ per cent.

The ratio of the best speed to the speed without a load was

for turbine No. 1 as 1 to 1·61,
    „    No. 2 as 1 to 1·79,
    „    No. 3 as 1 to 1·77.

The quantity of water consumed with turbine No. 1 increased as the speed diminished, owing to the negative value of the angle of inflow $a_1$, while with turbines Nos. 2 and 3, in which $a_1$ was positive, the contrary occurred. This is in accordance with theory.

*Turbine No. 4.*

$a = 78°.$
$a_1 = -63.$
$a_2 = 65°.$
$r = 8·333$ Vienna inches.
$z = 12.$
$z_1 = 24.$
$c_1 = 3·333$ Vienna inches.
$e_2 = 3·333$ Vienna inches.
Depth of guide passages at outflow, 8·17 Vienna lines.
Depth of wheel-buckets at outflow, 8·8 Vienna lines.
Depth of wheel-buckets at inflow, 9·33 Vienna lines.

$\dfrac{A}{A_2} = 0·418.$   $A = 0·1887 \times 0·9$ Vienna square feet.

|  | $h$ | $Q$ | $n$ | $\epsilon_e$ | Remarks. |
|---|---|---|---|---|---|
|  | Vienna feet. | Vienna cubic ft. |  |  |  |
| 1 | 9·34 | 3·06 | 289 | 0 | Turbine with ordinary |
| 2 | 9·22 | 3·78 | 186 | 0·693 | bucket form. |
| 3 | 9·20 | 3·89 | 171 | 0·711 |  |
| 4 | **9·20** | **3·94** | **164** | **0·715** | **23·20** = $c$. |
| 5 | 9·19 | 4·00 | 156 | 0·711 |  |
| 6 | 9·17 | 4·05 | 141 | 0·705 | Gradually decreasing |
| 7 | 9·16 | 4·17 | 126 | 0·674 | depth of passages between |
| 8 | 9·14 | 4·30 | 112 | — | wheel-vanes. |

Feet per second.
Best speed as observed . . . = 11·922
(1) Speed for inflow without shock . = 13·224
(2) Speed for vertical outflow . . = 8·779
(3) Speed calculated by
$w = \frac{1}{2}(c_2 \sin a_2 + c \sin a)$ . . = 15·738
Mean of (1) and (2) . . . = 11·001

*Turbine No. 5.*

$a = 76° 0'$.

$a_1 = - 61° 0$.

$a_2 = 65° 0'$.

$r = 8\cdot333$ Vienna inches.

$e_1 = 3\cdot333$ Vienna inches. $z = 12$.

$c_2 = 3\cdot333$ Vienna inches. $z_1 = 24$.

Depth of guide passages at outflow, 9·5 Vienna lines.

Depth of wheel-buckets at outflow, 8·5 Vienna lines.

Depth of wheel-buckets at inflow, 9·5 Vienna lines.

$\dfrac{A}{A_2} = 0\cdot504$.  $A = 0\cdot220 \times 0\cdot9$ Vienna square feet.

|   | $h$ | $Q$ | $n$ | $c_v$ | Remarks. |
|---|---|---|---|---|---|
|   | Vienna feet. | Vienna cubic ft. |   |   |   |
| 1 | 9·18 | 3·03 | 289 | 0 | Passages between wheel- |
| 2 | 9·14 | 3·60 | 184 | 0·623 | vanes of equal *horizontal* |
| 3 | 9·14 | 3·66 | 176 | 0·636 | depth throughout. |
| 4 | 9·13 | 3·71 | 167 | 0·641 |   |
| 5 | **9·13** | **3·77** | **159** | **0·645** | **19·040** $= c$. |
| 6 | 9·12 | 3·82 | 150 | 0·639 |   |
| 7 | 9·12 | 3·95 | 143 | 0·618 |   |
| 8 | 9·11 | 4·08 | 115 | 0·575 |   |

Feet per second.

Best speed as observed . . . = 11·471

(1) Speed for inflow without shock . = 11·464

(2) Speed for vertical outflow . . = 8·697

(3) Speed calculated by

$$w = \tfrac{1}{2}\,(c_2\,sin\,a_2 + c\,sin\,a)$$ . = 13·581

Mean of (1) and (2) . . . = 10·080

The experiments with turbines Nos. 4 and 5 were

made at Mariazell and intended to test the effect on their performance of different methods of constructing the buckets.

In turbine No. 4 the buckets were of the form usual in turbines of the type in question, with negative angle $a_1$; in No. 5 the wheel-vanes were so designed that the clear distance between them *measured parallel with the direction of rotation* was constant throughout.

Turbine No. 4 gave a maximum efficiency of $71\frac{1}{2}$ per cent., while No. 5 developed only $64\frac{1}{2}$ per cent.

With only $\frac{3}{4}$ of the guide passages open, the efficiency sank to about 40 per cent. in the case of No. 4, and 37 per cent. with $\frac{1}{2}$ the passages closed; under the latter conditions turbine No. 5 had an efficiency of 35 per cent.

The ratio of the best speed to the speed without a load was

for turbine No. 4 as 1 to 1·76,
„        No. 5 as 1 to 1·81.

### Turbine No. 6.

$a = 75^\circ 20'$.

$a_1 = -59^\circ 40'$.

$a_2 = 63^\circ 50'$.

$r - 0·4856$ Vienna feet.

$r_1 = 0·195$ Vienna feet.     $z = 12$.

$c_2 = 0·195$ Vienna feet.     $z_1 = 24$.

Depth of guide passages at outflow, 7·2 Vienna lines.

Depth of wheel-buckets at outflow, 6·0 Vienna lines.

Depth of wheel-buckets at inflow,  7·2 Vienna lines.

$\dfrac{A}{A_2} = 0·54.$   $A = 0·1174 \times 09$ Vienna square feet.

| | $h$ | $Q$ | $n$ | $\epsilon_o$ | Remarks. |
|---|---|---|---|---|---|
| | Vienna feet. | Vienna cubic ft. | | | |
| 1 | 17·61 | 2·41 | 585 | 0 | |
| 2 | 17·58 | 2·48 | 540 | 0·231 | |
| 3 | 17·55 | 2·55 | 493 | 0·410 | |
| 4 | 17·52 | 2·62 | 447 | 0·544 | |
| 5 | 17·49 | 2·69 | 400 | 0·633 | |
| 6 | 17·46 | 2·75 | 354 | 0·687 | |
| 7 | 17·45 | 2·79 | 331 | 0·697 | |
| 8 | **17·43** | **2·82** | **308** | **0·700** | **26·689** = $c$. |
| 9 | 17·42 | 2·86 | 285 | 0·694 | |
| 10 | 17·40 | 2·89 | 262 | 0·679 | |
| 11 | 17·37 | 2·96 | 216 | 0·626 | |

Feet per second.

Best speed as observed . . . = 15·645
(1) Speed for inflow without shock . . = 14·252
(2) Speed for vertical outflow . . . = 12·934
(3) Speed calculated by
$$w = \tfrac{1}{2} (c_2 \sin a_2 + c \sin a) \quad . \quad = 19\cdot375$$
Mean of (1) and (2) . . . = 13·593

*Turbine No. 7.*

$a = 75°\ 50'$.
$a_1 = -66°\ 0'$.
$a_2 = 61°\ 0'$.
$r = 0\cdot4856$ Vienna feet.
$c_1 = 0\cdot195$ Vienna feet. $z = 12$.
$c_2 = 0\cdot195$ Vienna feet. $z_1 = 20$.
Depth of guide passages at outflow, 6·9 Vienna lines.
Depth of wheel-buckets at outflow, 8·6 Vienna lines.
Depth of wheel-buckets at inflow, 6·9 Vienna lines.
$\dfrac{A}{A_2} = 0\cdot433.$   $A = 0\cdot1117 \times 0\cdot9$ Vienna square feet.

| | $h$ | $Q$ | $n$ | $\epsilon_e$ | Remarks. |
|---|---|---|---|---|---|
| | Vienna feet. | Vienna cubic ft. | | | |
| 1 | 17·45 | 2·84 | 430 | 0·485 | |
| 2 | 17·41 | 2·92 | 392 | 0·574 | Ordinary buckets. |
| 3 | 17·37 | 3·00 | 357 | 0·638 | |
| 4 | 17·33 | 3·08 | 320 | 0·670 | |
| 5 | **17·30** | **3·12** | **302** | **0·678** | **31·03** = $c$. |
| 6 | 17·28 | 3·16 | 283 | 0·676 | |
| 7 | 17·24 | 3·24 | 247 | 0·659 | |
| 8 | 17·20 | 3·32 | 210 | 0·616 | |

Feet per second.

Best speed as observed   .   .   . = 15·352

(1) Speed for inflow without shock  .   . = 13·034

(2) Speed for vertical outflow  .   . = 11·756

(3) Speed calculated by

$$w = \tfrac{1}{2} (c_2 \sin a_2 + c \sin a) \quad . \quad . = 20·927$$

Mean of (1) and (2)  .   .   . = 12·365

### *Turbine No. 8.*

The dimensions of this turbine were the same as those of No. 7, the only difference being in the form of the buckets.

| | $h$ | $Q$ | $n$ | $\epsilon_e$ | Remarks. |
|---|---|---|---|---|---|
| | Vienna feet. | Vienna cubic ft. | | | |
| 1 | 17·49 | 2·92 | 440 | 0·481 | Bucket with passages |
| 2 | 17·36 | 3·00 | 403 | 0·576 | of nearly equal |
| 3 | 17·32 | 3·08 | 367 | 0·641 | depth throughout. |
| 4 | 17·29 | 3·18 | 332 | 0·679 | |
| 5 | 17·27 | 3·20 | 315 | 0·691 | |
| 6 | **17·25** | **3·24** | **297** | **0·691** | **32·222** = $c$. |
| 7 | 17·24 | 3·28 | 279 | 0·690 | |
| 8 | 17·22 | 3·32 | 261 | 0·680 | |
| 9 | 17·19 | 3·40 | 225 | 0·645 | |

<div align="right">Feet per second.</div>

|  |  |  |  |  |
|---|---|---|---|---|
| Best speed as observed | . | . | . | $= 15\cdot100$ |
| (1) Speed for inflow without shock | . | . | . | $= 13\cdot533$ |
| (2) Speed for vertical outflow | . | . | . | $= 12\cdot205$ |

(3) Speed calculated by

$$w = \tfrac{1}{2}\left(c_2 \sin a_2 + c \sin a\right) \quad . \quad . = 21\cdot730$$

Mean of (1) and (2) . . . $= 12\cdot869$

If no allowance be made in calculating the effective value of $A$ for the obstruction of the guide passages by the passing wheel-vanes, then all of Rittinger's turbines have been designed for simultaneous inflow without shock and vertical outflow. Rittinger in designing the wheels appears to have neglected the obstruction caused by the wheel-vanes; this seems to be confirmed by the fact that turbines 4 and 5 were designed for a consumption of water per second of respectively 4·77 and 5·01 cubic feet (Austrian), whereas at the best speed the experiments showed that only 3·94 and 3·77 cubic feet were actually used, indicating probably an insufficient area of the guide orifices.

Turbines 6, 7, and 8 were also tested at Blankso. No. 6 had ordinary wheel-vanes, while Nos. 7 and 8, in other respects exactly alike, differed in the construction of the vanes. No. 7 was made with ordinary vanes and passages of decreasing depth towards the outflow, while No. 8 had passages of nearly equal depth throughout, measured perpendicular to the direction of flow—the object being to determine which method of design gave the better results.

The difference in efficiency of the three wheels was very slight.

The ratio of the best speed to the speed without a load was

<div align="center">for turbine No. 6 as 1 : 1·90.</div>

An inspection of the experimental and calculated results obtained with Rittinger's turbines, as given in a compendious form in Table A for the best speeds, shows very plainly the effect of varying ratios $\dfrac{A}{A_2}$ on the velocity of flow and speed.

The agreement between the theoretical and experimental values of $K_1$ is on the whole satisfactory, especially when it is considered that the effective area of outflow from the guide passages, from which the experimental velocity of flow must be calculated, is somewhat uncertain.

For turbines 1, 5, 6, 7, and 8, the speed for inflow without shock agrees most closely with the observed best speed; for turbine 4 the mean of the speeds for inflow without shock and vertical outflow gives the closest approximation to the experimental value. In all of these wheels the relative angle of inflow $a_1$ is negative; and for 1, 5, 6, 7, and 8, the observed speed is *greater* than that corresponding to inflow without shock.

With turbines 2 and 3 the actual speed agrees best with that calculated for vertical outflow. Each of these wheels has a very unusually large *positive* angle $a_1$ and exceptionally high ratios $\dfrac{A}{A_2}$. The observed speed is *less* than that calculated for inflow without shock.

In the absence of available data as to the thickness of the wheel-vanes at the inlet, it has been assumed that the obstruction of the guide orifices by the passing wheel-vanes reduces their area by 10 per cent., so that to arrive at the effective value of $A$, the measured area $A^1$ has been multiplied by 0·9. This would be rather too small a co-efficient for wheels of large or moderate size,

but for small turbines like those in question is probably not too low, since in a small motor the thickness of the vanes is relatively to the area of the passages greater than in a large one.

### Haenel's Experiments with an Axial Turbine.

About the year 1861 Edward Haenel, who is generally credited with having constructed the first turbines with back-vanes, carried out a series of trials with a turbine of his own design, which were very complete and instructive in their results.

Haenel, who was at that time manager of the Gräflich Stolberg'schen Maschinenfabrik at Magdeburg, made the experiments in question in conjunction with Mr. J. C. Bernhard-Lehmann, the present manager of the Maschinenfabrik von Queva at Erfurt.

## Description of the Turbine.

The turbine with which the trials were made was one of eight, all exactly alike, constructed to drive a mill at Rothenburg on the Saale. Fig. 130 (p. 362) shows a vertical section of this motor.

The guide apparatus consisted of an inner casing carrying the guide-vanes, and an outer casing connected with the supporting-frame. The vanes, thirty-two in number, were fastened by rivets to projections on the inner casing. The thickness of twenty-four of these vanes was $\frac{1}{4}$ inch; that of the remaining eight, $\frac{3}{8}$ inch. To the ends of the latter were attached radial wrought-iron bars projecting beyond the outer circumference of the vanes and fitting into corresponding recesses in the outer casing; these bars carried the weight of the guide-

wheel and regulating apparatus. Surrounding the outer casing was an annular channel, closed at the top by a wrought-iron ring, and divided by radial ribs into thirty-two compartments, each communicating with a guide passage by means of a hole drilled through the casing. The wrought-iron ring had an equal number of corresponding holes, in each of which was inserted a wrought-iron pipe reaching above the head-water level and communicating with the atmosphere. A boss with four arms was connected by set screws to the inner casing of the guide-wheel, and the spaces between the arms filled in by sectors of iron plate.

The boss contained a bearing for the turbine-shaft consisting of three blocks of lignum-vitæ held in cast-iron frames; these were adjustable by means of wedges attached to rods projecting above the upper water-level, so that they could be conveniently manipulated.

The wheel itself consisted of a cast-iron ring with four arms, wrought-iron vanes riveted on, and an outer casing of sheet-iron.

The ring was cast with projecting lugs to which the arms were riveted. There were thirty-two back-vanes constructed of iron $\frac{1}{4}$ inch thick.

The surfaces of all the vanes both of the guide apparatus and wheel were polished in order to reduce friction as much as possible.

The entrance of the water to the guide passages was controlled by a scroll-regulator consisting of two conical rollers diametrically opposite, carrying bands of gutta-percha, as elsewhere described. The regulator was worked by a spur-wheel and pinion, the latter keyed to a vertical shaft carried up above the head-water level. The gutta-percha bands were furnished with strips of wrought-iron on the outer circumference, to give stiffness and resist

the pressure of the water; these strips were $\frac{1}{8}$ inch thick, $\frac{7}{8}$ inch wide, and placed $\frac{1}{8}$ inch apart, each being attached by four copper rivets to the band, which was itself $\frac{1}{4}$ inch thick.

The turbine-shaft was of cast-iron, hollow, with the main-bearing above water, the weight being carried by a wrought-iron spindle fixed in a socket resting on the foundation.

The passages between the wheel-vanes were so designed as to be of equal section throughout, measured normally to the direction of flow of the water, so that the relative velocity of flow in the buckets would remain constant when the wheel worked under water. To attain this end, the buckets were made wider at the outflow than at the inflow, and back-vanes, as already stated, were employed.

The following were the leading dimensions and proportions of the turbine :—

| | | |
|---|---|---|
| Outer diameter of guide and wheel at inflow . . . . . | | = 5 ft. $7\frac{1}{2}$ in. |
| Inner diameter of guide and wheel at inflow . . . . . | | = 4 ft. $4\frac{1}{2}$ in. |
| Mean diameter of guide and wheel at inflow . . . . | . $d$ | = 5 ft. 0 in. |
| Outer diameter of wheel at outflow | . | = 6 ft. 3 in. |
| Inner diameter of wheel at outflow | . | = 3 ft. 9 in. |
| Width of wheel-buckets at inflow | . $c_1$ | = 1 ft. 3 in. |
| Width of wheel-buckets at outflow | . $c_2$ | = 2 ft. 6 in. |
| Depth of wheel . . . | . $h_o$ | = 1 ft. 0 in. |
| Angle of outflow from guide-vanes | . $a$ | = 67° 30′. |
| Angle of inflow of wheel-vanes . | . $a_1$ | = 45° 0′. |
| Angle of outflow of wheel-vanes on the concave side . . . . | . $a_2$ | = 63° 40′. |

A A

Angle of outflow of wheel-vanes on the
convex side . . . . . $a_2 = 67°\ 0'$.
Measured area of outflow from
guide passages . . . $A^1 = 3\cdot28824$ sq. ft.
Effective area of outflow from
guide passages . . . $A \doteq 2\cdot877984$ sq. ft.
Effective area of outflow from
wheel-passages . . . $A_2 = 6\cdot0155$ sq. ft.

$$\frac{A}{A_2} = 0\cdot47843 \text{ sq. ft.}$$

All dimensions given are in *Prussian* feet and inches.
1 lineal Prussian foot $= 1\cdot02972$ English foot.
1 square Prussian foot $= 1\cdot06032$ English square foot.
1 cubic Prussian foot $= 1\cdot09183$ English cubic foot.

## *Measurement of Power developed.*

The effective work done by the turbine was measured
with the ordinary "Prony" brake, applied to a brake-
pulley fitted on to the spur-wheel keyed to the turbine-
shaft, and made in halves bolted together.

The brake-lever was connected by a hinged rod with
a bell-crank lever, to one arm of which the weights were
suspended. One of the brake-blocks prolonged to form
the brake-lever was suspended by a chain to the roof.
The brake-pulley had an outer diameter of 65¾ inches
and a breadth of 6⅛ inches, with two flanges ¼ inch
deep. Water was the lubricant employed.

To determine the work actually developed by the
wheel, that portion of it spent in overcoming shaft-
friction had to be estimated and added to the value ob-
tained by brake-measurement. For this purpose the
weight of the different parts carried by the main-bearing

was ascertained, and the frictional resistance calculated with the co-efficient 0·075 given by Morin for footstep bearings. The diameter of the bearing-spindle was 3 inches, consequently the effective radius at which the resistance must be taken as acting was 1 inch. The downward pressure exerted by the water during its passage through the buckets was neglected in determining the work of friction, as being very small compared with the weight of the turbine. The total weight on the bearing was taken as 6,932 lbs. (Prussian).

The ratio of the actual work done by the wheel (work measured by brake *plus* work required to overcome friction) to the available energy gives the hydraulic efficiency $\epsilon$, as distinguished from the actual efficiency $\epsilon_e$.

## *Water Measurements.*

In determining the quantity of water used by the turbine, a float was employed for measuring the velocity of the fluid in the head-race. The latter was of exactly rectangular section, with horizontal floor, and 8 feet 11 inches wide.

The float was formed of two wooden discs, 3 feet in diameter and 2 inches thick, connected by twelve round bars of wood equidistant from each other in a circle of 32 inches diameter, the diameter of each bar being $1\frac{1}{4}$ inch; the total depth of the float was 2 feet 6 inches.

Detailed particulars of the manner in which the measurements were carried out are not given in the original account of the trials.

The time was observed occupied by the float in traversing a distance of 35 feet after it had attained a uniform motion.

The water-levels were measured in close proximity to

the turbine.   The level of the tail-water was indicated by
means of a float inserted in a pipe $1\frac{1}{2}$ inch diameter,
dipping into the tail-race and passing up through the
floor of the head-race.   To the float was attached by one
end a cord passing over a roller, and carrying at its other
end a pointer, which indicated the water-level on a scale.
The level of the head-water was read off on another scale
at the grating through which the water passed into the
turbine-chamber.

During the experiments no depression or vortex motion
of the water over the wheel was noticeable.   By means
of a sluice the lower water-level could be varied so as to
alter the head.

Observations were made at speeds varying from 0 to
the speed at which the wheel ran without a load; the
ratio to the latter of the best speed, as shown by the first
series of experiments, was, with all the guide passages
open, 0·482; with twenty passages, 0·507.   It must, how-
ever, be pointed out that the head was not in these cases
quite the same when the wheel ran without a load as
when working at the best speed, so that the comparison is
not strictly accurate; it will be seen that the ratio is in
both instances very nearly $\frac{1}{2}$.

## Comparison of Experimental and Theoretical Results.

For inflow without shock the ratio of the best velocity
at the mean diameter to the velocity of flow should be

$$\frac{w}{c} = \frac{\sin{(a + a_1)}}{\cos{a_1}},$$

$a_1$ being in this case negative.

For vertical outflow the same ratio should be ex-
pressed by

$$\frac{w}{c} = \frac{A}{A_2} \sin a_2,$$

and if the turbine has been designed according to the principles stated, both values should be identical.

Substituting the particular values for Haenel's turbine of $a$ and $a_1$, $a_2$ and $\frac{A}{A_2}$ in the two formulas, it will be found that

(1) $$\frac{\sin (a+a_1)}{\cos a_1} = 0\cdot5412$$

and

(2) $$\frac{A}{A_2} \sin a_2 = 0\cdot4344;$$

in calculating the latter quantity, the mean value of the angles of outflow on the concave and convex sides of each vane has been used.

It is clear from the above results that inflow without shock and vertical outflow do not occur at the same speed.

The actual ratio of the best speed as ascertained by experiment to the velocity of flow, with all the guide passages open, is in one case

<div align="center">0·51489,</div>

in another               0·57724,

with different heads; the mean of these two ratios is 0·54606, which agrees very well with the theoretical ratio for inflow without shock. A variation of 7 or 8 per cent. above or below the best speed has a very slight influence on the efficiency, so that for all practical purposes the values obtained on the assumption of inflow without shock are sufficiently close.

It is noticeable that with turbines which have not been designed for simultaneous vertical outflow and inflow

without shock, the latter condition usually agrees best with experiment. In such cases, some writers advocate the adoption of a mean between the values calculated on the two assumptions; for Haenel's turbine this would give

$$\frac{w}{c} = \frac{0\cdot5412 + 0\cdot4344}{2} = 0\cdot4878,$$

which is sufficiently near one experimental ratio, $0\cdot51489$, but does not agree very well with the other, $0\cdot57724$.

The ratio to the theoretical velocity, due to the head over the guide-orifices, of the velocity of flow as calculated from the *observed* efficiency by the formula

$$K_1 = \sqrt{\frac{\epsilon}{2\dfrac{A}{A_2}\, \sin a \, \sin a_2}} ,$$

is for the efficiency $0\cdot6949$, (the best result recorded in Table 3,) $K_1 = 0\cdot930$; for the highest efficiency given in Table 2, $K_1$ is practically the same. Comparing this with the experimental values of $K_1$, it will be seen that in the first case

$$K_1 = 0\cdot8484,$$

and in the second

$$K_1 = 1\cdot1035.$$

The last of these values indicates a pressure *below* that of the atmosphere in the clearance space between guide passages and wheel, since the velocity of flow is greater than that theoretically due to the head.

That the co-efficient $K_1$ should vary so considerably for the same wheel is explained by the probable supposition that the actual effective sections of the stream passing through the turbine were not at all times those represented by the dimensions of the wheel, but were altered by the formation of eddies or *dead* water within the

passages and buckets, not participating in the flow, but, so to speak, modifying the shape of the walls of the channels.

During the experiments recorded in Table 2, the immersion of the wheel was only about 0·5 foot, while the results given in Table 3 were obtained with an immersion of about 1·5 foot; the modifications in the effective sections of the stream must be ascribed to the difference in the immersion. In both cases the co-efficient $K_1$ shows that the wheel worked as a reaction turbine at the best speeds, since had free deviation taken place the co-efficient must have been less than 1 and greater than 0·8484. At higher speeds than the best with an immersion of 0·375 foot and less, the value of $K_1$ is what might be expected with free deviation, and is very nearly the same as the calculated value.

In a turbine constructed like that of Haenel, with buckets of equal section throughout, the relative velocity of flow and pressure is nearly constant throughout the bucket, the pressure being that of the atmosphere; hence the water leaves the guide passages at approximately atmospheric pressure, and, apart from differences in the hydraulic resistances, the velocity of flow must be the same as if free deviation took place.
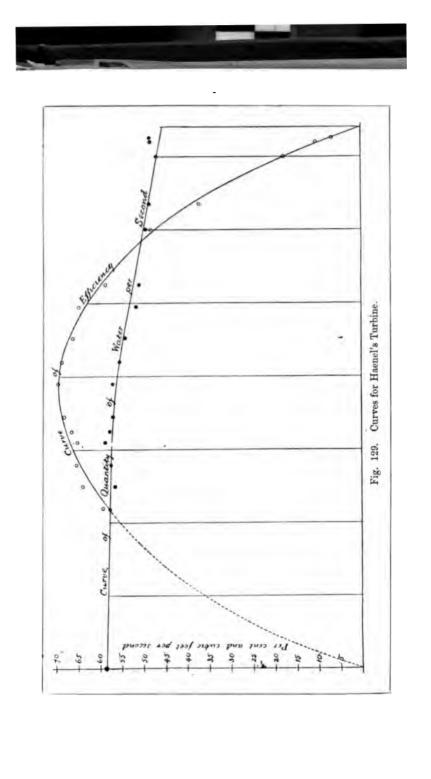
It will be seen on reference to the tables of results that as the speed of the wheel *increases*, the quantity of water consumed *decreases;* this, as explained in dealing with Rittinger's turbine trials, is due to the negative value of the angle $a_1$.
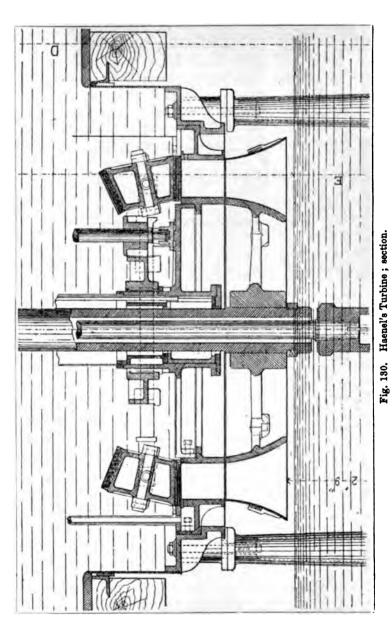
During one part of the experiments, the ventilating pipes communicating with the guide passages—referred to in the description of the turbine—were open, and during another part closed, but no appreciable difference was noticed in the performance of the wheel.

In Fig. 129 the efficiencies and quantities of water per
second consumed by the wheel are plotted as ordinates
with the number of revolutions per second as abscissæ,
and for these values the mean curves have been drawn.
The dotted portion of the curve of efficiency indicates only
the *probable* form, as no experimental data were obtained
below a certain speed.

In the following tables 1, 1A, 2, and 3, will be found
the results of the trials.

Fig. 129. Curves for Haenel's Turbine.

Fig. 130. Haenel's Turbine ; section.

## Results obtained with Haenel's Turbine.

### TABLE 1.

| Number of guide passages open. | Total head. h | (+) Clearance of wheel above tail-water or (−) depth of immersion. | Quantity of water used per second. Q | Number of revolutions per minute. n | Available energy of water per second. E_a | Power developed as measured by brake per second. W_e | Efficiency. e |
|---|---|---|---|---|---|---|---|
| | Feet. | Feet. | Cubic ft. | | Ft. lbs. | Ft. lbs. | |
| 3 | 4·2292 | − 1·9792 | — | 15·5 | 0 | 0 | 0 |
| 6 | 4·4063 | − 1·6970 | — | 30 | 0 | 0 | 0 |
| 8 | 4·3333 | − 1·8333 | — | 36 | 0 | 0 | 0 |
| 12 | 4·2292 | − 2 | — | 42 | 0 | 0 | 0 |
| 16 | 3·8333 | − 2·0417 | — | 44 | 0 | 0 | 0 |
| 16 | 4 | − 2·1666 | 18·790 | 32·5 | 4960·6 | 2858·9 | 0·5763 |
| 16 | 3·9583 | − 2·1666 | 18·914 | 32 | 4941·3 | 2814·8 | 0·5697 |
| 20 | 3·75 | − 2·0833 | — | 47 | 0 | 0 | 0 |
| 22 | 3·9375 | − 2·1666 | 28·308 | 32 | 7356·4 | 4691·4 | 0·6377 |
| 22 | 4 | − 2·1666 | 27·728 | 31·5 | 7320·2 | 4618·2 | 0·6309 |
| 22 | 4·2083 | − 1·9166 | 30·201 | 31 | 8388·3 | 5194·2 | 0·6192 |
| 22 | 3·6666 | − 2·3333 | 34·676 | 0 | 8391·5 | 0 | 0 |
| 24 | 3·8125 | − 2·1875 | — | 51 | 0 | 0 | 0 |
| 24 | 3·6666 | − 2·4166 | 24·730 | 50·5 | 5984·5 | 0 | 0 |
| 24 | 3·6042 | − 2·4166 | 31·580 | 27·417 | 7511·9 | 4593·6 | 0·6115 |
| 26 | 3·8333 | − 2·1805 | 33·841 | 32 | 8561·8 | 4959·6 | 0·5793 |
| 26 | 3·5104 | − 2·5 | 32·981 | 28·75 | 7641·3 | 4817·1 | 0·6304 |
| 26 | 3·625 | − 2·4583 | 25·646 | 52 | 6135·7 | 0 | 0 |
| 28 | 3·8125 | − 2·3333 | — | 54·5 | 0 | 0 | 0 |
| 28 | 4 | − 2·0833 | 36·716 | 32·25 | 9693 | 6416·7 | 0·6620 |
| 28 | 3·9792 | − 2·1042 | 36·716 | 30·5 | 9642·6 | 6387·9 | 0·6625 |
| 32 | 3·6666 | − 2·3333 | — | 55 | 0 | 0 | 0 |
| 32 | 3·6355 | − 2·3333 | 41·966 | 34·25 | 10069 | 6456 | 0·6412 |
| 32 | 3·9583 | − 2· | 46·162 | 33·5 | 12060 | 7719·0 | 0·6400 |
| 32 | 3·9583 | − 2· | 46·162 | 32 | 12060 | 8042·5 | 0·6669 |

## TABLE 1A.

| Number of guide passages open. | Total head. $h$ | (+) Clearance of wheel above tail-water or (−) depth of immersion. | Quantity of water used per second. $Q$ | Number of revolutions per minute. $n$ | Available energy of water per second. $E_a$ | Power developed as measured by brake per second. $W_e$ | Efficiency. $t_e$ |
|---|---|---|---|---|---|---|---|
| | Feet. | Feet. | Cub. ft. | | Ft. lbs. | Ft. lbs. | |
| 2 | 6·1042 | 0·1666 | — | 20·75 | 0 | 0 | 0 |
| 3 | 6· | 0·1666 | — | 49·4 | 0 | 0 | 0 |
| 4 | 6·2292 | 0·1666 | — | 50·5 | 0 | 0 | 0 |
| 4 | 6·2083 | 0·0833 | 4·928 | 31·5 | 2019·1 | 1121·6 | 0·5555 |
| 4 | 6·0833 | 0·125 | 4·980 | 27 | 1999·5 | 1131 | 0·5656 |
| 6 | 6·2083 | 0·0833 | 7·868 | 34·5 | 3223·8 | 1806·4 | 0·5604 |
| 8 | 6·2083 | 0·0833 | — | 59 | 0 | 0 | 0 |
| 8 | 6·1666 | 0 | 10·520 | 36·5 | 4281·5 | 2675·6 | 0·6249 |
| 8 | 6·2083 | 0 | 10·177 | 32 | 4169·9 | 2680·8 | 0·6429 |
| 12 | 6·2083 | 0 | — | 61 | 0 | 0 | 0 |
| 12 | 6·0971 | − 0·0833 | 17·023 | 34·5 | 6850·1 | 4335·5 | 0·6329 |
| 12 | 6 | 0 | 19·091 | 34·5 | 7559·7 | 0 | 0 |
| 14 | 5·875 | − 0·1666 | 20·354 | 49·25 | 7892 | 3713·4 | 0·4705 |
| 14 | 5·9375 | − 0·2086 | 20·508 | 35·25 | 8036·7 | 5315·6 | 0·6614 |
| 14 | 5·9375 | − 0·2086 | 20·806 | 33·75 | 8153·2 | 5301·5 | 0·6502 |
| 14 | 5·9583 | − 0·2086 | 21·441 | 32·166 | 8431·7 | 5052·6 | 0·5992 |
| 14 | 5·9166 | − 0·2086 | 22·292 | 30·166 | 8705 | 5054·4 | 0·5806 |
| 16 | 6·1875 | 0 | 24·967 | 64 | 10196 | 0 | 0 |
| 16 | 6· | − 0·1666 | 24·007 | 34·25 | 9506·6 | 6169 | 0·6489 |
| 16 | 6· | − 0·0833 | 27·537 | 0 | 10905 | 0 | 0 |
| 20 | 6·125 | − 0·0833 | — | 66 | 0 | 0 | 0 |
| 20 | 5·8333 | − 0·25 | 29·723 | 33·5 | 11443 | 7717·9 | 0·6745 |
| 24 | 6· | − 0·0833 | — | 67 | 0 | 0 | 0 |
| 24 | 5·8333 | − 0·25 | 31·076 | 34·5 | 11964 | 7587 | 0·6341 |
| 28 | 5·7917 | − 0·2817 | 39·011 | 49 | 14912 | 8825·8 | 0·5919 |
| 28 | 5·6666 | − 0·375 | 47·907 | 33 | 17917 | 11404 | 0·6365 |
| 32 | 5·875 | − 0·2083 | 46·162 | 69·5 | 17899 | 0 | 0 |
| 32 | 5·5417 | − 0·4583 | 56·889 | 33·5 | 20807 | 13331 | 0·6407 |
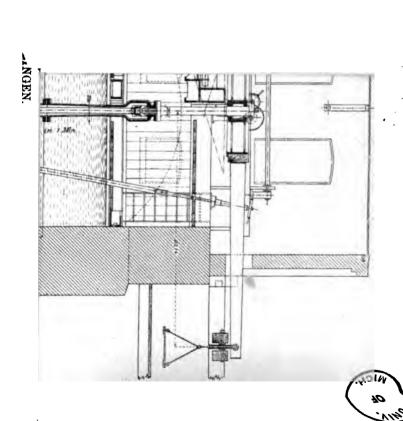
## Jonval Turbines at the Doubling and Sewing-Thread Mill, Goeggingen.[1]

The Goeggingen manufactory on increasing their works ordered two turbines from the Augsburg Maschinenfabrik to utilize the water-power of the Wertach.

The available fall is 13·45 feet, and the quantity of water per second 423·8 cubic feet, equivalent to 647 horse-power.

The makers of the turbines guaranteed an efficiency of 75 per cent. when all the passages in the wheel were open, and 70 per cent. when half were closed.

According to preliminary measurements, a minimum quantity of water of 176·58 cubic feet per second could be relied on, in which case the tail-water level was about 3·94 inches below the average, while, when floods occurred, the upper water-level rose by as much as 1 foot 7·69 inches; these circumstances had to be taken into account.

To meet these requirements two Jonval turbines of equal power were supplied, each of which could be regulated, in stages of $\frac{1}{12}$, down to half the greatest quantity of water.

Under the special conditions obtaining, automatic regulation is unnecessary, and the adjustment is effected through a series of sector-flaps hinged on the outside of the guide-vane casing, and operated by hand-wheels, as shown in the illustration (Fig. 131).

Both turbines have the same dimensions, namely, mean diameter 8 feet 0·46 inches, width of buckets 1·475 feet = 1 foot 5·72 inches (0·367 of mean radius). Under ordinary conditions they make forty-six revolutions per minute.

[1] *Zeitschrift des Vereins Deutscher Ingenieure*, vol. xxx., p. 781.

They have each thirty-six cast-iron guide-vanes and thirty-eight wheel-vanes of steel plate, and the depth of the guides and wheels is respectively 12·99 and 11·81 inches.

The wheel is surrounded by a welded wrought-iron ring, and keyed to the hollow cast-iron shaft by a complete truncated cone in halves.

The main bearing is of the overhead (Fontaine) type carried by a fixed wrought-iron spindle. This bearing is accessible through two openings in the hollow shaft, and the latter is in two lengths bolted together by flanges. The guide-casing has a dished top with an adjustable bearing of lignum-vitæ, through which the shaft passes.

Below the wheel is a short tube, from which the water flows out under a balanced circular sluice. This tube, in the event of the tail-water being very low, would act to a slight extent as a suction-tube. The position of the sluice can be regulated by hand. There are six flaps of wrought-iron arranged consecutively round one half of the circumference for closing the guide passages; each flap covers three passages. An arrangement is made by means of which air is allowed free access to the guide passages, when the latter are closed, through tubes in connection with the hollow hinges of the flaps.

The weight of the complete turbine with regulating apparatus and circular sluice is about twenty-six tons.

## Water Measurements.

The quantity of water passing through the turbine was measured by means of current meters, as it was impossible to use a weir or tumbling bay. Two instru-

ments were employed, one manufactured by Elliott Brothers and the other by Ertel and Son.

For the purpose of measurement the whole cross-section of the channel was divided into twenty-four rectangular parts, eight lengthways and three in depth, in the middle of each of which the velocity was observed. The total width of the channel was about 19 feet 8·22 inches, the depth 5 feet 4·72 inches.

Both the instruments used were carefully tested and their constants determined before use, at the hydrometric experimental station under the management of Dr. Bauernfeind at Munich.

As a further check on the accuracy of the observations, the curves of equal velocity for the section were constructed and from these the quantity of water flowing per second calculated. The agreement of the results obtained by the two methods was highly satisfactory.

## *Brake Tests.*

| | Proportion of guide passages open. | Head. | Quantity of water per second. | Available power. | Number of revolutions per minute. | Effective power measured on brake. | Efficiency. |
|---|---|---|---|---|---|---|---|
| | | $h$ . | $Q$ | $E_a$ | $n$ | $W_e$ | $e_e$ |
| | | Feet. | Cubic feet. | Horse-power. | | Horse-power. | |
| 1 | All | 13·583 | 210·155 | 324·19 | 45·52 | 269·46 | 0·832 |
| 2 | All | 13·509 | 214·202 | 328·44 | 44·92 | 265·91 | 0·810 |
| 3 | All | 13·823 | 213·827 | 330·11 | 39·34 | 266·69 | 0·808 |
| 4 | All | 13·675 | 213·860 | 331·89 | 51·76 | 266·11 | 0·801 |
| 5 | All | 14·540 | 229·780 | 379·23 | 45·40 | 293·13 | 0·773 |
| 6 | $\tfrac{9}{12}$ | 13·726 | 158·334 | 246·57 | 45·29 | 196·18 | 0·795 |
| 7 | $\tfrac{8}{12}$ | 13·709 | 161·489 | 251·20 | 40·42 | 196·86 | 0·784 |
| 8 | $\tfrac{5}{12}$ | 13·590 | 129·605 | 199·92 | 44·35 | 149·82 | 0·749 |
| 9 | $\tfrac{6}{12}$ | 13·800 | 111·169 | 174·08 | 45·63 | 130·68 | 0·751 |
| 10 | $\tfrac{4}{12}$ | 13·770 | 109·597 | 169·94 | 40·28 | 128·51 | 0·756 |
| 11 | Turbine held fast | 13·228 | 122·054 | | 0 | 0 | 0 |

The construction and dimensions (in millimètres) of the brake used for determining the power given off are clearly shown in the illustrations.
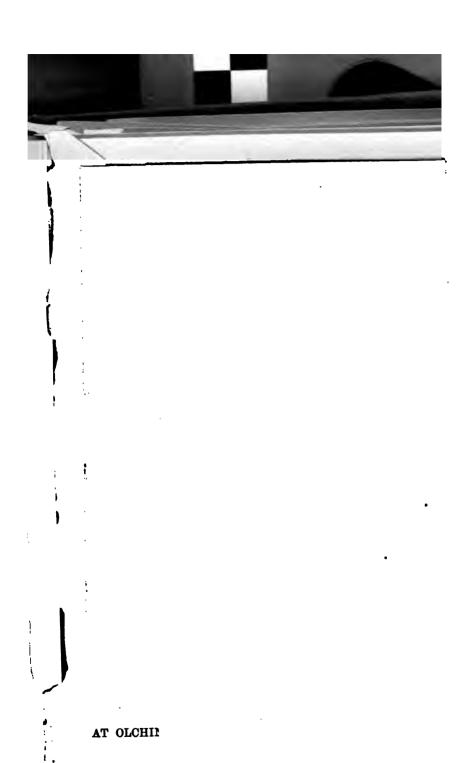
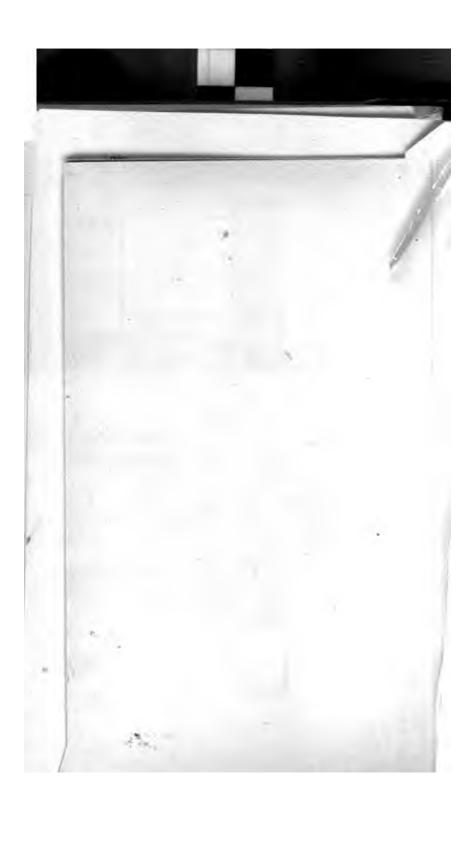In the preceding table the leading results of the trial are given.

### Experiments with 480 horse-power Turbines at the Wood-fibre Manufactory of the Muenchen Dachauer Company, at Olching.[1]

This installation, illustrated by Fig. 132, consists of two turbines, one of the Jonval and the other of the Girard type, with vertical axes, enclosed in wooden chambers; the latter were originally made for a smaller quantity of water, but to avoid costly alterations it was decided to construct the new motors, with which the tests were carried out, to suit existing arrangements. Under ordinary circumstances, for about nine months of the year the available quantity of water is 16 to 17 cubic mètres (565 to 600 cubic feet) per second; in dry seasons this sinks to 10 to 11 cubic mètres (350 to 390 cubic feet), and in very extreme circumstances, which, however, only last a short time, to 7 cubic mètres (250 cubic feet). The fall is 2·9 mètres (9 feet 6 inches), and invariable.

To suit these conditions, the installation was so designed that with the normal quantity of water both turbines work together with *full* admission. When the amount of water decreases, the Girard wheel (which is the smaller) is regulated by closing some of the guide passages.

[1] *Zeitschrift des Vereins Deutscher Ingenieure,* vol. xxxii., p. 125.

AT OLCHII

The manufacturer, Mr. J. G. Landes of Munich, guaranteed the following results :—

1. With both turbines working together at full power, with the normal quantity of water . . 75·0 per cent.

2. With the Jonval wheel full, and the water admitted to ⅔ of the guide passages of the Girard wheel . . . . . 74·6 per cent.

3. With the Jonval wheel full, and the Girard wheel with ⅓ admission . . . . . 73·8 per cent.

4. With the Jonval wheel full, and the other closed . . . 75·0 per cent.

The Jonval turbine was designed to develop at least 300 horse-power under all circumstances, and the other motor to utilize the remainder of the available power.

## *Dimensions of Turbines.*

| | JONVAL WHEEL. | | | GIRARD. |
|---|---|---|---|---|
| | Inner Division. | Outer Division. | Totals or Means. | |
| Angle of outflow from guides | 71° 10′ | 71° 10′ | 71° 10′ | 66° 0′ |
| Angle of outflow from buckets | 73° 25′ | 73° 20′ | 73° 23′ | |
| Angle of inflow into buckets . | 9° 25′ | 9° 25′ | 9° 25′ | 54° 0′ |
| | | | | 11·81 in. at top |
| Clear width of buckets . . | 16·34 in. | 13·78 in. | 30·12 in. | 29·14 in. at bottom |
| Mean diameter . . | 9 ft. 6·3 in. | 12 ft. 3·6 in. | 11ft. 1·86 in. | 9 ft. 6·38 in. |
| Area of guide passages . | 11·524 sq. ft. | 13·853 sq. ft. | 25·377 sq. ft. | |
| Outflow area of buckets . | 9·253 sq. ft. | 12·002 sq. ft. | 21·255 sq. ft. | |
| Number of guide-vanes . | 23 | 30 | | |
| Number of wheel-vanes . | 29 | 38 | | |
| Thickness of guide-vanes | 0·59 in. | 0·59 in. | | |
| Thickness of wheel-vanes | 0·51 in. | 0·51 in. | | |

The Jonval wheel is divided by a partition between the outer and inner casing into two concentric parts of unequal width, so proportioned that the power developed in each is nearly the same.

## Brake Tests.

For measuring the effective power of the motors a brake of the usual "Prony" type was employed; the brake-wheel had a diameter of 2·2 mètres (7 ft. 2·61 in.) and a width of 0·55 mètres (1 ft. 9·66 in.). Very careful arrangements were made for keeping the wheel and brake-blocks cool during the experiments. The only available place for applying the brake was on the horizontal first-motion shaft, driven direct by bevel-gearing from the vertical turbine-shafts, and to arrive at the power actually given off by the latter, it was necessary to allow for the friction of the gearing and horizontal shaft. This was done by calculation from the actual weight of the machinery, allowing a co-efficient of friction of 0·07 for the bearings, and 0·13 for the bevel-gearing working with iron upon iron.

## Water Measurements.

The measurement of the quantity of water used took place in the head-race by means of current meters. For this purpose the total section of the channel was divided into twenty-eight rectangles, fourteen in the width and two in the depth. Three instruments were used simultaneously at different points of the stream section, one by Elliott Brothers, another by Ertel and Son, and the third, with electric signal, by Amsler. In consequence of there being as yet no weir in the tail-race by which

the level of the water in the latter could be regulated, this level varied according to the quantity of water used, and, in consequence, during experiment No. 1, the Girard wheel was *immersed* to a depth of 2 inches, while in trials 3 and 4 the Jonval wheel was 3·86 inches *above* the tail-water level. This of course affected to some extent the efficiency of both wheels, but no allowance has been made for this in the results given, which are as follows :—

| Number of revolutions of *Jonval* wheel per minute. *n* | Number of revolutions of *Girard* wheel per minute. *n* | Horse-power measured on brake, after allowing for friction of horizontal shaft. *We* | Available head. *h* | Quantity of water used per second. *Q* | Available energy in horse-power. *Ea* | Efficiency. *ea* | Remarks. |
|---|---|---|---|---|---|---|---|
| | | Horse-power. | Feet. | Cubic feet | Horse-power. | Per Cent. | |
| 27·10 | 25·55 | 472·24 | 9·308 | 585·86 | 629·35 | 75·0 | Both tur- |
| 27·69 | 26·10 | 481·70 | 9·433 | 583·57 | 624·72 | 77·1 | bines to-gether. |
| 23·87 | — | 365·62 | 9·991 | 404·12 | 458·33 | 79·8 | Jonval tur- |
| 23·28 | — | 364·43 | 9·991 | 404·62 | 458·92 | 79·4 | bine alone. |

If the height of the lower edge of the Jonval wheel above the tail-water surface were allowed for by deducting that distance from the available head, the efficiency would be **83** per cent. instead of 79·8.

The object of dividing the Jonval wheel into two concentric portions in this case is partly to strengthen the vanes and partly to allow the vane angles at different diameters to be independently fixed. Each part can then be treated as a separate wheel, and constructed, if

necessary, with a degree of reaction differing from the other, although in the turbine in question this does not appear to have been done except to a slight extent.

The vane-angles are nearly the same at the mean diameter of each part; this could not be the case were there no division.

## Francis's Experiments.

The well-known American engineer, Mr. James B. Francis, carried out in 1851 a series of very complete experiments with two large turbines, of which a full account is given in his work, *Lowell Hydraulic Experiments.* One of the turbines in question was an outward-flow radial (Fourneyron) wheel, the other an inward-flow radial wheel; the former of these was made for the Tremont Mills, the latter for the Boott Cotton Mills, both at Lowell, Massachusetts, where the experiments were carried out.

### Experiments with Tremont Turbine.

### *Description of Turbine.*

The turbine tested was, as above stated, of the Fourneyron type with vertical shaft.

*The Wheel* consisted of a central plate of cast-iron and of two crowns, to which the buckets were attached. The lower crown was fastened to the central plate by set screws; the upper and lower crowns were precisely alike, and each $9\frac{1}{2}$ inches wide.

*The Wheel-vanes* were of wrought-iron $\frac{9}{64}$ inch thick, let into grooves in the crowns, and secured by tenons in mortices cut in the crowns, and riveted over. The vanes terminated $\frac{1}{4}$ inch short of the edges of the crowns,

for the purpose of allowing the wheel to be handled with less risk of injuring the ends of the vanes.

*The Guide-Vanes* were of plate-iron $\frac{1}{10}$ inch thick, secured by tenons to the bottom of a cast-iron cylindrical casing, to the flanged upper end of which was bolted the cast-iron mouth-piece of the curved supply-pipe. Up through the centre of the casing passed a vertical cast-iron tube protecting the shaft from contact with the water. The upper corners of the guides near the wheel were connected by a ring or garniture composed of thirty-three pieces of cast-iron, one to fill each space between the guide-vanes, riveted to the latter and to each other.

*The Regulating-Sluice* or gate was cylindrical, of cast-iron, passing between the orifices of the guide passages and the inlets to the buckets. The upper part of the cylinder was stiffened by a rib, to which were attached three brackets, each connected with a vertical wrought-iron rod; by these rods the gate was raised or lowered. The rods were actuated by an arrangement of levers, racks, and pinions. The weight of the gate was counterbalanced by weights attached to the levers.

The cylindrical portion of the gate had a thickness of one inch.

*The Supply-Pipe,* conducting the water from the head-race to the wheel, was in its upper part of wrought-iron plates $\frac{3}{4}$ inch thick riveted together, and had a diameter of 9 feet where the water entered horizontally, diminishing in the curved portion to its junction with the cast-iron mouth-piece before referred to.

*The Shaft* was of wrought-iron, 8 inches diameter for the main part of its length; it was suspended from a collar bearing and steadied laterally by bearings of cast-

iron lined with babbitt-metal, one in the cast-iron shaft-tube, the other in the floor of the turbine-house above the supply-pipe. The collar-bearing was carried upon a gimbal or hollow frame resting on adjusting screws supported by a cast-iron girder connected with the walls of the turbine-house. At its lower end the shaft had a neck running in a step made in three parts, and lined with case-hardened wrought-iron; this step was furnished with adjusting screws, by means of which the shaft could be moved slightly in any direction horizontally. This step did not bear any portion of the weight of the motor, but only served to retain the lower end of the shaft in place.

## *Dimensions of Turbine.*

The following were the leading dimensions and proportions of the turbine:—

| | |
|---|---|
| Angle of outflow from guides . | $a_1 = 62°$. |
| Angle of outflow from buckets . | $a_2 = 68°$. |
| Angle of inflow into buckets . | $a_1 = 0°$. |
| Diameter of wheel where water enters . . . . . | $2r_1 = 6·750$ ft. |
| Diameter of wheel where water leaves . . . . . | $2r_2 = 8·292$ ft. |
| Width of guide passages where water leaves . . . . | $c = 0·9707$ ft. |
| Width of buckets where water enters . . . . . | $c_1 = 0·9368$ ft. |
| Width of buckets where water leaves . . . . . | $c_2 = 0·9314$ ft. |
| Number of guide-vanes . . | $z = 33$. |
| Number of wheel-vanes . . | $z_1 = 44$. |

Thickness of guide-vanes $t = \frac{1}{10}$ in. $= 0\cdot0083$ ft.
Thickness of wheel-vanes

$$t_1 = t_2 = \tfrac{9}{64}\text{ in.} = 0\cdot0117\text{ ft.}$$

Measured outflow area of guide
  passages . . . . $A^1 = 6\cdot5371$ sq. ft.
Measured outflow area of
  buckets . . . . $A_2^1 = 7\cdot6869$ sq. ft.

With respect to the outflow areas $A^1$ and $A_2^1$ of the guide passages and buckets, some explanation is necessary of the manner in which these have been calculated.

Francis determined these areas by ascertaining the mean values of the shortest distances $a\ b$ and $c\ d$, Fig. 133, between two adjacent vanes, and multiplying them with the width of the passages or buckets and their corresponding number. It is open to question whether or not this method gives the true areas. The ends of the guide-vanes are comparatively slightly curved, and the actual direction of the stream leaving each passage is assumed to be parallel with the tangent at $a$, the extreme end of the vane-curve on the *concave* face of the vane. This stream would consequently be bounded by the parallel (dotted) lines through $a$ and $b$, and its sectional area on leaving the orifices would be *less* than if all the filaments of water followed similar courses relative to the wheel. As has been previously pointed out, it would be reasonable to suppose that the fluid occupies the largest section possible, and, in the case of the turbine in question, this must happen when it follows similar curves through all

Fig. 133.

points of the circumference. Under such conditions the area $A^1$ would be found by the well-known formula:

$$A^1 = e \; (2 \; \Pi \; r \; cos \; a \; - \; z \; t).$$

Experiment, as will be subsequently shown, proves that calculated results do not agree so well with practice on the latter hypothesis as when Francis's method of computing the areas is adopted, and therefore the dimensions given above and in Table A have been calculated by Francis's method. (*Vide* p. 122 *et seq.*)

In the Tremont turbine the guide-vanes did not extend to the outer circumference of the casing, as room had to be left for the regulating-gate between the guide-vanes and buckets. On this account no allowance has been made for the obstruction of the wheel-vanes in estimating the effective area of outflow $A$ from the guides, as the distance of the edges of the wheel-vanes from the guide passage orifices is very considerable. The co-efficient of contraction assumed is 0·9, and with this the effective area of the guide passages

$$A = 5\cdot8834 \text{ square feet.}$$

The same co-efficient has been used in calculating the outflow area from the buckets

$$A_2 = 6\cdot9182 \text{ square feet.}$$

From the dimensions given the following ratios result:

$$\frac{A}{A_2} = 0\cdot850 \; ; \frac{r_1}{r_2} = 0\cdot814 \; ; \frac{c_1}{c_2} = 1\cdot006.$$

## Method of conducting Experiments.

The chief quantities to be ascertained were as usual the actual power developed, the quantity of water passing

through the wheel, and the head, but in addition to these an arrangement was made for observing the absolute direction in which the water left the wheel, and the temperature was also noted.

For each class of data a separate observer was appointed, and the time of each observation recorded.

## Measurement of Head.

The level of the water in the head-race was observed immediately before entering the supply-pipe, and the level of the tail-race in the wheel-pit. For these observations hook-gauges were employed, enclosed in boxes to which the water was admitted only through a hole in the bottom, in order to prevent the surface where the measurements were taken from being affected by oscillations and other disturbances. Both gauges had the same zero point, so that the difference in the readings gave the desired head at once.

## Measurement of Power developed.

For ascertaining the effective power given off by the wheel a Prony brake was employed, applied to a pulley attached to the vertical shaft of the turbine. The friction-pulley was of cast-iron, 5 feet in diameter, 2 feet wide on the face, and 3 inches thick. The brake was of maple wood, made in two blocks drawn together by bolts, one of the blocks being prolonged to form the lever. The end of the lever was supported in a scale at the end of one arm of a vertical bell-crank lever, to the other arm of which the weights were hung. To regulate the motion of the brake and prevent sudden shocks, a hydraulic regulator or dashpot was employed

in connection with the bell-crank lever in a manner now well known.

To ascertain the velocity of the wheel a counter was attached to the top of the vertical shaft, so arranged that a bell was struck at the end of every fifty revolutions of the wheel. The friction-pulley was lubricated and cooled by four jets of water, the mean quantity used during two trials being 0·0288 cubic feet per second.

In some of the experiments with heavy weights and low velocities, oil was used instead of water, as the friction was found to be greater with the former.

### *Observation of Absolute Direction of Outflow.*

For ascertaining the absolute direction in which the water left the wheel, the following apparatus was employed: a rectangular vane was placed near the circumference of the wheel, keyed to a vertical shaft which turned freely on a step resting on the wheel-pit floor. The upper end of the shaft carried a pointer which indicated the position of the vane on a graduated semicircular scale divided into 180 degrees. When the vane was parallel with a tangent to the circumference of the wheel drawn through the point nearest to the axis of the vane, and the latter pointed in the direction of the motion of the wheel, the pointer indicated 0°; when the vane was in a radial position relative to the wheel the pointer indicated 90°.

### *Measurement of Quantity of Water.*

The quantity of water passing through the turbine was gauged by a weir erected for the purpose at the mouth of the wheel-pit. To quiet the surface of the

water before reaching the point at which the measurements in connection with the weir were made, a grating was placed across the wheel-pit. The measuring-weir had two bays of nearly equal length, one being 8·489 feet, the other 8·491 feet.

The crest of the weir was of cast-iron, planed on the upper edge and on the up-stream face for a width of $1\frac{1}{8}$ inch below the top; the upper edge was $\frac{3}{8}$ inch thick and practically horizontal. The up-stream corner was sharp. The ends of each bay were of wood, and of the same form as the crest near the edges. The ends of the bays projected a mean distance of 1·235 feet from the walls of the wheel-pit, and from the central pier dividing the weir into two parts.

The crest of the weir was about 6·5 feet above the floor of the wheel-pit which formed the canal of approach.

The depth of water or head over the weir was observed by means of a hook-gauge enclosed in a box, the distance of the gauge from the weir being about 5·5 feet.

The formula used for calculating the quantity of water from the head $h$ over the weir was that of Francis:

$$Q = 3\text{·}33 \left(L - 0\text{·}1 \, n \, h\right) h^{\frac{3}{2}}.$$

## RESULTS OF EXPERIMENTS WITH THE TREMONT TURBINE.

| 1 Number of the experiment. | 4 Height of the regulating gate in inches. | 9 Number of revolutions of the wheel per second. $\frac{n}{60}$ | 10 Useful effect, or the friction of the brake in pounds avoirdupois raised one foot per second. $W_e$ | 13 Total fall acting upon the wheel in feet. $h$ | 14 Depth of water on the weir in feet. | 15 Quantity of water which passed the weir, in cubic feet per second. $Q$ | 16 Total power of the water in pounds avoirdupois raised one foot per second. $B$ | 17 Ratio of the useful effect to the power expended. $e$ | 18 Velocity due to the fall acting on the wheel in feet per second. $A$ | 19 Velocity of the interior circumference of the wheel in feet per second. $v$ | 20 Ratio of the velocity of the wheel to the velocity due to the interior circumference of the fall acting on the wheel. $K_2$ | 23 Direction of the water leaving the wheel as indicated by the vane. deg. | 23 (min.) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 4 | 11·49 | 1·59651 | 33348·3 | 12·554 | 2·0383 | 156·6470 | 122663·3 | 0·27187 | 28·4169 | 33·8553 | 1·19138 | 18 | 3 |
| 6 | ,, | 1·46149 | 51680·6 | 12·653 | 1·9989 | 152·2682 | 120174·8 | 0·43005 | 28·5287 | 30·9921 | 1·08635 | 22 | 56 |
| 8 | ,, | 1·30933 | 66845·5 | 12·720 | 1·9536 | 147·2942 | 118864·8 | 0·57199 | 28·6041 | 27·7653 | 0·97067 | 29 | 49 |
| 10 | ,, | 1·18460 | 77154·6 | 12·800 | 1·9315 | 144·8734 | 115667·0 | 0·66704 | 28·6939 | 25·1203 | 0·87546 | 35 | 37 |
| 13 | ,, | 1·78404 | 0· | 12·510 | 2·0089 | 163·4313 | 127527·3 | 0· | 28·3670 | 37·8319 | 1·33366 | 41 | 26 |
| 14 | ,, | 1·06744 | 83069·8 | 12·856 | 1·9098 | 142·5180 | 114284·2 | 0·73475 | 28·7566 | 22·6359 | 0·78716 | 46 | 18 |
| 16 | ,, | 0·99945 | 86314·4 | 12·890 | 1·9048 | 141·9762 | 114150·8 | 0·75614 | 28·7946 | 21·1940 | 0·73604 | 48 | 26 |
| 18 | ,, | 0·94507 | 87392·7 | 12·880 | 1·8908 | 140·4657 | 112848·8 | 0·77442 | 28·7835 | 20·0409 | 0·69626 |  |  |
| 20 | ,, | 0·91116 | 87819·8 | 12·886 | 1·8866 | 140·0066 | 112532·3 | 0·77040 | 28·7902 | 19·3219 | 0·67113 | 50 | 37 |
| 22 | ,, | 0·89713 | 88076·2 | 12·898 | 1·8834 | 139·6678 | 112364·5 | 0·78384 | 28·8036 | 19·0243 | 0·66048 |  |  |
| 23 | ,, | 1·78094 | 0· | 12·429 | 2·0834 | 161·6944 | 125355·0 | 0· | 28·2750 | 37·7662 | 1·33567 | 58 | 10 |
| 25 | ,, | 0·88496 | 88189·9 | 12·899 | 1·8775 | 139·0291 | 111859·4 | 0·78840 | 28·8047 | 18·7662 | 0·65150 |  |  |
| 30 | ,, | 0·85106 | 88278·9 | 12·903 | 1·8697 | 138·1892 | 111218·1 | 0·79375 | 28·8092 | 18·0474 | 0·62645 | 61 | 54 |
| 31 | 11·48 | 1·78971 | 0· | 12·539 | 2·0891 | 162·3283 | 126960·2 | 0· | 28·3999 | 37·9521 | 1·33635 |  |  |
| 32 | ,, | 0·83624 | 88320·8 | 12·915 | 1·8704 | 138·2668 | 111384·0 | 0·79294 | 28·8225 | 17·7330 | 0·61525 |  |  |
| 34 | ,, | 0·78401 | 87947·8 | 12·941 | 1·8687 | 138·0869 | 111463·0 | 0·78903 | 28·8515 | 16·6254 | 0·57624 | 86 | 12 |
| 35 | ,, | 0·74211 | 87066·5 | 12·939 | 1·8652 | 137·7076 | 111138·7 | 0·78340 | 28·8493 | 15·7371 | 0·54649 | 99 | 25 |

| No. | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 37 |  | 0·64568 | 83965·5 | 12·940 | 1·8412 | 135·1415 | 109077·1 | 0·76978 | 28·8504 | 13·6922 | 0·47459 | 131 | 18 |
| 38 | ,, | 1·78571 | 0· | 12·500 | 2·0834 | 161·6944 | 126071·1 | 0· | 28·8557 | 37·8674 | 1·33544 | 139 | 45 |
| 40 | ,, | 0·60000 | 82109·8 | 12·973 | 1·8380 | 134·7976 | 109077·0 | 0·75277 | 28·8872 | 12·7235 | 0·44045 | 147 | 26 |
| 41 | ,, | 0·53232 | 78492·2 | 12·977 | 1·8282 | 133·7538 | 108265·8 | 0·72499 | 28·8916 | 11·2882 | 0·39071 |  |  |
| 43 | ,, |  | 0· | 12·797 | 1·8460 | 135·6636 | 108280·4 | 0· | 28·6906 | 0· |  |  |  |
| 45 | ,, | 1·78833 | 0· | 12·471 | 2·0864 | 162·0237 | 126034·8 | 0· | 28·3228 | 37·8168 | 1·33521 | 60 | 52 |
| 46 | ,, | 0·83207 | 88603·9 | 12·954 | 1·8737 | 138·6244 | 112009·3 | 0·79104 | 28·8660 | 17·6447 | 0·61126 | 66 | 27 |
| 48 | ,, | 0·80321 | 88240·1 | 12·948 | 1·8723 | 138·4890 | 111832·0 | 0·78904 | 28·8598 | 17·0327 | 0·59020 | 89 | 44 |
| 50 | ,, | 0·76893 | 87865·8 | 12·952 | 1·8694 | 138·1559 | 111613·5 | 0·78723 | 28·8638 | 16·3058 | 0·56492 | 12 | 0 |
| 51 | 8·55 | 1·48831 | 39452·4 | 12·758 | 1·9173 | 143·3819 | 114080·7 | 0·34589 | 28·6468 | 31·4548 | 1·09802 | 22 | 19 |
| 53 | ,, | 1·14177 | 74827·2 | 12·909 | 1·8656 | 137·7518 | 110917·6 | 0·67462 | 28·8159 | 24·2121 | 0·84023 | 28 | 56 |
| 55 | ,, | 1·02459 | 80215·4 | 12·950 | 1·8586 | 137·0026 | 110664·7 | 0·72485 | 28·8816 | 21·7272 | 0·75281 | 37 | 47 |
| 57 | ,, | 0·93071 | 81911·6 | 12·965 | 1·8408 | 135·0974 | 109252·2 | 0·74975 | 28·8783 | 19·7365 | 0·88344 | 47 | 80 |
| 59 | ,, | 0·86538 | 82195·5 | 12·999 | 1·8240 | 133·3014 | 108082·5 | 0·76049 | 28·9161 | 18·3511 | 0·63463 | 59 | 39 |
| 61 | ,, | 0·80429 | 81786·2 | 13·026 | 1·8117 | 131·9960 | 107246·3 | 0·76260 | 28·9461 | 17·0556 | 0·58922 | 94 | 4 |
| 63 | ,, | 0·70779 | 79788·1 | 13·028 | 1·8013 | 130·8932 | 106366·6 | 0·75012 | 28·9484 | 15·1091 | 0·51848 |  |  |
| 64 | 5·65 | 1·71808 | 0· | 12·720 | 1·9742 | 149·5470 | 118652·1 | 0· | 28·6041 | 36·4332 | 1·27370 |  |  |
| 66 | ,, | 1·54799 | 290454·7 | 13·077 | 1·7160 | 121·9885 | 100194·6 | 0·30046 | 29·1057 | 32·8262 | 0·87363 | 6 | 80 |
| 68 | ,, | 1·35135 | 531424 | 13·176 | 1·6829 | 118·5511 | 96699·5 | 0·56592 | 29·0028 | 28·6564 | 0·85216 | 12 | 32 |
| 70 | ,, | 1·08460 | 587271 | 13·253 | 1·6409 | 114·2599 | 93904·9 | 0·63703 | 29·1123 | 22·9997 | 0·81823 | 21 | 17 |
| 74 | ,, | 0·92348 | 587762 | 13·311 | 1·6139 | 111·5197 | 92188·4 | 0·65520 | 29·1973 | 19·5831 | 0·79828 | 39 | 2 |
| 76 | ,, | 0·77482 | 562531 | 13·326 | 1·5793 | 108·0452 | 89707·1 | 0·64127 | 29·2611 | 16·4306 | 0·76979 | 69 | 27 |
| 77 | 9·96 | 0·68627 | 460561 | 13·412 | 1·5541 | 105·5341 | 87720·9 | 0·54757 | 29·2776 | 14·5530 | 0·49707 | 144 | 56 |
| 80 | 2·875 | 0·45662 | 862076 | 13·883 | 1·5034 | 100·5410 | 84110·0 | 0·78100 | 29·3719 | 9·8830 | 0·32966 | 55 | 52 |
| 82 | ,, | 0·85470 | 861644 | 12·912 | 1·8620 | 137·3618 | 110380·8 | 0·78350 | 28·7868 | 18·1246 | 0·62961 | 74 | 28 |
| 84 | ,, | 0·78918 | 0· | 13·347 | 1·8544 | 136·5469 | 109973·0 | 0· | 28·8192 | 16·7351 | 0·58069 | 0 | 30 |
| 86 | ,, | 1·24902 | 21256·6 | 13·395 | 1·2914 | 80·4534 | 66979·0 | 0·33204 | 29·3006 | 26·4865 | 0·90396 | 4 | 32 |
| 88 | ,, | 0·96022 | 279851 | 13·478 | 1·2492 | 76·6213 | 64018·1 | 0·46314 | 29·3533 | 20·3622 | 0·69369 | 20 | 9 |
| 90 | 1·00 | 0·67189 | 24462·9 | 13·556 | 1·1960 | 71·8750 | 60424·6 | 0·42661 | 29·4441 | 14·2480 | 0·48390 | 81 | 40 |
| 91 | ,, | 0·46243 | 0· | 13·516 | 1·1497 | 67·8158 | 57342·0 | 0· | 29·5292 | 9·8061 | 0·33208 |  |  |
| 92 | ,, |  | 4998·8 | 13·985 | 1·0623 | 60·3593 | 50886·6 | 0·14993 | 29·4856 | 0· | 0·43806 |  |  |
|  | ,, | 0·61958 | 34277 | 14·001 | 0·7798 | 38·2210 | 33340·8 | 0·10176 | 29·9928 | 13·1386 | 0·48677 |  |  |
|  | ,, | 0·68886 | 7815·6 | 14·020 | 0·7846 | 38·5699 | 33683·5 | 0·24042 | 30·0099 | 14·6079 | 0·27370 |  |  |
|  | ,, | 0·38760 |  |  | 0·7853 | 37·1733 | 32508·0 |  | 30·0303 | 8·2193 |  |  |  |

## Results of Experiments.

The more important results of the trials of the Tremont turbine are given in the preceding table, compiled from the original.

For the *best speed* the experimental results will be compared with those obtained by calculation on three assumptions as regards the method of computing the areas of outflow $A$ and $A_2$.

(1) When determined on the assumption that all filaments of water follow the same relative course:

$$A = 7\cdot6424 \text{ square feet,}$$
$$A_2 = 4\cdot4809 \quad \text{,,} \qquad \text{,,}$$

and

$$\frac{A}{A_2} = 1\cdot705.$$

(2) With $A$ as above and $A_2$ as determined by Francis:

$$A = 7\cdot6424,$$
$$A_2 = 6\cdot9182 \text{ square feet (allowing}$$

for contraction), and

$$\frac{A}{A_2} = 1\cdot105.$$

(3) When both areas are estimated in the manner adopted by Francis:

$$A = 5\cdot8834,$$
$$A_2 = 6\cdot9182,$$
$$\frac{A}{A_2} = 0\cdot850.$$

The following table shows side by side the experimental data and the figures obtained by calculation under all three hypotheses.

The angles, it should be remembered, vary according to

the way in which the outflow areas are computed, and the values for them employed in calculation must be those corresponding to the *mean direction of flow*, not necessarily nor generally the vane-angle.

| | $A = 7\cdot6424.$ | | | | $A = 5\cdot8834.$ | |
|---|---|---|---|---|---|---|
| | (1) | | (2) | | (3) | |
| | $\dfrac{A}{A_2} = 1\cdot705.$ | | $\dfrac{A}{A_2} = 1\cdot105.$ | | $\dfrac{A}{A_2} = 0\cdot850.$ | |
| | Experimental. | Calculated. | Experimental. | Calculated. | Experimental. | Calculated. |
| Co-efficient of flow $K_1 =$ | 0·628 | 0·586 | 0·628 | 0·748 | 0·815 | 0·856 |
| Co-efficient of velocity $K_2 =$ | 0·6264 | | 0·6264 | | 0·6264 | |
| [1] For inflow without shock $K_2 =$ | | 0·564 | | 0·564 | | 0·734 |
| For radial absolute outflow $K_2 =$ | | 0·848 | | 0·523 | | 0·523 |
| Mean value $K_2 =$ | 0·6264 | 0·706 | 0·6264 | 0·543 | 0·6264 | 0·628 |
| | | | | | | |
| *Purely Theoretical values.* | | | | | | |
| For inflow without shock $K_2 =$ | | 0·526 | | 0·672 | | 0·756 |
| For radial absolute outflow $K_2 =$ | | 0·792 | | 0·623 | | 0·549 |
| Mean value $K_2 =$ | | 0·659 | | 0·647 | | 0·652 |

From the above figures it will be seen that the *third* method of determining the outflow areas gives by far the best agreement with experiment both as regards the velocity of flow and the velocity of rotation. The *mean* of the calculated speeds for inflow without shock and radial outflow agrees very closely with experiment; the difference between these speeds shows that the wheel has not been designed for simultaneous fulfilment of both conditions.

A further confirmation of the correctness of the third method of calculation adopted is afforded by the observed directions of outflow of the water from the wheel at certain speeds.

On reference to the table of results obtained with the

[1] Calculated from *observed* quantity of water.

Tremont turbine, it will be seen that at the *best speed* (line 30) the angle at which the water left the wheel, as indicated by the vane used, was 58° 10′; as determined by graphical construction from what has been termed the experimental velocity of flow and the actual speed, this angle is about 64°. This is a very fair agreement under the circumstances, and proves that the data from which the velocity of flow has been ascertained are nearly correct, as well as the corresponding relative angle of outflow $a_2$.

At a velocity of rotation of 0·545 of the velocity due to the head, the angle of absolute outflow observed was 99° 25′; while according to calculation (from observed data) radial outflow (corresponding to an angle of 90°) would have occurred for $K_2 = 0·523$.

It is remarkable that the purely theoretical mean values of $K_2$ for all the three 'methods of calculation are very nearly the same.

The quantity of water passing through the turbine as calculated theoretically by the third method is greater than by the first.

### Experiments with Turbine at Boott Cotton Mills.

This turbine, like that at Tremont, was made to the designs of Mr. Francis in 1849, being one of a pair intended to develop about 230 horse-power with a fall of 19 feet. It was of the radial inward-flow type.

*The Wheel* was ·of cast-iron with plate-iron vanes ¼ inch thick, the upper of the two crowns enclosing the buckets being attached to a central plate with boss keyed to the shaft and curved downwards towards the centre so as to form a kind of inverted cone, in order to

guide the water after leaving the buckets from the horizontal to the vertical direction.

*The Guide Apparatus* was formed by two flat rings or crowns, containing the guide-vanes between them, surrounding the wheel. The guide-vanes were of plate-iron $\frac{3}{16}$ inch thick.

*The Regulating-Gate* was a cast-iron cylinder, as in the case of the Tremont turbine, passing between the guide-orifices and buckets. This gate was made to slide on the circumference of a disc supported from four brackets resting on columns, the disc having the object of relieving the wheel from the pressure of the column of water above it. The gate was kept water-tight by means of leather packing against the circumference of the disc; the latter also contained the lower bearing for the turbine-shaft.

The water was conducted to the wheel-pit (or forebay) by a wrought-iron riveted pipe about 130 feet long and 8 feet in diameter, constructed of plates $\frac{3}{8}$ of an inch thick.

*The Shaft* was of wrought-iron suspended from a collar-bearing, and steadied laterally by two bearings adjustable by means of set screws. The top of the fore-bay was closed to prevent the water rising to its natural level by about 6 or 7 feet, so that the shaft had to pass through a stuffing-box.

## *Dimensions of Turbine.*

The following were the leading dimensions and proportions of the turbine :—

Angle of outflow from guides . . $a = 78°$.
Angle of outflow from buckets . $a_2 = 75°$.

| | | |
|---|---|---|
| Angle of inflow into buckets . . | $a_1 =$ | 30°. |
| Diameter of wheel where water enters . . . . | $2\,r_1 =$ | 9·338 ft. |
| Diameter of wheel where water leaves . . . . | $2\,r_2 =$ | 7·987 ft. |
| Width of guide passages where water leaves . . . | $e =$ | 1·0066 ft. |
| Width of buckets where water enters . . . . | $e_1 =$ | 1·0066 ft. |
| Width of buckets where water leaves . . . . | $e_2 =$ | 1·2300 ft. |
| Number of guide-vanes. . . | $z =$ | 40. |
| Number of wheel-vanes . . | $z_1 =$ | 40. |
| Thickness of guide-vanes $t = \tfrac{3}{16}$ in. | $=$ | 0·0156 ft. |
| Thickness of wheel-vanes $t_1 = t_2 = \tfrac{1}{4}$ in. | $=$ | 0·0208 ft. |
| Measured area of guide passages . | $A_1 =$ | 5·904 sq. ft. |
| Measured outflow area of buckets . | $A_2{}^1 =$ | 6·8092 sq. ft. |

The areas $A^1$ and $A_2{}^1$ are those obtained by taking the shortest distance between two vanes, measured from the extremity of one of them, as the depth of the outflow section of each passage or bucket. This gives a *larger* area than the assumption that every filament of water follows the same relative course. The angle of outflow is given by the direction of a perpendicular to the plane of the outflow section in the centre of the latter, as elsewhere explained; the angles of flow are therefore not the same as the vane-angles.

In the present case, owing to the considerable space between the extremities of the guide-vanes and the entrance to the buckets to allow room for the regulating-gate, some uncertainty exists as to the true value of the angle *a*. If the water continued to follow the direction in which it issues from the guide passages, a portion of

it, instead of coming in contact with the buckets, would simply pass round the out-
side of the wheel, as indi-
cated in the sketch, Fig.
134. This of course is
impossible, as fresh water
keeps flowing in, so that
the fluid must be deflected
to some extent before en-
tering the wheel; what
this deflection, and there-

fore the *absolute* direction of inflow is, it is impossible to say with certainty. It has, in the present instance, been assumed that the angle *a*, which the direction of flow of the water immediately before entering the buckets forms with a radial line to any point of entry, is *the same* as the angle at which it leaves the guides; if the course remained unchanged, the former would be *greater* than the latter—in the case of some portions of the fluid, 90°.

The effective area of outflow from the guide passage, after allowing for contraction, is

$$A = 5\cdot3136 \text{ square feet},$$

while $\qquad A_2 = 6\cdot1283 \quad \text{,,} \quad \text{,,}$

From the dimensions given, the following ratios result : ·

$$\frac{A}{A_2} = 0\cdot867,$$

$$\frac{r_1}{r_2} = 1\cdot169,$$

$$\frac{e_1}{e_2} = 0\cdot82,$$

EXPERIMENTS ON THE BOOTT CENTRE-VENT WATER-WHEEL.

| 1 | 4 | 8 | 10 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Number of the experiment. | Height of the regulating-gate in inches. | Number of revolutions of the wheel per second. $\frac{n}{60}$ | Useful effect, or the friction of the brake in pounds avoirdupois raised one foot per second. $Wu$ | Total fall acting upon the wheel in feet. $h$ | Depth of water on the weir in feet. $H$ | Quantity of water passing the wheel in cubic feet per second. $Q$ | Total power of the water in pounds avoirdupois raised one foot per second. $E$ | Ratio of the useful effect to the power expended. | Velocity due to the fall acting on the wheel in feet per second. $V$ | Velocity of the exterior circumference of the wheel in feet per second. | Ratio of the velocity of the exterior circumference of the wheel to the velocity due to the fall acting on the wheel. |
| 1 | 3 | 0·59289 | 23211·8 | 14·603 | 1·2964 | 67·532 | 61493·4 | 0·37747 | 30·648 | 17·393 | 0·56750 |
| 2 | ,, | 0·87829 | 12073·5 | 14·672 | 1·2619 | 64·887 | 59364·4 | 0·20338 | 30·721 | 25·766 | 0·83871 |
| 3 | ,, | 0·72314 | 20032·3 | 14·568 | 1·2821 | 66·432 | 60347·0 | 0·33195 | 30·612 | 21·214 | 0·69301 |
| 4 | ,, | 0·54455 | 22447·2 | 14·158 | 1·2845 | 66·614 | 58821·6 | 0·38161 | 30·178 | 15·975 | 0·52937 |
| 5 | ,, | 0·49929 | 22630·8 | 14·197 | 1·2899 | 67·029 | 59351·7 | 0·38129 | 30·219 | 14·647 | 0·48470 |
| 6 | ,, | 0·44944 | 22026·8 | 14·143 | 1·2881 | 66·889 | 59002·2 | 0·37332 | 30·162 | 13·185 | 0·43714 |
| 7 | ,, | 0·25445 | 16129·1 | 14·244 | 1·2943 | 67·368 | 59858·1 | 0·26946 | 30·269 | 7·465 | 0·24661 |
| 8 | ,, | 1·01420 | 0· | 14·300 | 1·2115 | 61·083 | 54486·4 | 0· | 30·329 | 29·753 | 0·98101 |
| 9 | 6 | 1·01010 | 22952·9 | 14·288 | 1·5145 | 84·998 | 75732·8 | 0·30308 | 30·316 | 29·633 | 0·97746 |
| 10 | ,, | 0·95986 | 28807·9 | 14·226 | 1·5308 | 86·355 | 76608·0 | 0·37604 | 30·250 | 28·159 | 0·93086 |
| 11 | ,, | 0·93220 | 31814·1 | 14·197 | 1·5395 | 87·080 | 77093·2 | 0·41267 | 30·219 | 27·347 | 0·90496 |
| 12 | ,, | 0·90090 | 34476·0 | 14·193 | 1·5467 | 87·685 | 77607·2 | 0·44424 | 30·215 | 26·429 | 0·87470 |
| 13 | ,, | 0·85627 | 38243·0 | 14·194 | 1·5539 | 88·285 | 78143·8 | 0·48939 | 30·216 | 25·120 | 0·83134 |

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 14 | ,, | 0·69444 | 45135·3 | 13·778 | 1·5762 | 90·166 | 77480·4 | 0·58254 | 29·770 | 20·372 | 0·68433 |
| 15 | ,, | 0·59791 | 46402·8 | 13·606 | 1·5943 | 91·697 | 77812·1 | 0·59634 | 29·584 | 17·540 | 0·59291 |
| 16 | ,, | 1·17647 | 0· | 13·945 | 1·4180 | 77·112 | 67076·7 | 0· | 29·950 | 34·513 | 1·15237 |
| 17 | 9 | 1·10565 | 19779·8 | 13·518 | 1·6418 | 95·762 | 80736·3 | 0·24499 | 29·488 | 32·436 | 1·09997 |
| 18 | ,, | 0·99338 | 35931·0 | 13·373 | 1·6734 | 98·490 | 82145·2 | 0·43741 | 29·329 | 29·142 | 0·99362 |
| 19 | ,, | 0·90090 | 48212·7 | 13·396 | 1·6955 | 100·418 | 83728·2 | 0·57582 | 29·325 | 26·429 | 0·90125 |
| 20 | ,, | 0·82493 | 56195·7 | 13·379 | 1·7184 | 102·421 | 85571·4 | 0·65671 | 29·354 | 24·200 | 0·82442 |
| 21 | ,, | 0·78628 | 59226·4 | 13·379 | 1·7230 | 102·825 | 85800·0 | 0·69029 | 29·336 | 23·066 | 0·78629 |
| 22 | ,, | 0·74627 | 61168·8 | 13·342 | 1·7308 | 103·517 | 86138·0 | 0·71013 | 29·295 | 21·893 | 0·74731 |
| 23 | ,, | 0·72464 | 62065·4 | 13·322 | 1·7337 | 103·769 | 86218·8 | 0·71986 | 29·273 | 21·258 | 0·72620 |
| 24 | ,, | 0·71111 | 62752·2 | 13·334 | 1·7328 | 103·689 | 86229·3 | 0·72274 | 29·286 | 20·861 | 0·71232 |
| 25 | ,, | 0·69869 | 63199·3 | 13·304 | 1·7389 | 104·229 | 86483·7 | 0·73077 | 29·253 | 20·497 | 0·70087 |
| 26 | ,, | 1·25392 | 0· | 13·702 | 1·5981 | 92·018 | 78648·0 | 0· | 29·688 | 36·785 | 1·23907 |
| 27 | 12 | 0·70922 | 74979·3 | 13·400 | 1·8316 | 112·525 | 94057·5 | 0·79716 | 29·359 | 20·806 | 0·70868 |
| 28 | ,, | 0·69930 | 75347·2 | 13·431 | 1·8367 | 112·987 | 94662·2 | 0·79596 | 29·393 | 20·515 | 0·69796 |
| 29 | ,, | 0·67935 | 74580·9 | 13·331 | 1·8320 | 112·562 | 93603·9 | 0·79677 | 29·283 | 19·929 | 0·68058 |
| 30 | ,, | 0·67189 | 75153·2 | 13·378 | 1·8368 | 112·996 | 94296·4 | 0·79699 | 29·335 | 19·711 | 0·67193 |
| 31 | ,, | 0·66007 | 75208·3 | 13·386 | 1·8377 | 113·071 | 94415·2 | 0·79457 | 29·343 | 19·364 | 0·65990 |
| 32 | ,, | 0·64865 | 75249·2 | 13·383 | 1·8387 | 113·164 | 94471·1 | 0·79053 | 29·340 | 19·029 | 0·64856 |
| 33 | ,, | 0·63636 | 75103·3 | 13·356 | 1·8379 | 113·090 | 94219·0 | 0·79753 | 29·311 | 18·668 | 0·63692 |
| 34 | ,, | 0·62435 | 75202·0 | 13·381 | 1·8443 | 113·673 | 94881·9 | 0·79154 | 29·338 | 18·316 | 0·62431 |
| 35 | ,, | 0·61350 | 73993·4 | 13·405 | 1·8511 | 114·293 | 95571·2 | 0·78687 | 29·364 | 17·998 | 0·61291 |
| 36 | ,, | 0·59242 | 0· | 13·321 | 1·8476 | 113·969 | 94703·1 | 0·78132 | 29·272 | 17·379 | 0·59871 |
| 37 | ,, | 0· | 0· | 13·537 | 1·8087 | 110·454 | 93270·1 | 0· | 29·508 | 0· | 0· |
| 38 | ,, | 0· | 0· | 13·575 | 1·8072 | 110·325 | 93422·4 | 0· | 29·550 | 0· | 0· |
| 39 | ,, | 1·28505 | 0· | 13·596 | 1·6884 | 99·795 | 84635·0 | 0· | 29·573 | 37·698 | 1·27477 |

The method of conducting the experiments and the
apparatus employed were practically the same in the
case of the Boott turbine as for the Tremont turbine, and
require no special description.

### Results of Experiments.

The more important results of the trial of the turbine
at Boott are given in the preceding table.

Comparing experimental with calculated results, for
the best speed it will be seen that while the observed
value of $K_1 = 0\cdot726$, the value as calculated from the
observed efficiency, after allowing for shaft friction, is
$K_1 = 0\cdot659$.

The experimental co-efficient of speed $K_2 = 0\cdot637$;
that calculated from the observed velocity of flow, for
the assumption that the inflow is without shock, is $K_2 =
0\cdot797$; the agreement in this case is not good, the actual
velocity of rotation being much *less* than that corre-
sponding to inflow without shock.

For radial outflow $K_2 = 0\cdot711$, a much closer corre-
spondence.

The purely theoretical value of $K_2$, calculated with the
theoretical value of $K_1$, is for inflow without shock,
$$K_2 = 0\cdot724,$$
for radial outflow        $K_2 = 0\cdot645$.
The latter of these figures is very near that obtained by
experiment, the purely theoretical values thus showing
a closer agreement with the facts than those calculated
from the observed velocity of flow.

The imperfect agreement of the calculated with the
experimental results is probably due to the actual
absolute direction of flow of the water on entering the
wheel being different to that assumed; the true angle

*a* less than 78°. This would make the calculated speed less than the velocity above stated, and at the same time increase the calculated value of $K_1$, bringing it into closer agreement with actual results.

The difference between the values of the co-efficient $K_2$ as determined for inflow without shock and radial outflow shows that the turbine was not exactly designed in accordance with the principles adopted by the author; the difference is not sufficient to cause any serious discrepancy between theory and practice. In such cases, as elsewhere pointed out, it is safest to take the mean of the values of $K_2$ for inflow without shock and radial outflow.

### Holyoke Testing Station.

At Holyoke in Massachusetts, in the United States, there is a large permanent installation for testing turbines, the only one of the kind in the world, of which the following account is taken from the pamphlet published by the Holyoke Water Power Company, to whom the establishment belongs:—
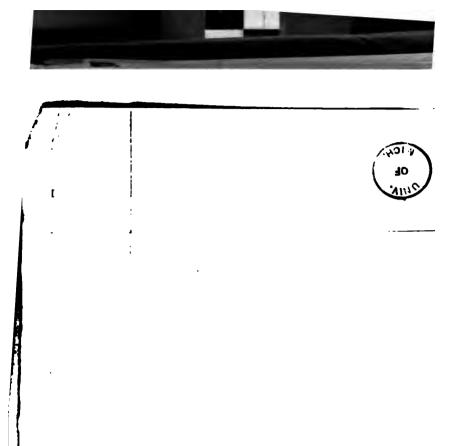
"The Holyoke Water Power Company controls the flow of the Connecticut river at Holyoke, Mass., on a fall of nearly 60 feet. Above its dam is a drainage area of 8144 square miles, as measured on the best maps in existence at the present time. At this date (1883) there are nearly 15,000 horse-power in use by day, and over 8000 by night, of which part are 'permanent powers,' held by the parties using them under indentures, and subject to annual rental, and the balance are 'surplus powers,' held by the parties using them subject to withdrawal at short notice. Surplus powers are paid for from day to day, and according to the amount used. In times

of drought, if used after prohibition, the parties so using are liable to a heavy penalty. There are at date 139 water-wheels in use in Holyoke, of which fifty-nine run about ten hours per day, and eighty run from Sunday midnight to Saturday midnight, or 144 hours per week. The number of distinct establishments operated by these wheels is about seventy. Observations are taken, giving the opening of the speed-gates of each wheel and the head acting upon it, once a day, and once during each night.

"These are carefully preserved, and from them are computed the amounts discharged by each establishment during the quarter year, and the ' surplus power' thus shown to have been used is paid for at the end of the quarter.

"This system results in economy in the use of water, where otherwise there would be great wastefulness. In times of low water it restricts the use of water, and, when need be, confines all parties to their indentured quantities.

"For the purpose of making the necessary experiments on the wheels of their tenants before they are set in the mills, the Holyoke Water Power Company has built at a large outlay a permanent testing flume. . . . Wheels are tested here both for power and for amount of water discharged. They are usually tested at five or six different openings of the speed-gate, ranging from wide open to the opening at which the discharge is one-half that at full opening, and at five or six different velocities of revolution at each gate opening, making some thirty odd experiments in all on each wheel. The final result is, that for all practical purposes the water-wheel has been converted into a water-meter, and its discharge may be known under any of the conditions under which it is found in the mill. Besides this, its efficiency, or the value of the wheel as a water-motor, is also known. Owing to the permanent and special arrangements made for testing

3 and the large number of wheels tried, the company
)le to carry out the tests at much less expense than
be possible if the experiments were made in the
)r manufactory where the wheel is used, and the
is that large numbers of wheels are sent from all
of the United States to be tested at Holyoke.

1e underground portion of the testing flume (*vide*
:35) consists in the main of a trunk or penstock
inging the water; a sort of vestibule *B*; an ante-
)er *C*; the wheel-pit *D*; and the tail-race *E*. The
*A* is of boiler-iron, about 9 feet in diameter, laid
)wn in the ground so as to pass longitudinally under
:ntre of a street. . . . The object of the vestibule
simply to afford opportunity for the application of
.ead-gates (sluices) *G G*. Besides these, there is a
gate at the point of entry of the trunk into the canal
:e it takes the water. A small trunk *F*, about 3
n diameter, takes water from this vestibule inde-
ntly of the gates *G*, and leads it to a turbine-wheel
: in an iron casing, in the chamber or pit *C*; so that
rheel can run even when *C* is empty. This wheel
.rges through the floor at the bottom of *C*, thence
gh the arch *L* and the supplementary tail-race *K*
:he second level canal. It is used to operate the
shops, also to lift and lower the gates *G*. The
)er *C* is bounded on one side by a tier of stop-plank
1 on another side by a tier of stop-plank *M*. The
: of the stop-plank *L* is to afford a waste-way out of
iamber *C*. This is of special use in regulating the
t of the water when testing under low heads; also
im off the oil floating on the water, which has
ed down from the dynamometer. The water thus
d over the plank *L* falls directly into the tail-race
d passes into the second level. The stop-planks *M*

come into use when testing scroll or cased wheels. In
that event, $D$ is empty of water, and the wheel-case in
question is attached to the planks $M$, over a proper open-
ing cut through the same. Large case-wheels could be
set in the centre of $D$ and the water be led to them by a
short trunk or penstock, leading from an opening cut in
the planks $M$. Flume wheels are set in the centre of the
floor of $D$, and $D$ is filled with water. They discharge
through the floor of $D$ (not shown on the drawing) and
out of the three culverts $N N N$ into the tail-race $E$. At
the down-stream end of this tail-race is the measuring-
weir $O$, the crest being formed of a piece of planed
wrought-iron. It can be used with or without end
contractions. The depth of the water on the weir is
measured in a quarter-cylinder $P$, set in a recess $Q$,
fashioned into the sides of the tail-race. These recesses
are water-tight, and the observer is thus enabled to stand
with the water-level about breast-high, or at a convenient
height for accurate observation. The methods of measur-
ing water over this weir are those described in *Lowell
Hydraulic Experiments*, by James B. Francis. . . . In
this country (America) these experiments are well known,
and the formula which was derived from them has met
with general acceptance. . . . A platform $R$ surrounds
the tail-race, and is suspended from the iron beams that
roof it in. Above the tail-race is a street. The wheels
to be tested arrive on this street, are lifted from the
wagon by a travelling windlass that runs out on a
framework over the street, are run into the building and
lowered into the wheel-pit $D$. Winding stairs lead into
a passage-way that leads in turn to the platform $R$. In
the well-hole of these stairs is set up the glass tube which
measures the head of water upon the wheel. It is con-
nected with the pit $D$ or the chamber $C$ by means of

pipes running through a cast-iron pipe $T$, built into the masonry dam which forms the down-stream end of the wheel-pit $D$.

"The power is weighed by a Prony brake, consisting of a cast-iron pulley surrounded by a hollow brass band, through which a stream of water is allowed to circulate, and a bent lever and weights to hold the band and pulley in its place. Five sizes of pulley and band are used, to weigh different sizes of wheels. The limits of power weighed have been from 11·4 to 230 horse-power. Heads have varied from 4 to 19 feet. To enable the observer at the brake-wheel, the one at the head-gauge, and the one at the measuring-weir to take simultaneous observations at intervals of one minute, an electric clock is set up in the testing-flume which rings three bells simultaneously at intervals of one minute, or of half-a-minute if desired.

"The whole structure is built in a durable and efficient manner. The pits and tail-race are all lined with brick laid in cement.

"The stone masonry was intended by careful work and grouting to be water-tight without the brick lining, and the brick lining was then carefully laid up with joints full of mortar as an extra precaution. As a consequence the front of the wall forming the down-stream side of the pit $D$ is barely damp with 20 feet head of water upon it. The floor of the pit $D$ is built so tight that an exact measurement of the leakage of the wheel-gate could be made if desired."
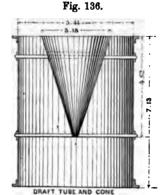
The following trials of "Collins," "Boyden," and "Hercules" turbines were carried out at Holyoke, and the author is indebted to the engineer of the Holyoke Water Power Company for most of the dimensions and other particulars regarding the construction of these turbines.

The greatest length of the measuring-weir at Holyoke is about 20 feet.

In the following tables the dimensions are given in feet, and the available and effective power in horse-power unless otherwise stated.

### Tests of a "Collins" Turbine.

In 1883 experiments were made at Norwich, Connecticut, on a turbine of the Jonval type constructed by Messrs. J. P. Collins and Co., to determine the efficiency of the wheel with and without a suction-tube (or "draft-tube," as it is called in America).

**Fig. 136.**



DRAFT TUBE AND CONE

The suction-tube was tried first with an interior inverted cone, as shown in Fig. 136, to give a gradually increasing area, and, secondly, without this cone.

Weir 8 feet long, with end contractions.

## Dimensions of Turbine.

$a = 72\frac{1}{4}°.$     $A_1 = 2\cdot912$ sq. ft.   $2\,r_m = 4\cdot170$ ft.

$a_1 = 12\frac{1}{4}°.$   $A_2 = 2\cdot822$ „

$a_2 = 71°.$     $z = 24.$

$c = 0\cdot836$ ft.   $z_1 = 30$ (brass vanes).

Effective area $A$, after allowing for obstruction by wheel-vanes, $= 2\cdot889.$

$$\frac{A}{A_2} = 1\cdot023.$$

## 60″ *Collins Turbine (Jonval type).*

| Proportional gate opening. | Total head. | Quantity of water per second. | Number of revolutions per minute. | Horse-power measured on brake. | Efficiency. | Ratio of velocity of rotation to $\sqrt{2gh}$. | Depth on weir. | |
|---|---|---|---|---|---|---|---|---|
| | $h$ | $Q$ | $n$ | $W_e$ | $e$ | $K_2$ | | |
| 1·000 | 16·58 | 64·31 | 82·00 | 96·24 | 79·71 | 0·551 | 1·867 | |
| ,, | ·57 | 64·35 | 85·25 | 96·41 | 79·85 | 0·573 | ·868 | |
| ,, | ·55 | 64·40 | 89·17 | 97·03 | **80·40** | 0·600 | ·869 | |
| ,, | ·56 | 64·40 | 93·00 | 96·65 | 80·03 | 0·625 | ·869 | |
| ,, | ·56 | 64·49 | 99·00 | 95·02 | 78·57 | 0·665 | ·871 | |
| 0·748 | ·79 | 58·50 | 81·50 | 86·69 | 77·94 | 0·544 | ·750 | |
| ,, | ·78 | 58·50 | 85·67 | 87·46 | 78·68 | 0·572 | ·750 | |
| ,, | ·77 | 58·50 | 90·00 | 87·48 | 78·75 | 0·601 | ·750 | |
| ,, | ·77 | 58·54 | 95·67 | 87·73 | 78·92 | 0·639 | ·751 | |
| ,, | ·77 | 58·44 | 105·00 | 86·65 | 78·08 | 0·701 | ·749 | |
| 0·600 | ·86 | 53·61 | 77·90 | 69·53 | 67·93 | 0·519 | ·650 | |
| ,, | ·89 | 53·61 | 83·50 | 70·44 | 68·70 | 0·556 | ·650 | |
| ,, | ·89 | 53·61 | 86·00 | 68·87 | 67·17 | 0·572 | ·650 | Leakage |
| ,, | ·90 | 53·61 | 89·75 | 69·68 | 67·92 | 0·597 | ·650 | 0·48 cubic |
| ,, | ·95 | 53·71 | 94·25 | 70·87 | 68·75 | 0·626 | ·652 | foot per |
| 0·503 | 17·18 | 45·14 | 69·67 | 56·22 | 64·02 | 0·460 | ·468 | second. |
| ,, | ·18 | 45·10 | 75·67 | 57·82 | 65·91 | 0·499 | ·467 | |
| ,, | ·19 | 45·06 | 80·17 | 57·83 | 65·93 | 0·529 | ·466 | |
| ,, | ·19 | 45·06 | 84·75 | 57·51 | 65·57 | 0·559 | ·466 | |
| ,, | ·18 | 45·06 | 88·67 | 56·37 | 64·31 | 0·585 | ·466 | |
| 0·303 | ·41 | 34·25 | 71·00 | 34·72 | 51·42 | 0·465 | ·218 | |
| ,, | ·42 | 34·21 | 76·00 | 36·24 | 53·70 | 0·498 | ·217 | |
| ,, | ·41 | 34·17 | 79·00 | 36·22 | 53·77 | 0·518 | ·216 | |
| ,, | ·40 | 34·12 | 83·50 | 35·73 | 53·15 | 0·548 | ·215 | |
| ,, | ·43 | 34·21 | 88·67 | 35·23 | 52·18 | 0·581 | ·217 | |
| 0·161 | ·84 | 24·80 | 53·00 | 19·12 | 38·16 | 0·343 | 0·984 | |
| ,, | ·84 | 24·72 | 59·17 | 19·53 | 39·11 | 0·383 | ·982 | |
| ,, | ·85 | 24·70 | 65·67 | 19·27 | 38·60 | 0·425 | ·981 | |
| ,, | ·85 | 24·67 | 71·17 | 18·71 | 37·52 | 0·461 | ·980 | |
| ,, | ·85 | 24·70 | 75·67 | 18·04 | 36·14 | 0·490 | ·981 | |

## 60" *Collins Turbine*, with suction-tube.

| Proportional gate opening. | Total head. $h$ | Quantity of water per second. $Q$ | Number of revolutions per minute. $n$ | Horse-power measured on brake. $W_e$ | Efficiency. $e_e$ | Ratio of velocity of rotation to $\sqrt{2gh}$. $K_2$ | Depth on weir. | |
|---|---|---|---|---|---|---|---|---|
| | | | *With inverted cone.* | | | | | |
| 1·000 | 16·58 | 65·13 | 77·50 | 97·12 | 79·46 | 0·521 | 1·880 | |
| ,, | ·56 | ·18 | 86·00 | 101·46 | 83·05 | ·578 | ·881 | |
| ,, | ·57 | ·30 | 90·25 | 102·62 | **83·78** | ·606 | ·884 | |
| ,, | ·58 | ·37 | 95·87 | 102·56 | 83·60 | ·644 | ·885 | |
| ,, | ·58 | ·37 | 106·00 | 101·73 | 82·92 | ·712 | ·885 | Leakage |
| 0·548 | ·96 | 52·09 | 74·00 | 69·66 | 69·66 | ·492 | ·613 | 0·30 cubic |
| ,, | ·98 | ·01 | 79·25 | 70·73 | 70·76 | ·526 | ·612 | foot per |
| ,, | 17·19 | ·36 | 87·33 | 73·14 | 71·79 | ·576 | ·619 | second. |
| ,, | ·01 | 51·99 | 92·25 | 71·62 | 71·54 | ·612 | ·611 | |
| ,, | ·01 | 22·01 | 99·50 | 69·95 | 69·85 | ·660 | ·612 | |
| 0·297 | ·40 | 36·11 | 67·00 | 38·50 | 54·13 | ·439 | ·257 | |
| ,, | ·63 | ·26 | 75·67 | 39·78 | 54·98 | ·493 | ·261 | |
| ,, | ·61 | ·22 | 82·12 | 39·16 | 54·23 | ·535 | ·260 | |
| ,, | ·61 | ·19 | 87·12 | 38·34 | 53·15 | ·568 | ·259 | |
| ,, | ·63 | ·22 | 93·25 | 37·05 | 51·26 | ·608 | ·260 | |
| | | | *Without inverted cone.* | | | | | |
| 1·000 | 16·56 | 64·57 | 78·00 | 97·75 | 80·75 | 0·524 | 1·869 | |
| ,, | ·57 | ·70 | 85·50 | 100·87 | 83·12 | ·575 | ·871 | |
| ,, | ·56 | ·88 | 89·87 | 102·18 | 84·01 | ·604 | ·875 | |
| ,, | ·55 | ·99 | 96·00 | 102·70 | **84·34** | ·646 | ·877 | |
| ,, | ·56 | ·88 | 105·50 | 101·25 | 83·25 | ·709 | ·875 | Leakage |
| 0·548 | 17·01 | 51·04 | 68·87 | 64·83 | 65·97 | ·457 | ·591 | 0·30 cubic |
| ,, | ·00 | ·04 | 76·00 | 67·83 | 69·05 | ·504 | ·591 | foot per |
| ,, | 16·99 | 50·99 | 82·50 | 69·09 | 70·45 | ·548 | ·590 | second. |
| ,, | 17·00 | ·92 | 89·75 | 69·68 | 71·10 | ·595 | ·589 | |
| ,, | ·02 | ·89 | 98·50 | 69·25 | 70·62 | ·653 | ·588 | |
| 0·297 | ·53 | 34·58 | 62·37 | 35·84 | 52·23 | ·407 | ·221 | |
| ,, | ·53 | ·53 | 69·92 | 36·76 | 53·64 | ·457 | ·220 | |
| ,, | ·52 | ·53 | 77·00 | 36·72 | 53·61 | ·503 | ·220 | |
| ,, | ·51 | ·53 | 82·00 | 36·09 | 52·73 | ·536 | ·220 | |
| ,, | ·51 | ·66 | 89·33 | 35·50 | 51·66 | ·584 | ·223 | |

It will be seen that without a suction-tube the maximum efficiency was             80·40 per cent.;
with a suction-tube and inverted cone     83·78 ,, ,, ;
and with a suction-tube without cone     84·34 ,, ,, ;
the suction-tube, therefore, increased the efficiency, but no advantage was obtained by using the inverted cone, although it might have been anticipated that this would act as a diffuser. The reason for this failure was probably due to the enlargement of the area not being sufficiently gradual, so that practically there was the same sudden change of velocity of the water as when no inverted cone was used.

### Test of a 96-inch Collins Turbine.

The following are stated to be the best results ever recorded at Holyoke with a turbine of the Jonval type; no details of the dimensions of the wheel are given, but the results are interesting as affording the material for a fair comparison between the parallel-flow and the mixed-flow turbine so much in vogue in America. Both types of wheel are tested under much the same conditions at Holyoke, and the same method of observation and calculation for determining the quantity of water used is employed, so that the effect of any important errors arising from differences in the method pursued is thus eliminated.

## 96″ *Collins Turbine* (*Jonval type*).

| Proportional gate opening. | Total head. | Quantity of water per second. | Number of revolutions per minute. | Horse-power measured on brake. | Efficiency. |
|---|---|---|---|---|---|
| | $h$ | $Q$ | $n$ | $W_e$ | $e_e$ |
| 1·00 | 16·59 | 113·46 | 63·38 | 131·49 | 85·06 |
| 0·778 | 16·75 | 101·67 | 63·00 | 160·83 | 83·27 |
| 0·631 | 16·95 | 87·66 | 58·93 | 133·56 | 79·26 |
| 0·547 | 17·12 | 77·19 | 54·32 | 111·36 | 74·64 |
| 0·422 | 17·26 | 65·36 | 49·65 | 84·40 | 65·96 |

### Experiments with "Boyden" Turbine.

The following are the particulars of dimensions and results of tests made with a Boyden turbine, a wheel of the Fourneyron type, for a mill of the Merrick Thread Company, on April 26th, 1882.

### *Dimensions of Turbine.*

[1] $a = 61°$.　　$A = 6·814$ sq. ft.　　[3] $d_g = 73·45$ in.

$a_1 = 0°$.　　$A_2 = 5·660$ sq. ft.　　[2] $r_1 = 73·60$ in.

[2] $a_2 = 61°$.　　　　　　　　　　　　[2] $r_2 = 90·00$ in.

$e = 9·875$ in.

$e_1 = 8·64$ in.　　Height of buckets at lowest point.

$e_2 = 9·125$ in.　　Height of buckets at outer end.

$z = 34$ (brass vanes).

$z_1 = 54$ (brass vanes).

The weir was 11 feet long and practically tight, the leakage being only 0·05 cubic foot per second.

[1] Mean angle of flow ; vane-angle = 66°.
[2] Mean angle of flow ; vane-angle = 64°.
[3] Diameter at guide-orifices.

## Boyden Turbine (*Fourneyron type*).

| Proportional gate opening. | Total head. | Quantity of water per second. | Number of revolutions per minute. | Horse-power measured on brake. | Efficiency. | Ratio of velocity of rotation to $\sqrt{2gh}$. | Depth on weir. | |
|---|---|---|---|---|---|---|---|---|
| | $h$ | $Q$ | $n$ | $W_e$ | $e_e$ | $K_2$ | *North.* | *South.* |
| 1·000 | 16·61 | 145·35 | 57·00 | 216·79 | 79·17 | 0·560 | 1·789 | 1·421 |
| ,, | ·57 | 146·18 | 59·33 | 216·55 | 78·82 | ·584 | ·790 | ·433 |
| ,, | ·60 | 147·10 | 63·50 | 222·04 | 80·17 | ·624 | ·798 | ·439 |
| ,, | ·62 | 148·32 | 66·50 | 220·28 | 78·79 | ·653 | ·807 | ·448 |
| ,, | ·64 | 149·87 | 71·12 | 222·50 | 78·66 | ·697 | ·824 | ·454 |
| 0·883 | ·66 | 142·04 | 55·67 | 203·19 | 75·71 | ·546 | ·761 | ·399 |
| ,, | ·63 | 142·82 | 58·75 | 207·23 | 76·93 | ·577 | ·765 | ·407 |
| ,, | ·65 | 143·27 | 62·50 | 210·87 | 77·94 | ·614 | ·769 | ·410 |
| ,, | ·64 | 144·93 | 65·17 | 209·88 | 76·73 | ·640 | ·783 | ·421 |
| ,, | ·63 | 145·74 | 68·37 | 209·71 | 76·29 | ·671 | ·790 | ·426 |
| 0·773 | ·74 | 136·12 | 56·00 | 188·94 | 73·11 | ·548 | ·716 | ·353 |
| ,, | ·72 | 137·40 | 58·75 | 189·21 | 72·61 | ·576 | ·722 | ·367 |
| ,, | ·69 | 138·18 | 62·33 | 191·17 | 73·09 | ·611 | ·729 | ·372 |
| ,, | ·73 | 139·65 | 66·17 | 192·81 | 72·76 | ·648 | ·736 | ·388 |
| ,, | ·73 | 140·66 | 69·25 | 193·29 | 72·42 | ·678 | ·749 | ·390 |
| 0·662 | ·80 | 128·86 | 55·75 | 169·28 | 68·95 | ·545 | ·654 | ·303 |
| ,, | ·81 | 130·69 | 59·67 | 172·04 | 69·04 | ·583 | ·658 | ·312 |
| ,, | ·81 | 130·89 | 62·00 | 171·15 | 68·58 | ·606 | ·665 | ·324 |
| ,, | ·80 | 132·22 | 66·17 | 172·51 | 68·47 | ·647 | ·673 | ·337 |
| ,, | ·76 | 133·22 | 69·67 | 170·95 | 67·51 | ·682 | ·685 | ·340 |
| 0·442 | 17·10 | 105·65 | 50·63 | 113·36 | 55·32 | ·490 | ·456 | ·127 |
| ,, | ·08 | 106·55 | 54·17 | 114·64 | 55·54 | ·525 | ·461 | ·137 |
| ,, | ·04 | 107·51 | 58·50 | 114·84 | 55·27 | ·568 | ·464 | ·151 |
| ,, | ·10 | 108·03 | 59·17 | 116·15 | 55·44 | ·573 | ·470 | ·153 |
| ,, | ·09 | 108·50 | 60·33 | 111·03 | 52·79 | ·585 | ·474 | ·157 |
| ,, | ·08 | 110·67 | 66·50 | 114·23 | 53·28 | ·645 | ·490 | ·177 |

## Tests of a 36-inch "Hercules" Turbine.

### Inward and Downward Flow.

These experiments were made in 1883 at Holyoke for the manufacturers of the wheel. The performance at the first test was so much better than the makers had anticipated, that it was decided to repeat the test. As the result of the second trial was equally good, the con-

D D

structors feared there might be some fault in the arrangements or the method of calculation. Professor Thurston was therefore requested to investigate the matter, and if possible ascertain whether the reported values of the efficiency might be safely accepted. The third trial, made under Professor Thurston's personal superintendence, practically confirmed the results previously obtained. In the following table only the particulars of the final test are given.

The weir was 20 feet long without end contractions.

## *Dimensions of Turbine.*

$a$     $= 75\frac{1}{4}°$, measured with centre line of guides.
$a_1 = - 8°$
$A^1 \; = 4\cdot752$ sq. ft.      $d_1$ mean $= 36$ in.      $z \; = 24.$
$A_2^1 = 7\cdot925$ sq. ft.                           $z_1 = 17.$

## *Description of Turbine.*

The "Hercules" turbine is of the inward and downward-flow type; the water on entering the wheel has a downward inclination, but the radial component preponderates. During its passage through the wheel the course of the fluid becomes more vertical, but only a portion of it near the outer circumference assumes entirely the axial direction. Fig. 137 shows a perspective view of the wheel itself, and Fig. 138 an outside view of the guide apparatus and casing. In Chapter XII., in connection with the
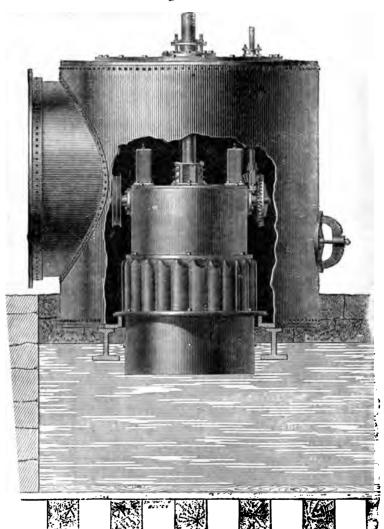
Fig. 137.

Fig. 138.



" Hercules " Turbine.

subject of "American Turbines," the shape of the buckets is shown in axial section through the wheel.

The outer edges of the bucket-vanes are inclined, so that the outer surface of the wheel where the water enters forms a truncated cone, the mean diameter of which is 3 feet. At the inflow the vanes are tapered almost to a knife-edge.

The regulation is effected by a cylindrical gate between the guide passages and wheel. Both guide passages and buckets are divided by transverse partitions into several parts, so as to form practically as many turbines super-imposed, which can be shut off in succession by the gate. This reduces the loss incidental to working at part gate. The outflow areas as given are calculated on the assumption that all the elementary filaments do *not* follow similar relative paths, but that the stream from each bucket or guide passage has the outflow area corresponding to the shortest distance between two adjacent vanes.

As with all radial turbines, to allow for contraction, the *effective* outflow area from the guides has been taken as only 90 per cent. of the measured area. Since there is a clear space between the guide orifices and the inflow orifices of the buckets (to allow room for the gate), the area of outflow, where measured, is not diminished by the obstruction of the wheel-vanes, and therefore no allowance has been made for this in computing the effective area.

A comparison of calculated with experimental results for the 36-inch "Hercules" turbine will be found under the heading of "American Turbines."

## 36" "*Hercules*" Turbine.

| Proportional gate opening. | Total head. | Quantity of water per second. | Number of revolutions per minute. | Horse-power measured on brake. | Efficiency. | Ratio of velocity of rotation to $\sqrt{2gh}$. | Depth on weir. |
|---|---|---|---|---|---|---|---|
| | $h$ | $Q$ | $n$ | $W_e$ | $e$ | $K^2$ | |
| 1·000 | 16·82 | 100·20 | Still | 0 | 0 | 0 | 1·305 |
| ,, | ·90 | 89·87 | 123·50 | 145·30 | 84·39 | 0·588 | ·214 |
| ,, | ·94 | 89·66 | 129·10 | 145·59 | 84·55 | ·614 | ·212 |
| ,, | ·95 | 89·00 | 135·33 | 146·02 | 85·38 | ·644 | ·206 |
| ,, | ·96 | 88·33 | 140·62 | 145·72 | **85·80** | ·669 | ·200 |
| ,, | ·98 | 87·79 | 144·80 | 143·88 | 85·14 | ·688 | ·197 |
| 0·806 | 16·84 | 80·56 | 112·33 | 128·73 | 83·70 | ·536 | ·129 |
| ,, | ·85 | 79·88 | 117·00 | 129·09 | 84·60 | ·558 | ·123 |
| ,, | ·88 | 79·38 | 123·00 | 129·71 | 85·39 | ·586 | ·118 |
| ,, | ·88 | 78·63 | 130·25 | 131·01 | 87·07 | ·621 | ·111 |
| ,, | ·92 | 78·00 | 137·00 | 130·28 | **87·08** | ·652 | ·105 |
| ,, | ·94 | 77·38 | 143·00 | 129·01 | 86·82 | ·680 | ·099 |
| 0·647 | 17·05 | 68·53 | 111·50 | 107·39 | 81·07 | ·529 | ·014 |
| ,, | ·05 | 67·83 | 120·00 | 110·46 | 84·25 | ·569 | ·007 |
| ,, | ·09 | 67·25 | 126·50 | 111·04 | 85·22 | ·599 | ·001 |
| ,, | ·15 | 66·52 | 134·67 | 111·65 | 86·33 | ·637 | 0·994 |
| ,, | ·16 | 65·72 | 140·40 | 109·55 | 85·69 | ·664 | ·986 |
| 0·489 | ·24 | 56·11 | 106·60 | 83·18 | 75·85 | ·503 | ·888 |
| ,, | ·25 | 55·54 | 115·60 | 85·27 | 78·50 | ·545 | ·882 |
| ,, | ·22 | 54·80 | 123·00 | 85·48 | 79·90 | ·581 | ·874 |
| ,, | ·28 | 54·07 | 130·00 | 84·79 | 80·05 | ·613 | ·866 |
| ,, | ·27 | 53·31 | 136·00 | 82·90 | 79·43 | ·641 | ·858 |
| 0·379 | ·62 | 45·64 | 104·00 | 64·66 | 70·93 | ·485 | ·774 |
| ,, | ·61 | 45·20 | 112·75 | 65·29 | 72·36 | ·526 | ·769 |
| ,, | ·65 | 44·50 | 122·67 | 65·06 | 73·06 | ·572 | ·761 |
| ,, | ·66 | 43·76 | 129·40 | 63·89 | 72·93 | ·603 | ·753 |
| ,, | ·65 | 43·24 | 135·00 | 61·72 | 71·34 | ·629 | ·747 |
| 1·000 | 9·59 | 68·01 | 88·67 | 60·00 | 81·14 | ·561 | 1·009 |
| ,, | ·62 | 67·41 | 93·33 | 60·31 | 82·03 | ·589 | ·003 |
| ,, | ·61 | 67·02 | 96·50 | 60·19 | 82·43 | ·612 | 0·999 |
| ,, | ·65 | 66·31 | 103·33 | 60·47 | 83·36 | ·651 | 0·992 |
| ,, | ·65 | 65·62 | 109·00 | 59·80 | 83·30 | ·687 | 0·985 |

### Trial of a " Humphrey " Turbine.

In December 1880, the "Humphrey" Machine Com-
pany, of Keene, New Hampshire, agreed to supply the
Tremont and Suffolk Mills with a new turbine of their
own design to replace a Fourneyron turbine of about
160 horse-power, the new wheel being intended to develop
about 270 horse-power with an average fall of 13 feet.
The payment for this was to depend upon the result of a
trial carried out by the well-known hydraulic engineer,
Mr. James B. Francis.

## *Description of Turbine.*

The "Humphrey" turbine in question is of the inward
and downward (mixed) flow type shown in Figs. 139, 140,
and 141.

The wheel $A$ has thirteen vanes $a\,a$, &c., which project
below the casing.  $B$ is a stationary cylindrical casing
enclosing the wheel; it has twelve openings $q$, through
which the water enters.  $C$ is the guide apparatus, which
contains twelve guide-vanes, and at the same time serves
as a regulator; it is enclosed by the curved sides $v\,v$, and
runs on spherical rollers $b$.  It can be turned by pinions
$D\,D$ gearing with a toothed ring on the circumference,
which receive their motion from a hand-wheel $G$ through
worm and bevel gearing $E$ and $F$.

The vanes $a$ are of gun-metal, and are secured to the
inner central casing by flanges and screws.

The turbine-shaft runs in a collar-bearing at the upper
end, and at the lower end in a footstep-bearing, which is
also capable of supporting the weight, if necessary, in the
absence of the collar-bearing.

The water from the head-race *R* enters the turbine through the wrought-iron tube *T*, which was employed for the old Fourneyron wheel, and is consequently rather too small for the new one. The estimated loss of head from this cause amounts to about four inches; but during the experiments the head was measured immediately over the wheel, so that the source of loss referred to was eliminated.

## *Brake Measurements.*

During the trial the main driving-wheel was removed from the turbine-shaft, and in its place the brake-pulley *H* substituted.

The latter is of cast-iron, containing a number of channels for the cooling water supply. *J* represents the brake-lever, which acts on the bell-crank lever *M* through the rod *K* suspended on knife-edges *L* and *L'*. The lever *M* rests on the knife-edge *N* and carries the pan *l*; the weight of the latter and of the bell-crank lever are balanced by the weight *P* hanging from the lever *O*.

The diameter of the brake-pulley was 5·44 feet, and the breadth 2·7 feet. The effective length of the brake-lever *J* was 15·9 feet, and the lengths of the arms of the bell-crank lever, *L' N* and *N i* respectively, 6·015 and 12·013 feet, so that the lever-arm to be used in the calculation was 31·755 feet, equivalent to a circumference of 199·523 feet.

## *Water Measurements.*

In order to observe the level of the water in the turbine-chamber immediately below the wheel, where

the fluid is in a state of violent agitation, a pipe $c\,c$ was laid on the floor of the chamber, both ends of which were connected with a water-tight vessel $d$ containing a scale. The pipe $c\,c$, of $1\frac{1}{2}$ inches internal diameter, was perforated on its upper side by twenty-two holes of about $\frac{3}{16}$ inch diameter.

It was agreed that as upper water-level was to be taken the level shown in the pipe $e$ connected with the chamber immediately above the wheel.

The pipe $e$ terminated in a glass-tube, parallel with which was placed a scale $f$.

The head introduced into the calculations is the difference between the levels in $e$ and $d$.

The quantity of water used was measured by two weirs $U\,U$, both of which extended over the whole width of the channels conducting the water to them, so that there was no end contraction; the widths were respectively 11·22 and 10·98 feet. To prevent any disturbance of the flow over the weir by the water in the wheel-chamber, gratings were introduced into the tail-race between the weirs and the chamber, which caused an appreciable depression of the water-level and a nearly uniform flow. The height of the water below the weirs was so regulated that the level was sufficiently deep below the sill not to influence the outflow.

The quantity of water passing over the weirs was calculated by Francis's well-known formula,

$$Q = 3\cdot33\,L\,H^{\frac{3}{2}},$$

in which

      $L$ denotes the length of the weir,

      $H$ the head of water above the sill, measured some distance from the weir, both dimensions in feet.

The head of water over the sills of the weirs was

measured by means of hook-gauges in water-tight vessels, connected by pipes with the water above the weirs. The hook-gauges were adjustable by micrometer screws, and allowed of very accurate observations.

The leakage during the tests was found to be only 0·01 cubic foot per second at most.

## Dimensions of Turbine.

Outer diameter 8·1.  $r_1 = 4·05.$
Inner diameter 2·0.

The *mean* diameter of outflow is uncertain, owing to the peculiar construction of the wheel, which allows the water on the outer side to follow its own course.

$$a = 75°. \qquad e = 2·086 \text{ feet.}$$
$$a_1 = 10°. \qquad e_2 = \text{indefinite.}$$
$$a_2 = \text{not given.}$$

## Humphrey Turbine.

| Opening of regulator. | Head of water. | Quantity of water per second. | Available power. | Number of revolutions per minute. | Velocity of circumference of wheel. | Work performed measured on brake. | Efficiency. | Ratio of velocity at circumference, to $\sqrt{2gh}$. |
|---|---|---|---|---|---|---|---|---|
| | $h$ | $Q$ | $E$ | $n$ | $w_1$ | $W_e$ | $e_e$ | $K_2$ |
| Per cent. | Feet. | Cubic feet. | Ft. lbs. | | | Ft. lbs. | Per cent. | |
| 100 | 12·478 | 207·82 | 161744 | 47·424 | 20·113 | 132475 | 81·9 | 0·7301 |
| 82·5 | 12·839 | 181·59 | 145421 | 51·600 | 21·884 | 111527 | 16·69 | 0·7831 |
| 77·24 | 12·983 | 180·27 | 145985 | 48·000 | 20·357 | 111733 | 76·54 | 0·7245 |
| 68·21 | 13·103 | 166·80 | 136326 | 45·840 | 19·441 | 99084 | 72·68 | 0·687 |
| 52·90 | 13·456 | 131·52 | 110387 | 50·64 | 21·477 | 67363 | 61·02 | 0·7508 |
| 40·66 | 13·688 | 110·70 | 94520 | 45·588 | 19·339 | 53065 | 56·14 | 0·6702 |
| 27·94 | 14·069 | 79·26 | 69558 | 55·680 | 23·614 | 18517 | 26·62 | 0·8073 |
| 19·15 | 13·998 | 59·02 | 51534 | 9·704 | 21·070 | 8264 | 16·04 | 0·7225 |
| 11·38 | 14·088 | 37·91 | 34013 | 37·206 | 15·777 | 3093 | 9·09 | 0·5336 |

The preceding table has been compiled from the original table of results, which has been much condensed, only the best out of several sets of observations at each opening of the regulator being given.

By mutual agreement between the users and makers of the turbine tested, it was agreed to measure the head available immediately over the wheel, in order to eliminate the loss due to friction in the conducting-pipe, which belonged to a former turbine of less power, and was consequently too small for the quantity of water supplied.

In doing this, however, no allowance has been made for the velocity of the water; at the point where the pressure was measured this velocity was about 3·458 feet per second in the first experiment, in which the maximum efficiency was obtained, equivalent to a head of about 0·2 foot, which should be added to the head given in the table to arrive at the effective available head over the turbine. The corresponding efficiency in that case is 80·59 instead of 81·9 per cent., but as no allowance has been made for friction of bearings the *hydraulic efficiency* will probably be somewhat in excess of the latter value.

### Experiments with "Little Giant" Turbine.

This series of tests is the only one made in Europe with an American turbine of which the author has been able to obtain any record; it was conducted by Professors Veith and Kronauer of Zürich.

The turbine is of the mixed-flow type, the water entering from the outside radially, and leaving axially; it is divided in the middle by a partition at right angles to the axis, which separates the water entering into two portions, which flow out on opposite sides of the wheel. That part of each bucket through which the water passes

after it has assumed an axial direction of flow stands out from the side of the wheel, in shape like a semicircular cowl; the construction is plainly shown in Fig. 142.

The water in its passage through the buckets appears to undergo a rather abrupt deflection of its course from the radial to the axial direction. The turbine is furnished with a guide apparatus.

Fig. 142.



### (1) *Turbine half open.*

| | Fall. $h$ | Quantity of water per second. $Q$ | Head above the weir. $H$ | Number of revolutions per minute. $n$ | Effective work. $W_e$ | Available energy. $E$ | Efficiency. $e_e$ | Proportion of velocity of rotation to $\sqrt{2gh}$. $K_1$ |
|---|---|---|---|---|---|---|---|---|
| | Feet. | Cubic feet. | Feet. | | Horse-power. | Horse-power. | | |
| 1) | 5·479 | 0·404 | 0·151 | 340 | 0·1568 | 0·2505 | 0·62 | 0·44 |
| 2) | 5·479 | 0·444 | 0·161 | 323 | 0·1655 | 0·2742 | 0·60 | 0·42 |
| 3) | 5·479 | 0·404 | 0·151 | 370 | 0·1517 | 0·2505 | 0·60 | 0·48 |
| 4) | 5·560 | 0·388 | 0·147 | 386 | 0·1183 | 0·2436 | 0·48 | 0·50 |

With a load of 1·213 pounds on the brake-lever the turbine came to a standstill, and the quantity of water flowing through was 0·459 cubic foot, with a head of 5·446 feet.

### (2) *Turbine fully open.*

| Fall. | Quantity of water per second. | Head above the weir. | Number of revolutions per minute. | Effective work. | Available energy. | Efficiency. | Proportion of velocity of rotation to $\sqrt{2gh}$. |
|---|---|---|---|---|---|---|---|
| $h$ | $Q$ | $H$ | $n$ | $W_e$ | $E$ | $e_e$ | $K_2$ |
| Feet. | Cubic feet. | Feet. | | Horse-power. | Horse-power. | | |
| 1) 5·381 | 0·954 | 0·269 | 424 | 0·2446 | 0·5824 | 0·42 | 0·55 |
| 2) 5·623 | 1·024 | 0·282 | 390 | 0·4206 | 0·6528 | 0·65 | 0·50 |
| 3) 5·610 | 1·095 | 0·295 | 310 | 0·3972 | 0·6904 | 0·57 | 0·40 |
| 4) 5·610 | 1·095 | 0·295 | 294 | 0·4063 | 0·6904 | 0·58 | 0·38 |
| 5) 5·610 | 1·130 | 0·302 | 264 | 0·3925 | 0·7200 | 0·55 | 0·34 |

With a load of 3·307 pounds on the brake-lever the turbine came to a standstill, and the quantity of water flowing through was 1·095 cubic foot, with a head of 5·446 feet.

The above results are in each case the mean of four tests.
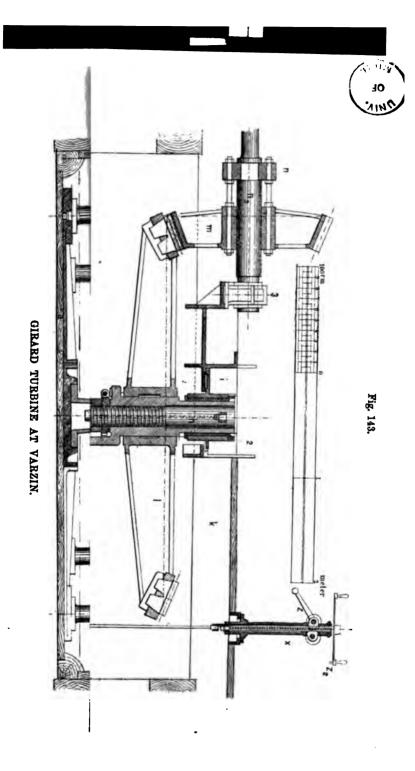
## *Water Measurements.*

The water used was measured by means of a weir, the fall taking place over the whole width. The quantity of water $Q$ was determined by the formula

$$Q = 0{\cdot}431\ L\,H\,\sqrt{2\,gH},$$

where $L$ is the width of the weir and $H$ the head above its edge ; $L = 1{\cdot}978$ foot.

The depth of water behind the weir during the experiments was 0·8525 foot.

GIRARD TURBINE AT VARZIN.

Fig. 143.

### Trial of Girard Turbines at Varzin.

At the Campmühle at Varzin (the property of Prince Bismarck) two Girard turbines, each of 200 horse-power, are employed to drive a wood-pulp manufactory. The turbines were constructed by the Gräflich Stolberg'schen Machine Works at Magdeburg.

The fall available is 13 feet $1\frac{1}{2}$ inches, and the quantity of water for each turbine 176·6 cubic feet per second. The makers guaranteed an efficiency of 75 per cent. with full admission, and 72 per cent. with half admission. To test the performance of these motors experiments were made by Professor Dr. Zeuner, which from the care and completeness of method with which they were carried out may be regarded as in all respects a pattern of what such trials should be.

## Description of the Turbines.

Both turbines are exactly alike in their construction (shown in Fig. 143), and enclosed in a wooden chamber divided by a partition into two independent compartments. Each motor can be shut off separately by a sluice from the head-race.

The axes of the wheels are vertical, and power is transmitted to the horizontal first-motion shaft by bevel gearing.

The turbines are of the axial or parallel-flow type, with ventilated buckets.

The guide apparatus rests on a circular cast-iron frame supported by eight cast-iron columns standing on a bedplate on the floor of the tail-race; the water enters the guide apparatus through a wrought-iron cylindrical casing connecting it with the head-race and also supported by the columns.

The guide-vanes are of steel-plate cast into the lower part of the walls of the guide-casing, which are here cylindrical; towards the top this casing is enlarged— the section being bell-mouthed—and divided into eight segments by cast-iron radial partitions, each of which can be closed by a valve. These valves serve to regulate the turbine. The inner casing of the guide apparatus supports a frame with arms, the spaces between which are filled in by wrought-iron plates; the frame carries the lower bearing for the turbine-shaft. This bearing consists of a cast-iron cylinder with four compartments containing blocks of lignum-vitæ, which can be adjusted by metal wedges with screws; a leather collar held by the cover of the bearing prevents leakage, and the water necessary for lubrication enters through a hole in the cover.

The turbine-wheel itself consists of the cast-iron casing with steel vanes, and a boss and arms, cast separately, bolted to flanges on the casing. This is keyed to the shaft, and to prevent the possibility of falling if it should become loose, a ring made in halves is let into the shaft below the boss.

The shaft is hollow, of cast-iron, with the main bearing, supporting the weight, above water. Into the upper part of the shaft is fixed the wrought-iron prolongation, the end of which rests upon the step, while to it is keyed the bevel driving-wheel. The step itself is carried upon the top of a wrought-iron spindle fixed into a taper socket on the bed-plate.

The wrought-iron prolongation of the turbine-shaft is screwed, and can be adjusted to take up wear by means of a nut which is turned by a small worm and worm-wheel, the latter forming part of the nut.

The turbine is regulated by eight valves of cast-iron,

already referred to; these valves are made water-tight by india-rubber packing held in place by dished cast-iron plates; they can be raised and lowered by rods actuated from screwed spindles in connection with gearing.

The following are the leading dimensions, &c. of the turbine and gearing:—

| | | |
|---|---|---|
| Mean diameter of guides and wheel . . | 8 ft. | 11·1 in. |
| Radial width of guide passages at outflow . | 0 ft. | 8·11 in. |
| Radial width of wheel-buckets at inflow . | 0 ft. | 8·5 in. |
| Radial width of wheel-buckets at outflow . | 2 ft. | 1·59 in. |
| Depth of wheel . . . . . | 1 ft. | 1·78 in. |
| Number of guide-vanes . . . . | | 64. |
| Number of wheel-vanes . . . . | | 70. |
| Thickness of guide-vanes . . . . | | 0·20 in. |
| Thickness of wheel-vanes . . . . | | 0·31 in. |
| Calculated number of revolutions per minute . . . . . . | | 30. |

## Bevel Gearing.

| | |
|---|---|
| Number of teeth in bevel mortice-wheel on the turbine-shaft . . . . . | 136. |
| Number of teeth in iron pinion on first-motion shaft . . . . . . . | 54. |
| Breadth of teeth . . . . . . | 13·78 in. |
| Pitch of teeth . . . . . . | 3·47 in. |
| Number of revolutions per minute of first-motion shaft . . . . . . | 75. |

## Brake Measurements of Power.

The brake employed to measure the horse-power of the turbine was of the usual "Prony" type. The brake-pulley was keyed to the horizontal first-motion shaft, it being impracticable to attach it direct to the vertical turbine

shaft; the diameter of this pulley was 8 feet 2·4 inches, and the breadth 1 foot 3·75 inches.

The end of the brake-lever pressed against the platform of a weigh-bridge, by which the load acting on the former was registered.

Water was employed for the lubrication of the brake-blocks. During the trial of the turbine with full admission a large quantity of water was required to keep the blocks cool.

The weight of the brake-pulley was 6788 lbs., that of the brake-band, blocks and lever complete 2595 lbs. Part of this latter weight was carried by the shaft and part by the weighing-platform, and before commencing the trials the portion of the weight carried by the platform was carefully ascertained by supporting the brake on a knife-edge laid on the highest point of the pulley, the brake-band being slackened so that none of the blocks were in contact with the pulley.

The weighing-machine was then adjusted so as to balance the free end of the lever, and the weight required for this noted, and in calculating the horse-power deducted from the total pressure on the weigh-bridge.

To arrive at the power actually given off by the vertical turbine-shaft, it was necessary to make allowance for the friction of the horizontal first-motion shaft and of the gearing.

This was done by calculation. For determining the power absorbed by the friction of the bearings the weight carried by these and its approximate distribution was taken into consideration, as well as the pressure exerted by the bevel-wheels. For the shaft-friction the co-efficient $f_1 = 0.07$ was employed in the formula for the work lost,

$$L_1 = f_1 \frac{n\Pi}{60} P d,$$

where $P$ denotes the total pressure on a bearing, $n$ the number of revolutions per minute of the shaft, and $d$ the diameter of the neck.

For friction of the gearing, a co-efficient $f_2 = 0 \cdot 1$ was used in the formula,

$$L_2 = f_2 \frac{l}{a} \, W,$$

where $W$ is the power transmitted, $l$ the length of a tooth, and $a$ half the length of the arc of contact of the teeth.

## *Water Measurements.*

The river Wipper, which supplies the power for the turbines, is dammed by a weir, from which the water enters each turbine-chamber by two sluices. During the experiments only one of these was open, and for the purpose of measuring the water behind this sluice a conducting channel of timber was specially constructed, having a width of 7 feet 4 inches, a depth of sides of 8 feet $2\frac{1}{2}$ inches, and a length of 23 feet, nearly. A section of this channel, at a distance of $19\frac{1}{2}$ inches behind the sluice, was selected in which to measure the velocity of the current. This was done by means of "Woltmann" current meters which had been carefully tested. The section was divided into sixteen equal rectangular parts, four in the width and four in the depth; in the centre of each of these the velocity of flow was observed, and from the results the mean velocity calculated.

When the turbine was working with full admission the mean velocity was $3 \cdot 151$ feet per second, and $1 \cdot 481$ feet with half admission.

The depth of the water in the channel varied slightly

during the trial, and in order to take these variations into account, the duration of the trial, 1 hour 20 minutes, was divided into four periods, during each of which the average depth was ascertained and used in calculating the section of the stream.

As the variation in the depth was relatively slight, Professor Zeuner considered it justifiable to assume that it had no appreciable influence on the velocity of flow.

According to previous agreement, the *effective head*, measured to the lower edge of the wheel-vanes, was to be taken into account in determining the efficiency, and not, as should from a scientific point of view be the case, the total head from the upper to the lower water-level.

As only *one* of the sluices admitting water to the turbine-chamber was open, the inflow was one-sided, and in consequence of this a whirling motion was set up, causing a depression in the surface of the fluid above the motor. To avoid any uncertainties due to this, the head was measured to the level of the water in the conducting channel immediately before entering the chamber, and not, as often happens, in the latter.

## *Results.*

In the following tables the results of the trials with full and half admission are given. To the particulars given in the original tables the author has added the available *total* head, measured from the head-race to the tail-race level, and the efficiency $\epsilon$ referred to this head.

Some further supplementary experiments were also carried out by Professor Zeuner to ascertain (1) the effect of allowing the turbine to *wade*, that is, to work partly immersed, and (2) the reduction in the efficiency caused by running the wheel at a higher and lower speed than the best.

The result of these experiments are also stated in a tabular form.

It will be seen that the power of the turbine is reduced by increasing the immersion; this is what would be anticipated, since free deviation turbines should work clear of the tail-water.

The experiments on the variation of the power with the velocity show that 30 revolutions per minute, the speed for which the turbine was designed, really is the best speed, as the work done decreases above and below this.

*Results of Experiment on the Effects of Immersion.*

| Depth of immersion or clearance. | Number of revolutions per minute. | Effective horse-power. | Remarks. |
|---|---|---|---|
| *h* | *n* | *W*ₑ | |
| —0·446 | 29·5 | 105·571 | |
| —0·446 | 29 | 101·617 | |
| —0·478 | 29 | 99·448 | |
| —0·217 | 33 | 108·123 | With half admission. |
| + 0·118 | 30 | 109·604 | |
| + 0·373 | 30 | 111·842 | |
| + 0·520 | 30 | 113·646 | |

*Results of Experiment on Varying the Speed.*

| Depth of immersion or clearance. | Number of revolutions per minute. | Effective horse-power. | Remarks. |
|---|---|---|---|
| *h₂* | *n* | *W*ₑ | |
| + 0·380 | 25·5 | 216·545 | |
| + 0·380 | 30 | 221·120 | With half admission. |
| + 0·377 | 34·5 | 215·589 | |
| + 0·380 | 36 | 213·489 | |

## Results of Trial with Full Admission.

| Total available head. | Clearance over tail-water. | Effective head. | Quantity of water used per second. | Available horse-power from effective head. | Number of re-volutions per minute. | Effective horse-power deve-loped. | Efficiency re-ferred to effec-tive head. | Efficiency re-ferred to total head. |
|---|---|---|---|---|---|---|---|---|
| $h$ | $h_2$ | $h$ | $Q$ | $E_a$ | $n$ | $W_e$ | $e_e$ | $e$ |
| Feet. | Feet. | Feet. | Cubic feet. | | | | | |
| 13·03 | 0·220 | 12·81 | 174·713 | 253·964 | 29 | 202·81 | 0·798 | 0·784 |
| 13·19 | 0·276 | 12·92 | 177·136 | 259·594 | 30·37 | 209·35 | 0·806 | 0·789 |
| 13·40 | 0·354 | 13·05 | 180·163 | 266·713 | 30 | 209·13 | 0·784 | 0·763 |
| 13·55 | 0·387 | 13·16 | 182·811 | 273·023 | 30·62 | 216·36 | 0·792 | 0·769 |
| | | | | | Mean : | 209·41 | 0·795 | 0·776 |

### Remarks.

According to contract the fall was only to be measured to the lower edge of the wheel, that is, not to include the clearance above the tail-water; the efficiency $e_e$ calculated at the trials was referred to the head thus measured, which is called the effective head $h_1$. For purposes of comparison the total head $h = h_1 + h_2$ has been added to the table as well as the efficiency $e$ referred to it.

## Results of Trial with Half Admission.

| Total available head. | Clearance over tail-water. | Effective head. | Quantity of water used per second. | Available horse-power from effective head. | Number of re-volutions per minute. | Effective horse-power deve-loped. | Efficiency re-ferred to effec-tive head. | Efficiency re-ferred to total head. |
|---|---|---|---|---|---|---|---|---|
| $h$ | $h_2$ | $h_1$ | $Q$ | $E_a$ | $n$ | $W_e$ | $e_e$ | $e$ |
| Feet. | Feet. | Feet. | Cubic feet. | | | | | |
| 13·59 | 0·076 | 13·51 | 89·666 | 137·488 | 29·62 | 109·032 | 0·793 | 0·786 |
| 13·61 | 0·066 | 13·54 | 89·949 | 138·188 | 30·37 | 111·359 | 0·805 | 0·801 |
| 13·63 | 0·062 | 13·57 | 90·305 | 139·065 | 30 | 111·871 | 0·804 | 0·800 |
| 13·51 | 0·072 | 13·58 | 90·376 | 139·243 | 29·83 | 111·842 | 0·803 | 0·807 |
| | | | | | Mean : | 111·024 | 0·801 | 0·798 |

### Remarks.

A negative ($\pm$) sign before the figure giving the clearance denotes that the turbine-wheel was *immersed* to the depth stated.

### Radial Outward-flow Impulse Turbines at Terni.

At Terni in Italy all the machinery of the large steel-works there is driven by a number of turbines acting under a head of 595½ feet. All these motors, which were made by Messrs. J. J. Rieter and Co. of Winterthur (Switzerland), are of the radial impulse outward-flow type with horizontal axes, and vary in capacity from 20 to 1000 horse-power. In the following list the leading particulars are given :—

| Horse-power. | Effective head. | Quantity of water per second. | Revolutions per minute. | Inner diameter. |
|---|---|---|---|---|
| | Feet. | Cub. ft. | | Ft. in. |
| 1000 | 595·5 | 19·77 | 180 to 240 | 7 10½ |
| 800 | ,, | 15·89 | 200 | 8 2·4 |
| 500 | ,, | 9·89 | 240 | 6 5·9 |
| 350 | ,, | 7·06 | 200 | 7 10½ |
| 150 | ,, | 3·00 | 250 | 6 4·7 |
| 50 | ,, | 0·99 | 850 | 1 10¼ |
| 50 | ,, | 0·99 | 850 | 1 10¼ |
| 40 | ,, | 0·85 | 450 | 3 6·1 |
| 40 | ,, | 0·85 | 450 | 3 6·1 |
| 30 | ,, | 0·60 | 600 | 2 7·5 |
| 20 | ,, | 0·42 | 450 | 3 6·1 |

In Fig. 144 a perspective view of one of the smaller motors is shown, while Figs. 145 and 146 represent a section and plan of the 800 horse-power turbine employed for driving the rail-mill.[1] The following are the leading dimensions and particulars of this latter motor :—
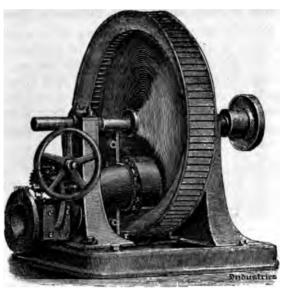
Angle of outflow from guide-
    passages .     .     .     .        $a = 70°.$
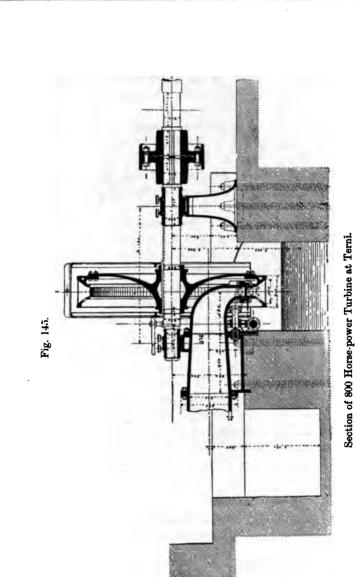
[1] For these illustrations the author is indebted to the courtesy of the editors of *Industries.*

Fig. 144.



| | |
|---|---|
| Angle of inflow into wheel-buckets . . . . | $a_1 = 54°.$ |
| Angle of outflow from wheel-buckets . . . . | $a_2 = 70°.$ |
| Internal diameter of wheel . | $2\,r_1 = 8$ ft. $2·4$ in. |
| External diameter of wheel . | $2\,r_2 = 9$ ft. $5$ in. |
| Width of guide-passages . . | $c = 4·91$ in. |
| Width of buckets at outflow . | $c_2 = 16·14$ in. |
| Number of guide-passages . | $z = 2.$ |
| Number of wheel-buckets . | $z_1 = 100.$ |
| Maximum proportion of circumference over which water is admitted . . . | $\omega = \frac{1}{36}.$ |
| Diameter of supply-pipe . . | $= 24$ in. |
| Diameter of auxiliary pipe . | $= 8$ in. |

Fig. 145.

Section of 800 Horse-power Turbine at Terni.

The radial velocity of the water leaving the wheel is 34 feet per second, representing a loss of only 3·6 per cent. of the available head.

The quantity of water is calculated as 85 per cent. of that theoretically due to the head at the guide-orifices; the velocity of flow as 92 per cent. of the theoretical velocity; the difference between 92 and 85 being due to the obstruction of the guide-orifices by the passing wheel-vanes.

The dimensions given in the illustrations are in mètres.

Since this turbine is used for a rolling mill train, it is necessary to stop and start at frequent intervals, the reversals being performed by gear on the train. To stop the flow of water under such high pressure suddenly would entail considerable risk of bursting the pipes, and to avoid this an auxiliary pipe 8 inches diameter has been provided, branching off from the main supply pipe on the pressure side of the throttle-valve. This auxiliary pipe is provided with a stop-cock geared to the spindle of the main throttle-valve in such a manner that it opens when the latter is closed, and *vice versâ*. The closing of the throttle-valve is effected through worm-gearing by hand. By this arrangement the motion of the water in the main supply-pipe is not entirely arrested when the throttle-valve is closed, but finds an outlet by the auxiliary pipe.

### Trial of Radial Outward-flow Impulse Turbine at Immenstadt.

## Description of Turbine.

This motor was constructed to drive a string manufactory at Immenstadt, and is of the same type as those

.It is intended to
rer with a head of
3·5 cubic feet per

teel-plate 0·2 inch
it there are besides
• support one side
is connected with
f the water-supply
rack on the back
-wheel and worm.
nflow edges of the
guide-passages as
from a governor or
the turbine are as

$r_1 = $ 7 ft. 10½ in.
$r_2 = $ 8 ft. 11·1 in.
$c = $ 4·32 in.
$c_1 = $ 4·72 in.
$c_2 = $ 15·75 in.
$z_1 = $ 110.
or through supply-
2 feet 4 inches, of
nd the remainder
ave a diameter of
about 13 feet, the
wrought-iron pipes
of 19 feet 8 inches,

.

ist-iron pipes, with
exceed about 4·125
it-iron pipes it is

The radial vel‹
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the main supply
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**Trial of Rad**


*D*

This motor w‹
tory at Immens

used at Terni previously described. It is intended to develop a maximum of 400 horse-power with a head of 570·8 feet, and a supply of water of 8·5 cubic feet per second.

The majority of the vanes are of steel-plate 0·2 inch thick, cast in with the wheel-casing; but there are besides thirty-six strong vanes of cast-iron to support one side of the casing from the other, which is connected with the arms and boss. The regulation of the water-supply is effected by a segmental slide with a rack on the back worked by a pinion through a worm-wheel and worm. The face of the slide fits against the inflow edges of the guide-vanes, and closes or opens the guide-passages as required; it can be controlled either from a governor or by hand. The principal dimensions of the turbine are as follows :—

| Inner diameter | | | $2\,r_1$ = | 7 ft. 10½ in. |
|---|---|---|---|---|
| Outer diameter | | | $2\,r_2$ = | 8 ft. 11·1 in. |
| Width of guide-passages | | $c$ = | | 4·32 in. |
| Width of buckets at inflow | | $c_1$ = | | 4·72 in. |
| Width of buckets at outflow | | $e_2$ = | | 15·75 in. |
| Number of wheel-vanes | | $z_1$ = | | 110. |

The water is conducted to the motor through supply-pipes having a total length of 4642 feet 4 inches, of which 3481 feet are of cast-iron and the remainder wrought-iron. The cast-iron pipes have a diameter of 21·66 inches, and are in lengths of about 13 feet, the metal being 0·71 inch thick. The wrought-iron pipes are 18·9 inches in diameter, in lengths of 19 feet 8 inches, and 0·2 to 0·47 inch thick.

The velocity of the water in the cast-iron pipes, with the largest possible supply, does not exceed about 4·125 feet per second, while in the wrought-iron pipes it is 5·44 feet per second.

## Water Measurements.

The quantity of water used by the wheel during the trial was measured over a weir or bay, the formula employed to determine the co-efficient being that of Braschmann, according to which

$$Q = \left(0.3838 + 0.0386 \div \frac{L}{B} + \frac{0.00174}{h}\right) L\, H \sqrt{2gH}$$

(*vide* "Measurement of the Quantity of Flowing Water"). As a check the results were compared with those obtained by Castel and with Weisbach's formula.

The head immediately before the guide-passages was measured by means of a pressure-gauge, and found (as above stated) to be 570·8 feet.

## Measurement of Power developed.

The power exerted by the turbine was measured with a brake in the ordinary way, and in addition to this the work expended on shaft-friction was also determined.

## Results of Trial.

The results of the experiments are given in the accompanying table. At the time they were carried out the maximum supply of water for which the wheel was designed (8·5 cubic feet) was not available, so that the turbine could not be tested to its full power.

In the account of the trial from which the preceding particulars have been taken, the vane-angles are not given, but as the effective head is very nearly the same as for the 800 horse-power turbine of the same type at Terni, it is safe to assume that the angles are also nearly identical with those of the latter :

$a$ = 70°, angle of outflow from guides.

$a_1$ = 54°, angle of inflow into buckets.

$a_2$ = 70°, angle of outflow from buckets.

## *Radial Outward-flow Girard Turbine with Horizontal Axis at Immenstadt.*

| Head. | Opening of regulator. | Quantity of water per second. | Available power. | Number of revolutions per minute. | Effective power measured on brake. | Efficiency. | Power absorbed by friction of shaft. |
|---|---|---|---|---|---|---|---|
| $h$ | | $Q$ | $E_{hp}$ | $n$ | $W_e$ | $c_e$ | $W_f$ |
| Feet. | | Cubic Feet | Horse-power. | | Horse power. | | Horse-power. |
| 570·84 | 10 | 2·111 | 136·8 | 211 | 81·5 | 0·595 | 5·0 |
| ,, | 15 | 3·350 | 217·1 | 211 | 144·3 | 0·665 | 4·8 |
| ,, | 20 | 4·375 | 283·4 | 214 | 196·8 | 0·694 | 4·7 |
| ,, | 25 | 5·187 | 336·0 | 216 | 253·9 | 0·755 | 4·5 |
| ,, | 15 | 3·156 | 204·5 | 240 | 129·2 | 0·632 | 5·6 |
| ,, | 15 | ,, | ,, | 213 | 137·5 | 0·672 | 4·9 |
| ,, | 15 | ,, | ,, | 194 | 140·2 | 0·686 | 4·4 |
| ,, | 31 | 6·804 | 440·9 | 210 | 336·8 | 0·764 | |

Head measured with pressure-gauge immediately before turbine.

From observed quantities and measurements the velocity of flow from the guide-orifices was found to be

$$c = 0{\cdot}948\sqrt{2gh} = \sqrt{\frac{2gh}{1 + 0{\cdot}113}}.$$

For calculating the effective area of the guide passages, the above co-efficient must of course be reduced by the usual amount to allow for obstruction by the wheel-vanes, the value given having been determined when the guide orifices were free.

## *Turbines at Schaffhausen for Electric Power Transmission.*[1]
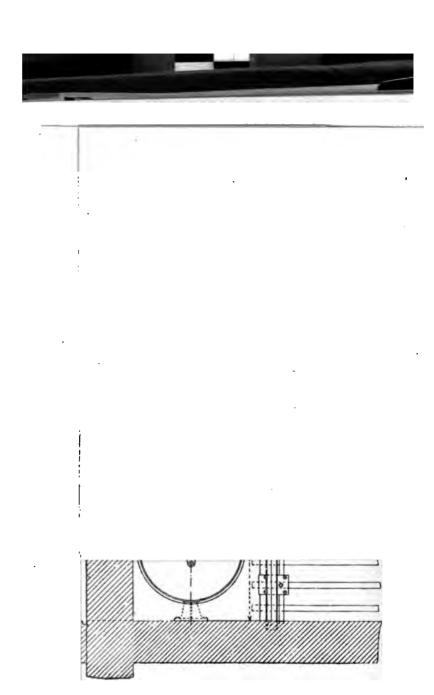
Five new turbines have recently been added to those already in existence for about thirty years, for utilizing the fall of the Rhine at Schaffhausen. The older turbines have always been and still are used for driving wire ropes which transmit the power developed to the opposite bank of the river. The new turbines drive dynamos from which the power is transmitted by electric cables to various points; each develops at least 300 H.P.

Two of the turbines were supplied by Messrs. J. J. Rieter and Co. of Winterthur. These work dynamos from which the current is transmitted through copper wires to Schaffhausen, on the opposite bank of the Rhine, for the purpose of driving a spinning manufactory. Of the remaining three turbines, supplied by Messrs. Escher, Wyss and Co. of Zürich, two are employed in providing electricity for aluminium works adjoining the turbine house.

In every case the dynamos are driven by ropes from counter-shafts connected by mortice bevel-gearing with the vertical turbine-shafts. Figs. 147, 148 illustrate the arrangement and design of Messrs. Rieter and Co.'s turbines. These are of the Jonval type, each with two concentric sets of buckets, an arrangement adopted on account of the variations occurring in the available head and quantity of water.

In time of flood, in summer, the head is only about 9 ft. 10 in., while the flow of water amounts to 388 cubic feet per second. When the river is low the maximum
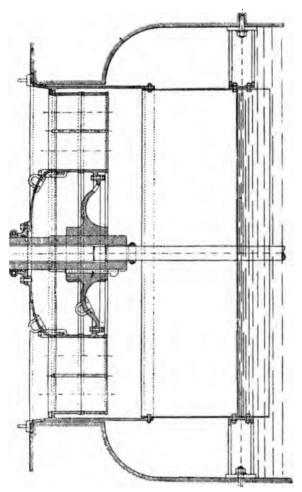
[1] *Zeitschrift des Vereins Deutscher Ingenieure,* vol. xxxvii., No. 46, November 18, 1893, p. 1416.
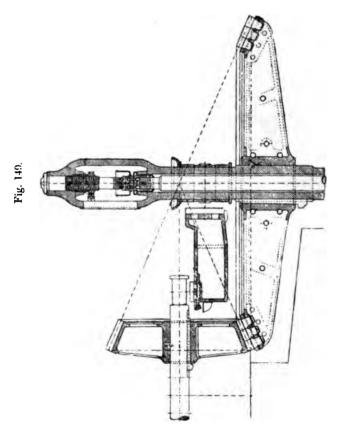
14 ft. 9 in., and the corresponding quantity of water

cubic feet per second; the change in the head is
the rise of the tail-water level. When the water

is low—with the maximum head—the outer set of buckets only is used, while in floods both sets come into action. When not required the inner buckets are closed by hand-covers.



Fig. 149.

The guide-ring fits into a cast-iron casing, let into the masonry, and carrying at its lower end a wrought-iron draft-tube. The lower edge of the wheel is 4 ft. 7 in.

above the tail-water level when the latter is at its lowest.

Each turbine wheel is keyed to a hollow cast-iron shaft supported on an overhead bearing. The shaft is made in two parts, with a total length of 32 ft. 1·84 in.; the outer diameter is 13 in., the inner diameter 7·47 in.

The wrought-iron column, about which the shaft revolves, fits into a cast-iron socket secured by four lewis-bolts to the floor of the tail-race.

The column carries at its upper end an oil-bath in which the shaft-pivot revolves, as clearly shown in Fig. 149. The lower end of the pivot is secured to a cast-iron washer, which runs on a similar washer attached to the top of the column; both washers have a diameter of 7·08 in., and the total load on their bearing surface, including water pressure, is 50,706 lbs.

To steady the shaft there are four bearings, of which two are below water and have lignum vitæ bushes.

The bevel mortice-wheel keyed to the upper end of the turbine-shaft has 132 teeth, with a pitch of 3·34 in., a pitch diameter of 11 ft. 8·32 in., and a breadth of 13·78 in. The pinion has 56 cast-iron teeth, with a pitch diameter of 4 ft. 11·53 in.

Each rope-pulley driving a dynamo has a diameter of 13 ft. 1·48 in., and 10 grooves for ropes each 1·97 in. diameter.

The water is admitted to the turbine-chambers by balanced gates turning about vertical spindles, as shown in Fig. 147.

The three turbines of Messrs. Escher, Wyss and Co. are designed for similar conditions, and are of the same type as those of Messrs. J. J. Rieter. They run, however, at a speed of 48 revolutions per minute, are smaller in diameter, and have a greater width of buckets.

Instead of working with a draft-tube, each turbine is placed at the lower end of a cylindrical casting forming part of the turbine-chamber.

With low water the wheel is just covered, but in times of flood the tail-water rises round the outside of the casing.

A section through the wheel and guide-casing is shown in Fig. 150.
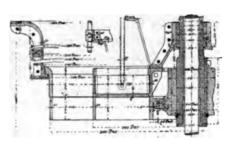
Fig. 150.



The inner set of guide-passages can be closed by vertical slides.

The dynamos driven from the Rieter turbines are six-pole compound dynamos, running at a speed of 200 revolutions per minute, and each developing 330 amp. with a pressure of 630 volts, equivalent in round numbers to 350 H.P.; of this, 565 H.P. is transmitted to the spinning manufactory.

The accompanying table, which has been compiled from data furnished to the author by the makers, contains the more important dimensions, ratios, and quantities for one of Messrs. Rieter's turbines.

The outer ring is so designed as to work at the best speed with the maximum head of 14·75 feet; its area is calculated for a quantity of water somewhat in excess of the minimum actually occurring, namely, 270 cubic feet

## Details of Jonval Turbine at Schaffhausen (by Messrs. J. J. Rieter and Co.).

| | Outer Ring. | | | Inner Ring. | | | Outer and Inner Ring combined. |
|---|---|---|---|---|---|---|---|
| | Outside. | Mean. | Inside. | Outside. | Mean. | Inside. | |
| $E_{hp}$ = Effective Horse-power | ... | 296 | ... | ... | ... | ... | 296 |
| $h$ = Heads, feet | ... | 14·75 | ... | ... | 9·833 | ... | 9·833 |
| $Q$ = Quantity of water per second, cubic feet | ... | 270 | ... | ... | 198 | ... | 420·4 calculated with mean co-efficient $K_1$ and lower head. |
| $a$ = Guide outflow angle | 66° | 63° | 60° | 64° | $59\frac{1}{2}$° | 54° | |
| $a_1$ = Wheel inflow „ | 40° | $36\frac{1}{2}$° | 33° | 20° | 17° | 14° | |
| $a_2$ = „ outflow „ | $71\frac{1}{2}$° | 69° | $66\frac{1}{2}$° | $67\frac{1}{2}$° | $63\frac{1}{2}$° | $59\frac{1}{2}$° | |
| $d$ Diameters, feet | 11·155 | 9·9083 | 8·6617 | 8·4975 | 7·185 | 5·8725 | |
| $e$ $e_1 = e_2 =$ Widths, feet | ... | 1·247 | ... | ... | 1·3125 | ... | |
| $z$ = Number of guide-vanes | ... | 40 | ... | ... | 40 | ... | |
| $z_1$ = Number of wheel-vanes | ... | 42 | ... | ... | 42 | ... | |
| $t$ = Thickness of guide-vanes, inches | ... | 0·59 | ... | ... | 0·59 | ... | |
| $t_1$ = Thickness of wheel-vanes, inches | ... | 0·55 | ... | ... | 0·55 | ... | |
| $h_o$ = Depth of guides, inches | ... | 10·63 | ... | ... | 10·63 | ... | |
| $h_o^1$ = Depth of wheel, inches | ... | 10·63 | ... | ... | 10·63 | ... | |
| $A^1$ ... sq. ft. | ... | 15·073 | ... | ... | 12·400 | ... | |
| $A^1_2$ ... „ „ | ... | 11·338 | ... | ... | 10·443 | ... | |
| $A$ ... „ „ | ... | 13·908 | ... | ... | 11·292 | ... | 25·200 |
| $A_2$ ... „ „ | ... | 11·338 | ... | ... | 10·443 | ... | 21·781 |
| $\dfrac{A}{A_2}$ = Area-ratios ... | ... | 1·227 | ... | ... | 1·081 | ... | 1·15 |
| $n$ = Revolutions per minute | ... | 46 | ... | ... | 46 | ... | 46 |
| $V$ = Velocity due to head, feet per second | ... | 30·816 | ... | ... | 25·164 | ... | |
| $K_1$ = Co-efficient of velocity | ... | 0·630 (1) | ... | ... | 0·697 (2) | ... | 0·663 Mean of (1) and (2). |
| $K_2$ = Co-efficient of speed | ... | 0·774 | ... | ... | 0·688 | ... | 0·814 |
| $w$ = Velocity of rotation, feet per second | 26·867 | 23·865 | 20·862 | 20·467 | 17·305 | 14·143 | 20·504 |
| $c$ = Velocity of flow, feet per second | ... | 19·414 | ... | ... | 17·540 | ... | 16·684 for lower [head. |
| $c_2$ = Relative velocity of outflow, feet per second | ... | 23·812 | ... | ... | 18·966 | ... | |

per second (the minimum being 240). The inflow takes place without shock, and the outflow is approximately vertical, $w$ and $c_2$ being nearly equal.

The inner ring is designed to give the maximum efficiency with the minimum head of 9·833 feet, and under these conditions is calculated to consume 198 cubic feet of water per second.

It is clear that in a double turbine of this class, when the diameter and width of the inner ring have been fixed, that of the outer ring is also determined within comparatively narrow limits, and this of course settles the velocity of rotation. In calculating the dimensions of the outer ring, a method somewhat different to that employed for a simple turbine has to be followed.

In the first place (as usual) $a$ may be chosen, but the other angles, the area ratio, and the diameter can only be determined by a process of trial and error.

If the inner ring is of normal design with $\dfrac{A}{A_2} = 1$ and $a_1$ a small angle, then for the outer ring $\dfrac{A}{A_2}$ must have a larger value, and $a_1$ must be greater in order that the conditions of inflow without shock and vertical outflow may be observed. This is the case in the present example. For the inner ring—as will be seen from the table—the area-ratio is 1·081, the angles $a$ and $a_2$ are rather less than is usual, $a_1$ slightly greater, while the diameter is a little under the normal value for continental practice. For the outer ring the ratio $\dfrac{A}{A_2}$ and all the angles are greater, the angle $a_1$ being quite abnormal.
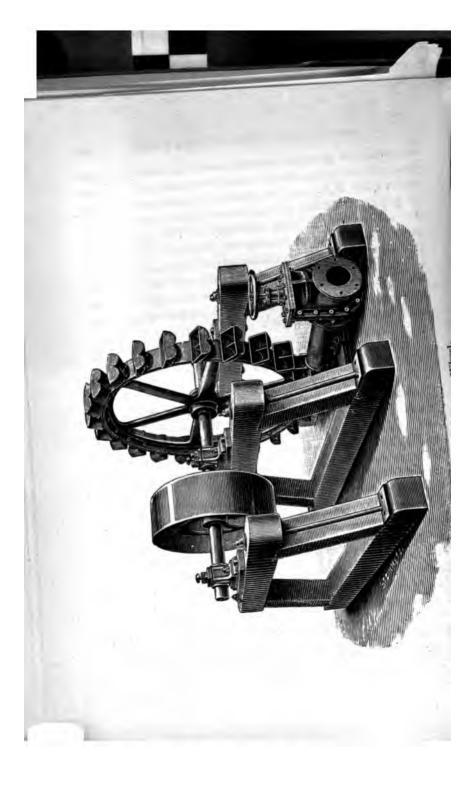
When the two rings are both at work with the maximum quantity of water and the minimum head, the inner ring only is working at the best speed. Under these conditions

the available power is greater than with the maximum head and minimum quantity, and the guaranteed effective power (296 H.P.) can be obtained with an efficiency of 68 per cent. When only the outer ring is working (with the maximum head), an efficiency of 73·5 per cent. is necessary in order to develop the required power. The preceding efficiencies are calculated for the actual maximum and minimum quantities of flow 388 and 240 cubic feet per second respectively, and not for the quantities for which the turbine is designed (which are rather greater in order to allow some margin).

### The Pelton Wheel.

The Pelton wheel, or, as it is popularly called, the "hurdy-gurdy," is a form of motor which has recently found great favour in the mining districts of California and other parts of America. Its construction is shown in Fig. 151.[1] It consists essentially of a series of double buckets attached to the circumference of a wheel; the water issues through a nozzle from the supply-pipe, and, striking the buckets in the middle, is deflected to both sides equally. The direction of the jet of fluid is tangential to the mean circumference, measured to the middle of the radial width of the buckets. A little consideration will make it evident that the Pelton wheel is an impulse turbine of the *axial* class, in which the angle $a$ is 90°, and $a_2$ also near 90°. Figs. 152 and 153 show respectively a perspective view of and a section through one of the buckets. The relative velocity of inflow is $c - w = c_1$, and the relative velocity of outflow—apart from friction—the same $c_2 = c_1$, since there is no fall in traversing the

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[1] For this illustration the author is indebted to the courtesy of the editors of *Engineering.*

buckets, and the velocity of rotation is the same where the water enters as where it leaves the wheel. To obtain the minimum absolute residual velocity, in this case $O: c_2 = w = c_1$, and hence $c = 2\ w$, or $w = \dfrac{c}{2}$ the best speed of the wheel.

As actually made, the relative angle of inflow $a_1$ is not quite $90°$, which for absence of shock should on theoretical grounds be the case, but to attain this it would be necessary to bring the middle of the bucket to a knife-edge to avoid loss by impact against a blunt surface. To do this would be impracticable, and the loss caused by making

Fig. 152.    Fig. 153.



the edge flat would probably more than counterbalance any advantage obtained from the correct construction of the angle $a_1$.

Very high results are claimed for these turbines, one of which is said to have shown an efficiency of over 87 per cent.; without feeling bound to accept this result as absolutely correct, the author sees no reason why a Pelton wheel should not have a very fair efficiency, although there are causes of loss peculiar to its construction which must be pointed out.

In the earlier wheels, owing to the angle $a_2$ being $90°$, the water on leaving a bucket has no velocity in an axial direction to enable it to clear the following bucket, and

must be carried round some distance by the wheel before acquiring from gravity the motion necessary for the purpose. This was pointed out in a paper read by Mr. Hamilton Smith, junr., before the American Society of Civil Engineers, in which the author stated that "an examination of a hurdy-gurdy with either flat, recessed, or curved buckets, while at work, shows that the wheel 'carries' over a large amount of water, the force of which is consequently lost, and in fact becomes an additional load to lift."

In more recently constructed Pelton wheels this defect has been remedied by making the angle $a_2$ rather less than 90°. Obviously, as in the case of other impulse turbines, the thickness (or depth) of the jet must be taken into account in determining $a_2$.

Another cause of loss consists in the obstruction of the jet of water driving the wheel, not only by the central dividing edges of the buckets in a plane at right angles to the axis, but also by the outer edges parallel with the latter.
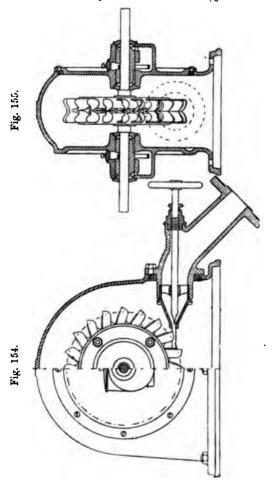
The usual method of regulating Pelton wheels is by throttling the jet, but recently nozzles have been introduced in which the jet is regulated by an accurately fitted internal cone, as illustrated by Figs. 154 and 155. For high powers several jets are sometimes employed, placed so far apart that a bucket supplied by the first jet is emptied before being acted on by the next jet.

In cases in which the water-supply varies at different times, the supply-pipes are so arranged that nozzles of different sizes can easily be fitted on to them.

The body of the wheel is generally of cast-iron, and the buckets of hard bronze. For small wheels the buckets are polished internally in order to reduce friction.

It is stated that Pelton wheels can be made ranging in

power from $\frac{1}{10}$ to 2000 h.p., and that the manufacturers guarantee an efficiency of from 80 to 85 %.

Fig. 155.

Fig. 154.

In designing a Pelton wheel it is obviously an advantage to pitch the buckets as far apart as practicable, in order

to reduce the loss caused by impact of the jet with the edges of the buckets to a minimum.

On the other hand, if the pitch be too great, it is not possible to give the buckets such a form that the energy of the jet is advantageously utilized in the extreme position of each bucket when the jet is acting on it from the greatest distance. The greater the pitch, the greater also is the distance through which the water has to pass before coming into contact with the buckets, and this increases the loss due to the resistance of the air. The best proportions, as regards these points, can only be determined by experience. It may be mentioned, that as at present made in America, a 6-foot wheel has 24 buckets.

The buckets are not fixed with their longer edges radial to the wheel, but so that, when any bucket is in the position in which the jet first acts upon it, the central edge forms an angle of *not over* 90° with the direction of the jet, measured on the side next the nozzle. If the angle in question were over 90°, the water would be deflected inwards (towards the wheel), and its free escape possibly impeded.

The width of the buckets should be at least seven times as great as the thickness of the jet (where it leaves the nozzle); and, with a given relative angle of outflow $a_1$ and given pitch, the jet where it leaves the bucket must be thin enough not to come into contact with the back of the succeeding bucket. This point has been fully dealt with in connection with the design of impulse turbines in general.

It is worth noting that with a Pelton wheel the water enters the buckets without shock at *all* speeds of the wheel, although the outflow is axial only at one speed. Among the various installations of Pelton wheels now at work the following may be mentioned—

At the Idaho Mines in Nevada, where these motors were first employed on a large scale, there are now eighteen Pelton wheels of various sizes, driving winding-drums, stamps, air-compressors, pumps, &c.

At the Treadwell Mine in Alaska, a 7-foot wheel working with a fall of about 490 feet and a water-supply of 10·5 cubic feet per second, develops about 500 horse-power, and drives 240 stamps, 96 ore-mills, and 13 ore-crushers. The nozzle usually employed has a diameter of $3\frac{5}{16}$ inches, but when the water-supply is plentiful a nozzle of 4 inches diameter is used, and the wheel gives off about 735 horse-power.

The largest Pelton wheel yet made is said to be one driving air-compressors at a mine in Costa Rica. This has a diameter of about 14 feet 6 inches, and at 95 revolutions per minute with a fall of 390·5 feet is stated to develop 120 horse-power.

In a recent installation of the "Pelton Water Wheel Company," the wheel works with a fall of 2106 feet, and has a circumferential velocity of 181 feet per second. In this case the body of the wheel is made of steel, instead of cast-iron, on account of the great stress to which it is subject.

In 1891 a test was made by the "Chester Hydraulic Engineering Company, Ltd.," of a very small Pelton wheel, driven by water from the mains of the London Hydraulic Power Company, supplied under a pressure of 740 lbs. per square inch. The wheel had a diameter of about 18 inches over the tips of the buckets, and worked at 1600 revolutions per minute. The efficiency as ascertained by means of a brake was 70 per cent. The pressure measured by a gauge in connection with the main close to the motor was 660 lbs. per square inch, equivalent to a head of 1522 feet. The difference between this pressure and that of

740 lbs. per square inch observed near the accumulator
was due to loss in the mains.

In California some trials with a Pelton wheel were
carried out by Mr. E. Browne. The diameter of the
wheel was about 15 inches, the width $1\frac{3}{8}$ inch. The fall
was 56 feet. With a nozzle of 10 millimetres internal
diameter, the efficiency—according to the *Engineering
and Mining Journal*—was 82·6 per cent., while with a
nozzle of 9 millimetres diameter, the efficiency reached
82·5 per cent.

## The Niagara Falls Installation.

Various projects for utilizing on a large scale some of
the water-power available at the Falls of Niagara have
long been under consideration, and at length one of these
is actually being carried out and is approaching its
completion.

The project in question contemplates as its leading
feature the installation of a Central Power Station, from
which the water-power utilized through turbines will be
transmitted and distributed by electricity to neighbouring
manufactories, and employed for lighting purposes.

The chief characteristic of the scheme, as carried out, is
a large tunnel tail-race, into which all the turbines erected
will discharge through branch tail-races.

Users of power on a large scale, instead of being sup-
plied electrically from the central station, may construct
turbines of their own discharging into the tunnel.

This installation is designed for utilizing 10,200 cubic
feet of water per second, with an available head of 140
feet, equivalent—with an assumed efficiency of 75 per
cent. for the turbines—to about 120,000 horse-power.

The head-race (or supply canal) starts at a point $1\frac{1}{2}$ miles above the American Fall; it has a width of 500 feet, a depth of 12 feet, and is 1500 feet long. Along the edge of this canal wheel pits, 160 feet deep, have been sunk for accommodating the turbines. From the bottom of these pits the branch tail-races drain into the tunnel. This tunnel is cut out of limestone rock, and it was expected that this would prove hard enough to render lining unnecessary. Experience has shown that such is not the case, and it has been found necessary to protect the rock with four rings of brickwork in cement. The tunnel has a horseshoe section, 21 feet high by 19 feet wide, with an area of about 385 square feet. The length is 6,700 feet, and the gradient 0·7 feet per 1000 feet. This unusually steep gradient is estimated to give a velocity of flow of 25 feet per second, which is extremely high.

The first two wheels for use in connection with the Central Power Station—for the Niagara Falls Power Company—are now approaching completion. They were designed by Messrs. Faesch and Piccard of Geneva (Switzerland), and are being made by the I. P. Morris Co. of Philadelphia.

Each of these wheels is to develop 5000 horse-power with a mean head of 136 feet.

The arrangement and construction of the turbines is shown in Figs. 156 to 162.[1]

Each turbine consists of two wheels of the Fourneyron type, keyed to the same shaft; into one of these the inflow is downwards, into the other upwards. The water is supplied through a steel pipe $7\frac{1}{2}$ feet diameter, Figs. 158 and 159, and flows through a cast-iron bend into the guide-vane chamber.

[1] The Author is indebted for these illustrations to the courtesy of the Editors of *Engineering*.

The lower wheel is so constructed that the pressure due to the head is not borne by the wheel itself, but by the guide-casing. On the other hand, the full pressure acts on the upper wheel and balances the superincumbent weight.
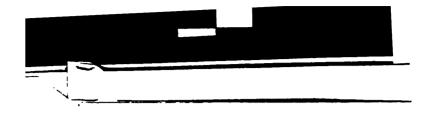
Each wheel with its guides is divided into three stages, and the regulation is effected by means of a balanced circular sluice surrounding the wheel. This sluice is actuated through levers and connecting rods from a governor—of peculiar construction patented by Messrs. Faesch and Piccard—placed at the top of the wheel-pit. The leading dimensions of the turbines are given in the illustrations. The water supply for each twin-turbine is 430 cubic feet per second, which, with a head of 136 feet measured to the centre between the pair of wheels and an efficiency of $75\frac{1}{3}$ per cent., will give 5000 horse-power.

The wheels will make 250 revolutions per minute; this gives a co-efficient of speed for the inflow-circumferences of the wheels, $K_2 = 0\cdot734$.

The main shaft of each turbine is a steel tube 38 inches in diameter, except at the guide-bearings, where it is solid, and has a diameter of only 11 inches. A thrust bearing carries the weight of the shaft and wheels when at rest; but when running it is relieved of its load by the pressure of the water on the upper wheel as already described.

To insure greater regularity in the speed, a fly-wheel $14\frac{1}{2}$ feet diameter and weighing 10 tons will be mounted on the shaft. As its peripheral velocity will be about 11,400 feet per minute, it is being made of wrought iron.
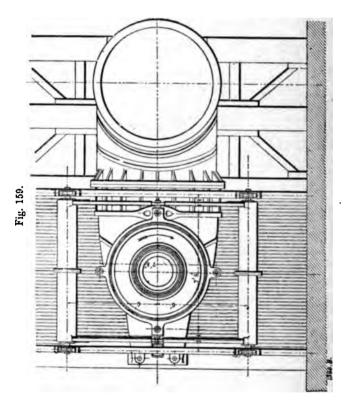
Before arriving at a decision as to the type of turbine to be adopted for the Niagara installation, the promoters referred the matter to an International Commission. This Commission invited competitive designs from some of the leading turbine-makers of the world, with the result, that the design considered to be first in order of merit was

Fig. 159.

Plan from underneath
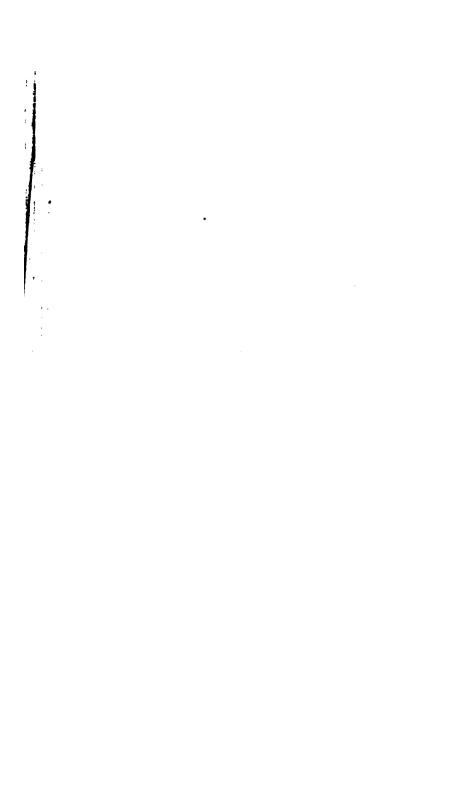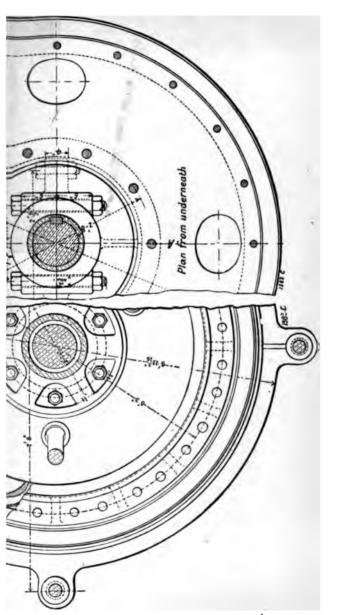
that of Messrs. Escher, Wyss and Co. of Zürich.   Owing, it is understood, to the refusal of this firm to allow their designs to be carried out by American manufacturers, the scheme of Messrs. Escher, Wyss and Co. has not been practically adopted; as being admittedly the best of those submitted to the Commission, it however deserves notice.

Messrs. Escher, Wyss and Co. proposed the employment of twin Jonval turbines, keyed to the same shaft, the lower one discharging upwards and the other downwards. The water, however, after leaving the wheel-vanes was to be so directed, by a curved continuation of the wheel-casing, that the final discharge into the tail-race should be *horizontal.*   With this arrangement the pressure of the water on the lower wheel would neutralize that on the upper wheel, and wholly or partially balance the weight of the revolving parts.   The bearing was to be of the over-head Fontaine type.   For regulating the turbine a balanced circular sluice throttling the discharge and controlled by an hydraulic governor was to be provided.   In this case also the speed would have been 250 revolutions per minute, and the power developed 5000 horse-power.

The first hydraulic motors actually at work in connection with the Niagara installation are three turbines supplied to the Niagara Falls Paper Company by Messrs. R. D. Wood and Co. of Philadelphia.   Each of these is designed to develop 1100 horse-power under a fall of 140 feet.   The type of wheel adopted is the inverted Jonval, in this case also with the object of balancing the weight of the shaft and gearing.   The regulation is effected by a circular sluice throttling the supply.   Each wheel has a diameter of 4 feet 8 inches, and is designed to run at a speed of 260 revolutions per minute.[1]

[1] A description with illustrations of these turbines is given in *Engineering,* vol. lvii., 1894, no. 1476, p. 480.

## Trial of a "Swain" Turbine at Boott Cotton Mills.[1]

In 1874, Mr. James B. Francis carried out a series of experiments on a "Swain" turbine, one of a pair, both of the same pattern, made to replace so-called centre-vent (inward-flow) wheels, which had been in use since 1849.

The apparatus used for measuring the power of the wheel was substantially the same as that employed in testing the above-mentioned centre-vent, and also the Tremont turbine previously described. For gauging the discharge of the water consumed, a weir without end contractions was adopted, with a length of 16·311 feet; in other respects the method of observation was similar to that followed in the earlier trials. The crest of the weir was 12·08 feet above the bottom of the canal of approach. The water had a free fall from the weir, the height of the water on the down-stream side of the weir being in no experiment less than 2·5 feet below the crest. For computing the quantity of water flowing over the weir, Francis's well-known formula

$$Q = 3\text{·}33\ L\ H^{\frac{3}{2}}$$

was used.

The experiments, of which the results are given in the adjoining table, were very carefully carried out in every respect, and perfectly independent of the makers of the turbines.

<hr>

[1] *The Journal of the Franklin Institute*, 1875, vol. xcix., p. 249.

## Description of the Turbine.

The "Swain" turbine tested was of the inward mixed-flow type, the outflow from the upper portion of the wheel being radial, while that from the lower portion had a varying vertical component, the outflow at the outer diameter being axial, but becoming more radial towards the centre.

The discharging edge of each bucket lay in a vertical plane passing through the axis of the wheel, being parallel to this axis from the under side of the crown to a point about 8½ inches below it, and from this point was continued in the form of a quadrant, with a radius equal to one-fifth of the outer diameter of the wheel, and having its centre in the cylinder forming the outer circumference of the wheel.

The regulating gate was formed by the lower side (or disc) of the guide-casing to which the guide-vanes were attached, the construction being such that the whole of this lower disc could be moved vertically, so as to increase or diminish the width of the guide-passages parallel with the axis of the wheel. In the upper crown of the guide-apparatus were slots to allow the passage of the vanes when the gate was raised.

Twenty-one of the guide-vanes were of bronze, 0·23 inch thick and 18·94 inches long, tapered at each end to 0·04 inch in thickness, with a bevel on each side one inch long; the remaining three of the guide-vanes were of cast-iron of sufficient thickness at the outer ends to allow the passage of the rods for adjusting the gate.

The inner edges of the guide-vanes were distant radially 1⅝ inch from the outer edges of the buckets. The water after leaving the wheel passed through a short

## RESULTS OF EXPERIMENTS WITH "SWAIN" TURBINE.

| Number of the experiment. | Height of the regulating gate. | Number of revolutions per second. $\frac{n}{60}$ | Useful effect or friction of the brake. $W_a$ | Total fall. $h$ | Depth of water on the weir. $H$ | Quantity of water passing the weir per second. $Q$ | Available power of the water. $E_a$ | Efficiency. $e_a$ | Ratio of the velocity of the exterior circumference to velocity due to the head. $K_2$ |
|---|---|---|---|---|---|---|---|---|---|
| | Inches. | | Foot lbs. | Feet. | Feet. | Cubic feet. | Foot lbs. | | |
| 3 | 3·25 | 1·0438 | 38038·2 | 13·912 | 1·2025 | 71·623 | 62022·1 | 0·6133 | 0·6583 |
| 14 | 6·50 | 1·0503 | 74992·3 | 13·316 | 1·6596 | 116·126 | 96257·8 | 0·7791 | 0·6770 |
| 19 | 13·00 | 1·1111 | 107810·0 | 12·666 | 2·0983 | 165·092 | 130166·2 | 0·8282 | 0·7344 |
| 29 | 2·00 | 1·0080 | 21572·5 | 14·283 | 0·9611 | 51·177 | 45518·7 | 0·4739 | 0·6274 |
| 34 | 3·00 | 1·0526 | 34806·5 | 13·977 | 1·1617 | 68·009 | 59189·5 | 0·5881 | 0·6623 |
| 49 | 4·00 | 1·0019 | 47672·5 | 13·699 | 1·3362 | 83·894 | 71562·5 | 0·6662 | 0·6478 |
| 51 | 5·00 | 1·0204 | 59179·7 | 13·482 | 1·4794 | 97·736 | 82048·8 | 0·7213 | 0·6537 |
| 69 | 6·00 | 1·0362 | 69263·5 | 13·281 | 1·6016 | 110·092 | 91044·1 | 0·7608 | 0·6689 |
| 78 | 7·00 | 1·1055 | 76932·3 | 13·102 | 1·6894 | 119·268 | 97309·2 | 0·7906 | 0·7184 |
| 90 | 8·00 | 1·0683 | 84794·3 | 12·968 | 1·7916 | 130·253 | 105184·6 | 0·8061 | 0·6979 |
| 103 | 9·00 | 1·0810 | 91961·2 | 12·836 | 1·8695 | 138·839 | 110956·3 | 0·8288 | 0·7098 |
| 113 | 10·00 | 1·0721 | 95583·9 | 12·680 | 1·9224 | 144·774 | 114292·8 | 0·8363 | 0·7082 |
| 118 | 11·00 | 1·0943 | 99266·1 | 12·640 | 1·9815 | 151·501 | 119226·3 | 0·8326 | 0·7227 |
| 123 | 12·00 | 1·1267 | 102435·2 | 12·521 | 2·0266 | 156·703 | 122158·8 | 0·8385 | 0·7490 |
| 128 | 13·08 | 1·8060 | 20237·7 | 13·099 | 1·7000 | 120·392 | 98203·9 | 0·2061 | 1·1738 |
| 129 | ,, | 1·6299 | 55189·4 | 12·880 | 1·8608 | 137·871 | 110581·6 | 0·4991 | 1·0683 |
| 130 | ,, | 1·4614 | 80804·4 | 12·603 | 1·9608 | 149·133 | 117041·9 | 0·6904 | 0·9683 |
| 131 | ,, | 1·3006 | 99109·8 | 12·480 | 2·0453 | 158·877 | 123471·8 | 0·8027 | 0·8660 |
| 132 | ,, | 1·2115 | 103522·0 | 12·432 | 2·0654 | 161·225 | 124814·5 | 0·8294 | 0·8083 |
| 133 | ,, | 1·1516 | 104652·0 | 12·372 | 2·0766 | 162·538 | 125223·8 | 0·8357 | 0·7701 |
| 134 | ,, | 1·1364 | 105401·9 | 12·408 | 2·0854 | 163·572 | 126387·3 | 0·8340 | 0·7588 |
| 135 | ,, | 1·1080 | 104857·4 | 12·366 | 2·0857 | 163·607 | 125986·6 | 0·8323 | 0·7412 |
| 136 | ,, | 1·0874 | 104952·0 | 12·368 | 2·0911 | 164·243 | 126496·7 | 0·8297 | 0·7273 |
| 137 | ,, | 1·0558 | 103885·0 | 12·385 | 2·0911 | 164·243 | 126670·6 | 0·8201 | 0·7057 |
| 138 | ,, | 1·0618 | 104472·3 | 12·380 | 2·0957 | 164·785 | 127037·5 | 0·8224 | 0·7098 |
| 139 | ,, | 1·0188 | 102160·1 | 12·372 | 2·0923 | 164·384 | 126646·6 | 0·8067 | 0·6813 |
| 140 | ,, | 0·9928 | 101416·6 | 12·386 | 2·0935 | 164·526 | 126899·0 | 0·7992 | 0·6635 |
| 141 | ,, | 0·9370 | 99238·3 | 12·381 | 2·0940 | 164·585 | 126893·2 | 0·7821 | 0·6264 |
| 142 | ,, | 0·8532 | 95034·8 | 12·399 | 2·0978 | 165·033 | 127423·8 | 0·7458 | 0·5700 |
| 143 | ,, | 0·6550 | 78892·7 | 12·399 | 2·0793 | 162·855 | 125741·9 | 0·6274 | 0·4376 |
| 144 | ,, | 1·8405 | | 13·173 | 1·6759 | 117·841 | 96666·1 | | 1·1928 |
| 145 | ,, | 0· | | 12·580 | 1·9840 | 151·788 | 118907·9 | 0· | 0· |
| 146 | ,, | 0· | | 12·556 | 1·9832 | 151·696 | 118609·3 | 0· | 0· |

*Remarks.*

At *part gate* only the results at the *best speed* are given; in the original, the results of trials at various speeds for each opening of the gate are recorded, but have been here omitted for the sake of brevity.

divergent suction-tube, about 5 feet 9 inches in diameter
and 21½ inches long.

The step upon which the wheel revolved was a cylinder
of white oak with conical ends, free either to revolve with
the wheel or remain stationary, and resting on a cast-
iron socket attached by three arms to the sides of the
suction-tube. This step was enclosed in the hollow con-
tinuation of the wheel-boss, and supplied with water for
lubrication by a pipe from below. The diameter of the
step appears to have been very considerable—about 12
inches—in proportion to that of the wheel, and makes it
probable that the loss by friction was greater than the
average for such turbines.

The wheel-vanes were of bronze cast into the cast-iron
wheel-casing.

### *Dimensions of Turbine.*

| | | |
|---|---|---|
| Outside diameter . | . | $2\,r_1$ = 6 ft. |
| Least inner diameter | . | $2\,r_2$ = 2 ft. 8 in. |
| Width of guide passages | . | $c$ = 13·08 in. |
| Width of buckets at inflow | | $c_1$ = 13·285 in. |
| Number of guide-vanes | . | $z$ = 24. |
| Number of wheel-vanes | . | $z_1$ = 25. |
| Thickness of guide-vanes | . | $t$ = 0·23 in. generally. |
| Thickness of guide-vanes | . | $t$ = 0·04 in. at ends. |
| Measured outflow area from guide passages | . | $A_1$ = 9·880 sq. ft. |
| Measured outflow area from buckets . | . | $A_2{}^1$ = 9·558 sq. ft. |
| *Mean* angle of outflow from guides . | . | $a$ = 65°. |
| *Mean* angle of inflow into buckets . | . | $a_1$ = 0°. |

*Mean* angle of outflow from
  buckets .        .        .        $a_2 = 65°$.

With respect to the mean angle of outflow, it must be
remarked that at the outer diameter, where the outflow
is axial, $a_2$ is about 64°, while for that portion of the
wheel where the outflow is radial, $a_2$ is about 68°.  From
the *outer* to the *inner* diameter, the depth of the passages
between the buckets, or the shortest distance between
them at the outlet, *decreases*, so that for a given width
a much larger proportion of the water leaves the wheel
through the *outer* part of the bucket orifices than through
the *inner*.  Taking this unequal distribution into account,
the mean *effective* value of the angle $a_2$ must be nearer 64°
than 68°, and has been estimated at 65°.

To arrive at some conclusion as to the *mean effective
radius* at which the water may be assumed to leave the
turbine, it is also necessary to have regard to the dis-
tribution of the water, on the assumption that the relative
velocity of outflow $c_2$ is the same at all points.  From a
study, as careful as the available information and illus-
trations permitted, of the proportions of the wheel, the
author has estimated the effective ratio of the outer to

the mean inner radius, $\dfrac{r_1}{r_{2m}}$, as 1·3, whence the mean

inner diameter is 4·615 feet.

The co-efficient of contraction for the guide-passage
orifices, from which all the water issues radially, may be
taken as somewhat less than that applicable to the bucket
orifices, from which a portion of the water flows in a more
or less axial direction; the values of the two co-efficients
may be assumed as respectively 0·9 and 0·95, in which
case the ratio of the effective outflow areas $A$ and $A_2$ is
practically 1, since

$$A = 0.9 \times 9.88 = 8.992 \text{ sq. ft.}$$
$$A_2 = 0.95 \times 9.558 = 9.080 \text{ sq. ft.}$$

Calculated from the preceding value of $A$ and the quantity of water, $Q = 162.538$ cubic feet, consumed at the best speed—as given in the table of results—the velocity of flow

$$c = 18.075 \text{ feet per second,}$$

and $\qquad K_1 = 0.644.$

The theoretical value for $K_1$ determined from the angles and proportions previously stated with an allowance of $3.4 \%$ for friction is

$$K_1 = 0.638.$$

The agreement in this instance is excellent, especially considering the uncertainty as to the actual distribution of the outflowing water and the co-efficients of contraction.

In the "Swain" wheel the outer casing is continued downwards as far as the lowest part of the buckets, instead of being, as in the "Hercules" wheel, merely a narrow ring; on this account the effective outflow area $A_2$ has been taken as nearly equal to the measured area, since the water after entering the buckets in much the same way as in the "Hercules" turbine, and making an abrupt turn at the outer edge, has room to spread itself out again before leaving the buckets, and thus fill the latter.

The *experimental* best velocity was

$$w_1 = 21.708 \text{ ft. per second,}$$

that for inflow without shock

$$w_1 = 16.376 \text{ ft.,}$$

that for radial outflow

$$w_1 = 21.289 \text{ ft.}$$
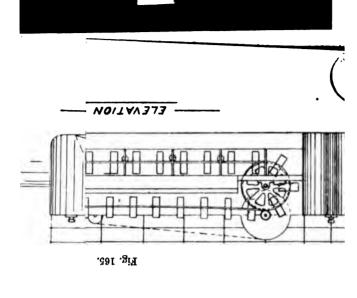
It will be seen that the last value agrees very closely with practice. It is obvious that the wheel was not designed for simultaneous inflow without shock and

vertical outflow.    The mean of the velocities on these two assumptions is 18·832 feet per second.

### THE PURDON-WALTERS STREAM MOTOR.

For utilizing the kinetic energy of a flowing stream, where no fall is available, floating water-wheels of the old-fashioned type are still used on many rivers with rapid currents.    Stream turbines are also occasionally employed. The latter are in all essential points similar to ordinary turbines, but must of course be arranged on horizontal shafts parallel with the general direction of the current.

Owing to the comparatively small amount of energy available in the form of velocity in a given stream-area, a turbine of very large diameter is necessary in order to develop a small amount of power, and obviously the diameter is limited by the depth of the stream.

In order to get over this difficulty a new form of hydraulic motor, belonging to the turbine class, the invention of Messrs. Purdon and Walters, has recently been brought into notice.    It may be best described as a turbine in which the vanes, instead of being attached to a wheel, are fixed between two endless pitch chains.    The two double chain-wheels, round which the chains pass, and which are driven by the latter, are supported by a pontoon or barge, through the lower part of which a channel is formed in which the vanes when under water move.    In this channel are fixed guide-vanes, which impart to the water flowing between them the desired direction.    The barge is moored across stream, so that the water flows through it at right angles to the motion of the chain. Half the moving vanes are immersed at any time, the axes of the chain-wheels being placed near the water-level. machinery to be driven is erected on the barge and

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ELEVATION

Fig. 165.



PLAN

PURDON-WALT

Fig. 163.

DIRECTION OF CURRENT

MOVEABLE FLAP

DECK FOR ELECTRIC G
OR OTHER MA

BRAKE
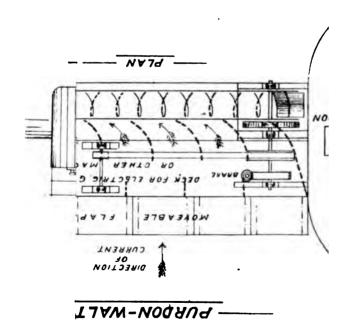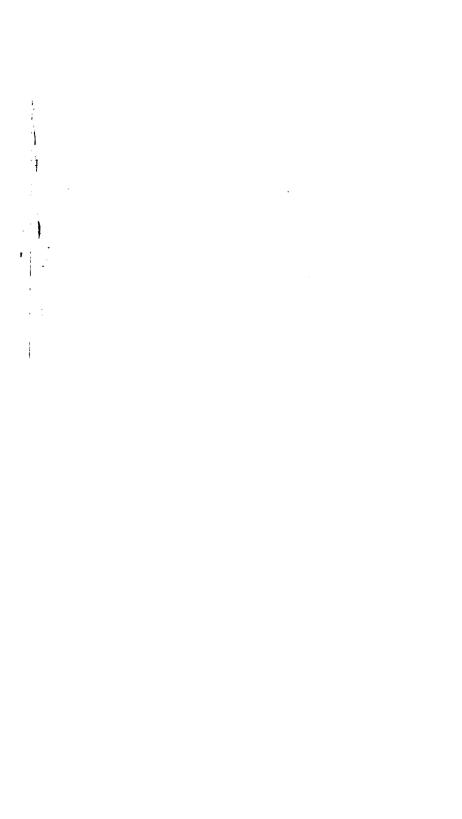
.ated through gearing from one of the chain-wheels,
, namely which is situated on the *concave* side of the
king vanes.

he chains are supported under water at regular distances
ertical rollers, and guided laterally by horizontal rollers,
ch take the pressure of the water on the vanes in the
of the stream. The construction will be easily under-
id from the accompanying illustrations (Figs. 163—166).
'he first motor designed by Messrs. Purdon and
lters was constructed with plane floats (instead of
ved vanes) and without guide-vanes, and the barge
placed so that the floats moved at an angle of 60° to
direction of the current. Trials carried out by Pro-
or Unwin, with an experimental motor thus constructed,
wed an actual efficiency—as measured by a brake—of
ut 11 per cent. with a current velocity of about 4·24 ft.
second. The theoretical hydraulic efficiency of such a
tor would, according to Professor Unwin, be 32 per
t., and it is certain that a large proportion of the loss,
resented by the difference between 32 and 11 per cent.,
due to mechanical friction.

it Professor Unwin's suggestion the motor was re-
igned and constructed in its present form. Preliminary
ls carried out at Chepstow by Professor Unwin are
l to have proved that the motor in its present shape
work with an efficiency of 25 per cent. at low velocities.
s is, for a stream motor, a very satisfactory result. It is
icipated, that by modifying in some particulars the
n of the vanes a still better efficiency can be obtained.
h a motor is of course not intended to compete with
inary turbines, but would be applicable in very many
es in which the employment of the latter would be out
he question. It is expected that the motor may prove
ful in tide-ways as well as in rivers having rapid currents,

and it is suggested that it could be advantageously employed in connection with the pontoons of floating bridges.

The leading idea in the design of the motor is to obtain a large sectional area of the stream entering the motor in as convenient a manner as possible.

For a stream motor there is of course no fall, and the available energy is only that due to the velocity of the current. It seems however probable that the machine, moored across the stream, would act to some extent as a dam, and cause a slight head to be formed on the up-stream side of the guide passages, so that in effect the motor would work under a fall. This fall would be influenced by the shape of the vanes and buckets them-selves, but in every case the available energy remains the same, and the maximum possible head could only be that equivalent to the velocity of the current.

The theory of the motor is essentially that of a turbine, but, in the absence of experimental data, the uncertainty as to the pressure conditions obtaining at the inflow and outflow render accurate calculations in advance impossible.

The following table gives the equivalent head and total energy per square foot of sectional area of a stream for velocities of from 1 to 6 miles per hour—

| Velocity of Stream. | | Equivalent Head. | | Total Energy. |
|---|---|---|---|---|
| Miles per Hour. | Feet per Second. | Feet and Decimals. | Inches and Decimals. | H. P. per Square Foot Sectional Area. |
| 1 | 1·467 | ·033 | ·43 | ·0055 |
| 2 | 2·933 | ·134 | 1·62 | ·0445 |
| 3 | 4·4 | ·300 | 3·60 | ·15 |
| 4 | 5·867 | ·534 | 6·42 | ·355 |
| 5 | 7·333 | ·834 | 10·07 | ·694 |
| 6 | 8·8 | 1·200 | 14·39 | 1·2 |

## *Fourneyron Turbine at St. Blaise.*

As an instance of an outward-flow turbine of exceptional proportions and for unusual conditions, one of the first wheels constructed by Fourneyron, and erected at St. Blaise in the Black Forest, deserves notice.

The turbine in question was driven by a fall of 354 feet at a speed of no less than 2300 revolutions a minute, and developed 30 effective horse-power. The outer diameter of the wheel was 12·99 inches, the inner diameter 7·47 inches.

Two larger turbines for the same fall were afterwards designed by Fourneyron, each with an outer diameter of 21·66 inches, to run at 2300 revolutions per minute and develop 60 horse-power. As might be expected at the enormous speed, the bearings had to be renewed every ten to fourteen days.

These early Fourneyron turbines discharged above water and were in reality impulse wheels. They were subsequently replaced by tangent wheels, and at a still later date by Girard turbines.

## *The Best Speed of Turbines as proved by Experiment.*

The Table A, in which the experimental best speeds of various turbines are compared with the results of calculation, shows that there is not in some of the cases analyzed a very close agreement between theory and practice but that in all of them the best speed as calculated does not deviate from the true best speed to a sufficient extent to cause any important difference in the efficiency—at the worst about 2 per cent.—it being

fortunately a peculiarity of turbines, that in the neigh-
bourhood of the most favourable velocity of rotation the
efficiency does not alter very rapidly with the speed.
The discrepancies between theory and practice are due
to several causes which have already been incidentally
mentioned, but call for further consideration in some
respects.

In the first place, the assumption that the best speed
is that at which the water enters with least resistance
and leaves with the smallest possible residual velocity, is
not strictly correct, since the velocity of flow varies with
the speed, and consequently the losses from friction,
bends, abrupt changes of section, &c., also vary and do
not attain their minimum value simultaneously with the
minimum values of the losses by shock and unutilized
energy. In the second place, many of the turbines in
question are not designed so that inflow without shock
and vertical outflow can take place simultaneously. In
the third place, the *assumed* conditions under which
inflow without shock occurs are not always those which
really correspond to inflow with least resistance. The
last of these causes tending to produce discrepancies
between theory and practice, is probably the most im-
portant. Professor Albert Fliegner of Zürich carried out
about 1879 a number of experiments[1] to ascertain to
what extent the direction of inflow *with least resistance*
deviates from that theoretically requisite for inflow with-
out shock. These experiments were made with stationary
guide passages and buckets so arranged that the direction
of the streams entering the buckets could be varied
relatively to the latter.

The streams issued from mouthpieces shaped like the

[1] *Zeitschrift des Vereins Deutscher Ingenieure*, 1879, vol. xxiii.,
p. 459.

| Thickness of guide-vanes at outflow. | (measured on brake). | | Theoretical velocity of flow due to head. | Number of revolutions per minute. | Remarks. | |
|---|---|---|---|---|---|---|
| $t$ | | | $V$ | $n$ | | |
| Feet. | ser. | | | | | |
| | ·297 | 0·69 | 20·074 | 99 | | 1 |
| | ·881 | 0·61 | 20·431 | 135 | | 2 |
| | ·232 | 0·66 | 20·190 | 151 | | 3 |
| | ·396 | 0·79 | 24·789 | 164 | | 4 |
| | ·910 | 0·69 | 24·692 | 159 | | 5 |
| | ·510 | 0·72 | 34·122 | 308 | | 6 |
| | ·804 | 0·61 | 33·994 | 302 | | 7 |
| | ·067 | 0·61 | 33·946 | 297 | | 8 |
| ·0208 (24) | ·987 | 0·67 | 18·168 | 33 | | 9 |
| ·0312 (8) | ·748 | 0·63 | 19·493 | 39 | | 9A |
| ,, | ·46 | 0·8 | 29·569 | 45·52 | | 10 |
| ·0491 | ·62 | 0·76 | 25·367 | 28·87 | | 11 |
| ,, | | | | | | 11A |
| ,, | | | | | | 11B |
| | ·03 | 0·88 | 32·653 | 89·17 | | 12 |
| | ·70 | 0·83 | 32·653 | 96·00 | Draft tubes. | 12A |
| | ·62 | 0·82 | 32·669 | 90·25 | Draft tube with inverted cone. | 12B |
| ·00833 | ·507 | 0·76 | 28·809 | 51·064 | | 13 |
| | ·04 | 0·85 | 32·702 | 63·50 | | 14 |
| ·0156 | ·623 | 0·59 | 29·311 | 38·182 | | 15 |
| ·025 | ·72 | 0·89 | 33·047 | 140·62 | At full gate. | 16 |
| | ·28 | 0·8 | 33·006 | | At 0·806 gate. | 16A |
| ·0033* ·0192 | ·278 | 0·88 | 28·224 | 69·096 | | 17 |
| ·0125 | ·863 | 0·8 | 28·344 | 47·424 | | 18 |

vertical out*ppcr* is fc

.

guide passages of turbines, and entered channels made
to resemble as nearly as possible the buckets of turbines·
The experiments were made with nine different forms of
channel, and for each of these ten mouthpieces were tried,
each inclined at various angles to the channel, which may
for convenience be termed the *turbine* bucket.

In carrying out the experiments the pressure $\xi$ imme-
diately above the orifice of the mouthpiece was measured
by a pressure gauge and the quantity of water flowing out
observed; the area of the orifice being known, the velocity
of flow $c_r$ could be calculated, and denoting by $h_o$ the
height of the orifice above the tail-water level, or the
depth of immersion (as the case might be), the available
head $h$ of the water leaving the mouthpiece would be
expressed by

$$h = \frac{c_r^2}{2g} + \xi + h_o.$$

The whole of the energy represented by this was ex-
pended in producing the velocity of outflow $c_2$ from the
turbine bucket and in overcoming the resistances of the
latter, including shock, so that

$$\frac{c_r^2}{2g} + \xi + h_o = (1 + \zeta)\frac{c_2^2}{2g},$$

where $\zeta$ is an experimental co-efficient of resistance or loss.

Since $c_2$ could be determined from the observed quantity
of water and the known outflow area of the bucket, the
only unknown quantity in the above equation was $\zeta$, and
this could be calculated in every case. Over 1000
experiments were made, from the results of which the
values of $\zeta$ were calculated. Each different bucket was
tried with several heads of water ranging from 3·28 to
98·4 feet, and the values of $\zeta$ plotted as ordinates with the
heads as abscissæ.

Professor Fliegner found that the co-efficient $\zeta$ diminished as the head $h$ increased, and that for any given head its minimum value did not occur for inflow without shock, but when the mouthpiece was placed at an angle $a_0$ differing from the vane angle $a_1$ of the turbine bucket.

In the following table the corresponding values of $a_2$, $a_1$, and $a_0$ as found by Fliegner are given :—

|      | $a_2$ | $a_1$ | $a_0$ |
|------|-------|-------|-------|
| I.   | 75°   | 30°   | 21°   |
| II.  | 60°   | 30°   | 19°   |
| III. | 75°   | 0°    | 15°   |
| IV.  | 60°   | 0°    | 12°   |
| V.   | 75°   | −30°  | $8\frac{1}{2}$° |
| VI.  | 75°   | −30°  | − 5°  |
| VII. | 75°   | −30°  | −10°  |
| VIII.| 75°   | −60°  | − 4°  |
| IX.  | 65°   | −45°  | −17°  |

In the case of buckets Nos. VI. and VII. the width was greater at the outlet than at the inlet, the form being otherwise the same as that of No. V.

In their *general tendency* these results agree with those recorded in Table A (which relate to actual turbines). From that table it will be seen that for nearly all the turbines with a *positive* angle $a_1$ the best velocity of rotation is *less* than that corresponding to inflow without shock, and consequently the best relative angle of inflow $a_0$ *smaller* than the vane angle $a_1$. For turbines with a negative value of $a_1$ the best speed is *greater* than indicated by theory, and consequently the angle $a_0$—in absolute value apart from its sign, positive or negative— under these circumstances also *less* than $a_1$.

For Jonval wheels of ordinary proportions, as constructed in Europe, with $a_1 = 0°$ and $a_2$ from 64° to 75°,

theoretical best speed agrees very well with
nent for practical purposes. For radial and mixe
wheels the deviations are somewhat more consid
, but this is *in part* due to the fact that they have not
. designed accurately for simultaneous inflow without
k and radial or vertical outflow.

further explanation of disagreements between calcu-
l and practical results is in some cases due to the
rtainty as to the real value of the outflow areas $A$
$A_2$ used in calculating the velocities of flow; this
rtainty arises from ignorance of the true allowance to
made for contraction and of that required for the
ruction of the guide passages by the wheel-vanes, and
sionally also is aggravated by causes already explained
nalyzing the performance of particular wheels, in
h there is a considerable space between the guide-
s and buckets, so that the absolute angle of outflow
m the former is indefinite. In radial wheels, too, as
dly pointed out, there may be some doubt as to the
er way of estimating the measured area of outflow $A_1$.
iegner's investigations show that the loss by shock is
so great as it should be according to Carnot's theory,
h was explained in connection with the losses in
tion turbines, and this fact causes a turbine to be in
tice less sensitive to deviations from the best speed
theory indicates.

or turbines which have *not* been designed for simul-
ous inflow without shock and vertical or radial outflow,
*mean* of the speeds corresponding to these two assump-
s is generally a sufficiently good approximation to the
speed for practical purposes. It is not difficult to see
the method of calculation might be modified to agree
the data furnished by Fliegner's experiments. If,
nstance, the relative vane angle of inflow were $a_1 = 0°$

and the relative vane angle of outflow $a_2 = 60°$; instead of introducing the velocity of rotation corresponding to inflow without shock for $a_1 = 0°$, the relative angle of inflow should be taken as $a_0 = 12°$, for which the resistance is a minimum, and the speed determined accordingly; $a_0$ simply takes the place of $a_1$ in the calculations. More experiments are required before this method can be generally adopted, as the data are insufficient; as far as they go Fliegner finds that the results may be expressed by the empirical formula,

$$a_0 = \frac{a_1 + a_2}{5} - 30 \frac{c_2 - c_1}{c_1}.$$

For most of the radial or mixed-flow turbines referred to in Table A, the best speed agrees more closely with that for radial or vertical outflow than with that for inflow without shock, even when allowance is made for the difference between $a_1$ and $a_0$. This must occur when the increased loss by unutilized energy due to a given deviation from the best speed in one sense is greater than the loss from shock resulting from an equal deviation in the opposite sense. To take an extreme example, suppose the velocity of rotation for vertical outflow to be 20 feet per second, while that corresponding to inflow with least resistance is 16 feet. If over 16 and up to 20 feet the loss from shock is inappreciable, then it is clear the best velocity will be nearer 20 than 16.

In the following short table, for more convenient reference, the corresponding experimental and calculated velocities of rotation are given for all the turbines mentioned in Table A, with the exception of Rittinger's. The calculated velocity is in every case except one the mean of the velocities for inflow without shock and vertical or radial outflow.

With impulse turbines fewer experiments extending over a considerable range of speed have been made than with reaction wheels, but there is no doubt that practice and theory are in substantial agreement in their case, which is in some respects simpler than that of reaction turbines, there being less uncertainty as to the true values of the dimensions and quantities on which the velocities of flow and rotation depend.

| Description of turbine. | Experimental velocity. | Calculated velocity. |
|---|---|---|
| Haenel's Turbine. Parallel-flow, buckets widened towards outflow . . . . . | 10·515 | 9·906 |
| Olching Turbine. Parallel-flow . . . . | 16·863 | 17·422 |
| "Collins" Turbine. Parallel-flow . . . | 20·960 | 22·337 |
| Tremont Turbine. Radial outward-flow . | 18·051 | 18·102 |
| "Boyden" Turbine. Radial outward-flow | 20·392 | 20·967 |
| Boott Turbine. Radial inward-flow . . | 18·608 | 22·102 |
| "Hercules" Turbine. Mixed inward-flow | 22·088 | 21·637 |
| "Swain" Turbine. Mixed inward-flow . | 21·708 | 18·832 |
| "Humphrey" Turbine. Mixed inward-flow . . . . . . . . . . . . | 20·113 | *17·533 |

* Velocity for inflow without shock.

# CHAPTER XII.

## AMERICAN TURBINES.

General remarks.—Distinctive features of American practice.—Holyoke tests: probable accuracy, agreement of Francis's and Braschmann's formulas.—Comparative analysis of mixed-flow and axial turbines : balance of advantages.—Desirability of Comparative tests by same methods and under similar conditions.—"Humphrey" and "Hercules" turbines : free deviation not possible ; evidence of reaction.—Explanation of performance from construction.—Dimensions of American turbines.--Peculiarities in angles and form of buckets, &c.— Diameters.— Number of vanes.— Comparison (numerical) of a mixed-flow with a parallel-flow turbine.—Object of investigation.—Dimensions assumed.—Velocity of flow.—Velocities of rotation.—Residual velocities at various points.—Loss from unutilized energy.—Mean residual velocity.—Conclusions.—Result of modified assumptions regarding outflow.—Estimation of efficiency of mixed-flow wheels.—Calculations for "Hercules" turbine and comparison with result of trials.—Calculation for Jonval turbine. —Explanation of superior efficiency of "Hercules" wheel at part gate.—Probable values of co-efficients of resistance.

AMERICAN turbines have already been incidentally referred to in the course of this work, and as in general their construction differs considerably from that usually adopted in Europe, and energetic measures are taken to introduce them on this side of the Atlantic by claiming in many cases extraordinary efficiency, some more detailed consideration of their peculiarities, in so far as information on the subject is available, is desirable.

American turbines are with few exceptions of the mixed-flow type, the direction of flow being generally inward and downward. Formerly outward-flow Fourneyron turbines appear to have been made, and many are still in use in the United States. Francis introduced the radial inward-flow wheel (invented, it is believed, by Professor J. Thomson), and this by an easy transition led to the mixed inward- and downward-flow turbine, of which the Leffel turbine seems to have been one of the first used in America.

As compared with European practice the following are the leading characteristics of American turbines: smaller diameter and greater number of revolutions for a given fall and power, greater relative width of buckets both at the inflow and outflow, smaller number of guide- and wheel-vanes, and regulation by means of pivoted guide-vanes or circular sluice between guide passages and wheel.

The majority, if not all American turbines, belong to the *reaction* system, and work drowned. There appears to be an idea on the part of some American makers of allowing the water to follow its own course, especially in *leaving* the wheel, thus permitting free deviation under water; as will be subsequently shown, however, in the case of two wheels in which this is supposed to occur, no such thing as free deviation in the ordinary sense of the term takes place, the phenomena observed being those incidental to reaction turbines.

In some American wheels the water leaves, not only in a downward (axial), but also partly in a radial outward direction, some of it being deflected through an angle approaching 180° in a path at right angles to that in which the driving force has to be exerted. In other wheels, again, the deflection of the fluid on its passage through the buckets is very abrupt, and it would seem

as if the designers of such motors had aimed at giving th
water the most tortuous course possible, with what objec
it is difficult to imagine.

Probably more trials of turbines have been made ii
America than in all the rest of the world put togethei
and of these a large number have been carried out a
the celebrated testing "flume" at Holyoke, described ii
another place (p. 391).

There can be little doubt of the relative accuracy of th
Holyoke tests, which bear all the evidence of carefu
observation, and—it may be noted in passing—it woul
not be in the interests of the Holyoke Water-power Com
pany to credit the motors of their customers, who pay fo
the water consumed by their turbines, with a greate
efficiency than that actually obtained.

It is, however, noticeable that the recorded efficiencie:
of turbines tested in America are all remarkably high a:
compared with the results obtained in Europe, and thi:
has not unnaturally given rise to the surmise that the
method of computing the quantity of water consumed i:
answerable to some extent for this, and that the co
efficients employed in the calculations, necessary when &
weir is used for measurement, are too small. The formula
applied at Holyoke is that of Francis, and the result:
given by it agree very well with those obtained with, fo
instance, Braschmann's formula, which is very generall}
used in Germany, so that there appears to be little pro-
bability of errors arising from this cause. On the othei
hand, it should be noted that in the best reliable trials
made in Europe with Jonval turbines, the quantity of
water consumed was measured by current-meters and not
over weirs, so that the details are not directly comparable
with those of American experiments.

American practice in the construction of turbines has

been rather savagely attacked by a late well-known continental writer on the subject, and it will be well to compare, carefully and dispassionately, the merits and faults of a wheel of the American type with those of the Jonval turbine generally accepted in Europe.

(1) In an inward- and downward-flow wheel the inflow of the water takes place without shock *over the whole width of the buckets,* while in a Jonval turbine this can only happen *at one diameter,* and consequently there is a loss from shock in the latter which is absent in the American type of motor.

(2) Owing to the relatively great width, in proportion to the diameter, of the buckets of American mixed-flow turbines at the outflow, *the deviations from the best speed—* the speed corresponding to vertical outflow—are greater than in a Jonval wheel of the usual proportions, and it has been inferred from this that the loss from unutilized energy must therefore also be greater; this, however, it will be shown, is not necessarily the case, the disadvantages arising from this cause being compensated by the superior efficiency due to inward-flow turbines, as previously explained in comparing the different types of wheel (p. 192).

Under similar conditions, for very narrow buckets the residual velocity of the water from an inward- and downward-flow wheel is less than from a parallel-flow wheel. As the width (of the outflow) is increased the average residual velocity becomes greater, but there is in many cases a sufficient margin between the values for the mixed-flow and for the parallel-flow turbines to compensate for a very considerable excess of width of the former over the latter at the outflow. This will be illustrated subsequently by a numerical example.

(3) It has been objected to the mixed-flow turbine that

H H

the water on entering the guides and passing through the buckets is twice uselessly deflected, whereby a certain loss is incurred. This is no doubt true, but there is reason to believe that this loss is unimportant in a well-designed wheel, although that portion of it due to downward deflection in the buckets themselves may be very appreciable where the curvature is abrupt, as in some American designs.

(4) As the course of the water while acting upon the vanes is partly radial, the downward pressure on the bearings is less in mixed- than in parallel-flow wheels, and is still further reduced by the lighter weight of the former, arising from their smaller diameter as generally made.

(5) For a given area-ratio $\dfrac{A}{A_1}$ the velocity of flow both in guide passages and buckets is lower in a radial or mixed-flow turbine than in an axial turbine, consequently the loss from hydraulic friction, &c., is also less, assuming the same experimental co-efficients to be applicable in both cases. This advantage may, however, be partly neutralized by the additional resistance due to sharp bends in a mixed-flow wheel.

This point will be more fully dealt with in the sequel by means of a numerical example.

Provided, therefore, it can be proved that the loss from residual velocity under given conditions is no greater in mixed-flow turbines, as proportioned in America, than in Jonval turbines, it is evident that the former have distinct theoretical advantages over the latter—first, in the absence of shock; secondly, in the diminution of friction due to less downward pressure.

In addition to these are the practical advantages of smaller diameter and consequent reduced cost.

It has been already noticed that the number of vanes in American wheels is generally less than in European turbines of the same diameter; provided the streams of water are sufficiently guided, this is an advantage, as it tends to reduce the friction. Whether the guidance afforded by the vanes is sufficient or not, depends mainly on the proportion between the length and depth of the passages; the passage in mixed-flow wheels is necessarily longer than in Jonval turbines, and hence the depth and pitch may safely be greater.

As the diameter of the shaft and bearings is inversely proportional to the cube root (according to some rules, the fourth root) of the speed for a given power, the work absorbed by friction with a certain pressure on the bearings is relatively greater as the diameter of the wheel is reduced. Thus the advantage due to the smaller pressure on the bearings of mixed-flow turbines is partially neutralized.

It will be seen on reference to the results of Mr. Bernhard-Lehmann's experiments on the shaft friction of various types of turbine, that while for inward-flow wheels this friction ranges from 0·9 to 1·7 per cent. only, for axial wheels the range is from 1·4 to 3·4 per cent. at full gate, and may be as high as 5·4 per cent. at half gate. At full gate therefore shaft friction alone may easily account for 2 per cent. efficiency.

Taking 83 per cent. efficiency as the best record of a Jonval turbine—somewhat higher efficiencies than this have been occasionally observed—an inward-flow wheel of *the same hydraulic efficiency* might thus show an actual efficiency of 85 per cent. From the causes already enumerated, however, the hydraulic efficiency may also be greater, so that an efficiency of 87 per cent. is by no means impossible.

Making every allowance for the merits of American hydraulic motors, the average results obtained at Holyoke still appear somewhat high as contrasted with trials made in Europe, and it would be satisfactory if some of the wheels reported to have given such admirable results at Holyoke could be compared under precisely the same conditions with Jonval turbines of the best European construction.

Apart from this, there is no doubt that *at least* as high efficiencies can be obtained with inward- and downward-flow wheels as with those having parallel-flow, while at the same time securing the practical advantages of greater lightness and lower cost.

The vanes of American wheels are generally thinner than those of European wheels, and at the ends are tapered almost to a knife-edge; frequently, also, they are constructed of brass or gun-metal.

Among the accounts of experiments with turbines will be found the results of trials with a "Humphrey" and a "Hercules" turbine, both of American design. In each of these there is reason to believe the water is intended, on leaving the wheel, to choose to some extent its own course, that is to say, to act with free deviation. On referring to the experimental value of $K_1$, it will be found that this is, for the "Humphrey" turbine, 0·621, and for the "Hercules" turbine, 0·625. These figures point to one of two conclusions: either the wheels work with so-called "reaction," or the efficiency must be exceedingly low, as the velocity of flow $c$ could only be reduced, in a wheel with free deviation, to the stated proportion of the theoretical velocity by an excessive amount of useless resistance.

The results of the trials show that the latter is not the case, since the efficiencies in both instances are very

good ; it is consequently clear that the motors are *reaction* turbines.

In the "Humphrey" wheel the vanes project below the revolving wheel-casing, and on the outside are practically unconfined, being connected merely by a narrow ring.

The stationary outer casing, however, entirely encloses the buckets, but there is a considerable space between the outer edges of the vanes and the interior surface of the stationary casing. It might naturally be supposed that this free space would interfere with the proper action of the water, which would escape outwards without doing work. The experiments prove that this does not happen to any great extent, and the explanation is, probably, that *the water itself*, between the outside of the wheel and the inside of the casing, acts as a wall, nearly stationary eddies being formed, which resist the outward pressure of the fluid in the wheel.

If free deviation really took place, the value of the co-efficient $K_1 = 0.621$ would indicate that the water on leaving the guide passages had already *lost* about **69** *per cent.* of its energy; with reaction the greater portion of this is still available as *pressure* to be converted into velocity during the passage of the fluid through the wheel-buckets.

## *Dimensions of American Turbines.*

The angles of the vanes of American wheels have much the same range as in European practice; in many cases there is a tendency to make the angle $a_2$ as great as possible, with a view to reducing the residual velocity of the water leaving the buckets, and in some instances

this seems to be carried so far, that the relative direction of the outflowing water is almost at right angles to the axis of the wheel. This, of course, is a mistake, as the absolute motion left in the fluid is not sufficient to carry it off, and a certain amount of pressure is then necessary for the purpose, which retards the flow and proportionately reduces the efficiency.

In many mixed-flow wheels the outflow orifices of the buckets do not lie in a plane, but each is of a semi-annular or crescent form, the edges of the vanes being curved. The vane-angle $a_2$ of outflow at any point must under such circumstances be measured in a plane defined by the tangent to the circumference through that point and the tangent to the vane-curve; the angle $a_2$ is then the angle formed by the latter, with a direction at right angles to the former.

The outer radius of American inward- and downward-flow turbines ranges from

$$r_1 = 0{\cdot}70 \, \sqrt{A} \text{ to } 1{\cdot}11 \, \sqrt{A};$$

from which it will be seen that the maximum in American design is about the same as the minimum in European practice for the *mean* radius of Jonval wheels—the outer radius being of course still greater, and at least $1{\cdot}25 \, \sqrt{A}$. For a given area of the passages, their width in American wheels exceeds that in Jonval turbines in proportion as the diameter is less.

As regards the number of guide-vanes and buckets the practice varies very considerably, but generally speaking these are fewer in number in American than in European turbines; the tendency in the former seems to be, to leave the water more at liberty to follow its own course, with the intention of thereby reducing the friction. This probably accounts for the loss from sudden

bends in the passages not being so great as might be anticipated ; in the sharp corners stationary eddies are formed, over which the flowing water passes with less resistance and loss than if it were compelled to follow exactly the curve of the vanes.

### Comparison of a Mixed-flow (inward and downward) with a Parallel-flow Turbine.

The object of this investigation is to show in how far —if at all—the disadvantages arising from great relative width of the buckets in an inward- and downward-flow turbine may be compensated by the more effective manner in which it utilizes the energy of the water as compared with a parallel-flow turbine. In the first instance the comparison will be confined entirely to the losses by unutilized energy.

As an example, a somewhat extreme case will be assumed, in which the mixed-flow turbine has the following proportions :—

Ratio of radius where water enters to greatest outflow radius . . . . $\dfrac{r_1}{r_2} = \dfrac{6}{5}$

Ratio of radius where water enters to mean outflow radius . . . . $\dfrac{r_1}{r_2{}^m} = \dfrac{12}{7}$.

Ratio of radius where water enters to least outflow radius . . . . . $\dfrac{r_1}{r_2{}^1} = \dfrac{3}{1}$

Ratio of width at outflow to mean outflow radius . . . . . . $\dfrac{c_2{}^m}{r_2{}^m} = \dfrac{6}{7}$.

The angle $a_2$ is (in the first instance) taken as constant for the whole width,

$$a_2 = 68°;$$

in consequence of this the distribution of the water is not uniform for the whole width, but the quantity flowing through a given portion is proportional to the diameter.

The relative loss from unutilized energy at any point —as elsewhere shown—is not represented simply by the square of the residual velocity, but by the product of that square and the corresponding proportional diameter. The quantity of water passing through at the mean radius $r_2^m$ may be taken as unity, and from the curve representing the losses at all points the *average loss* determined.

The angle $a$ is assumed

$$a = 69°;$$

the ratio

$$\frac{A}{A_2} = 0.75.$$

The Jonval turbine with which the mixed-flow wheel is to be compared has the same angles $a$ and $a_2$ and the same ratio $\dfrac{A}{A_2}$.

For calculating the velocity of flow, the efficiency of the turbines may be taken as equal, and then from the well-known formula

$$c = K_1 \sqrt{2gh} = \sqrt{2 \frac{r_1}{r_2} \frac{A}{A_2} \sin a \sin a_2}.$$

$$\frac{c_p}{c^m} = \frac{\text{Velocity of flow of parallel-flow wheel}}{\text{Velocity of flow of mixed-flow wheel}} =$$

$$\sqrt{\frac{2 \dfrac{r_1}{r_2^m} \dfrac{A}{A_2} \sin a \sin a_2}{2 \dfrac{A}{A_2} \sin a \sin a_2}};$$

or since $a$, $a_2$ and $\dfrac{A}{A_2}$ are the same for both turbines,

$$\frac{c_p}{c_m} = \sqrt{\frac{r_1}{r_2^m}} = \sqrt{1 \cdot 714} = 1 \cdot 316.$$

For a distribution of the water *proportional to the diameters*, the correct *average* value for $\dfrac{r_1}{r_2}$ to be substituted in the formula is actually the ratio of the outer radius $r^1$ to the mean outflow radius $r_2^m$, as above. For an *even distribution* of the water, however, such as occurs with a helical vane surface, the average value of $\dfrac{r_1}{r_2}$ would not be that given, but the mean of all the values of $\dfrac{r_1}{r_2}$ between the greatest and least outflow radius; this can be found by constructing a curve representing successive values of $\dfrac{r_1}{r_2}$ and determining the mean ordinate; the average ratio in that case is $1 \cdot 835$.

As only proportions are in question, it may for convenience be assumed that the velocity of flow for the mixed-flow wheel

$$c_m = 10;$$

then the relative velocity of outflow from the buckets

$$c_{2m} = \frac{A}{A_2} c_m = 0 \cdot 75 \times 10 = 7 \cdot 5.$$

The velocity of rotation $w_1$, relative angle of inflow $a_1$, and residual velocities, at the various diameters, can now be determined by graphical construction.

In Fig. 167, let $O\,D = r_1$, $O\,A = r_2^m$, $O\,B = r_2^1$ and $O\,C = r_3$, to any scale, so that the proportions be those given. Let each of the angles $O\,A\,a$, $O\,B\,b$ and $O\,C\,c = a_2 = 68°$.

According to the usual principles of design it may be assumed, to begin with, that the absolute direction of outflow at the mean radius $r_2{}'''$ is perpendicular to the direction of rotation, or, for exclusively downward outflow, parallel with the axis.

$A\,a$, $B\,b$ and $C\,c$ are each equal to $c_2 = 7\cdot5$, and $E\,a$, perpendicular to $A\,E$ the momentary direction of rotation of $A$, then represents the absolute velocity of outflow $u_m$ at the mean radius $r_2{}'''$, while $A\,E$ is the corresponding velocity of rotation. $O\,E$ prolonged intersects $B\,F$, $C\,G$ and $D\,H$ in $F$, $G$ and $H$ ($B\,F$, $C\,G$ and $D\,H$ being at right angles to $O\,D$), and $B\,F$, $C\,G$ and $D\,H$ are the velocities of rotation at the radii $r_2{}^1$, $r_2$ and $r_1$. $F\,b$ and $C\,G$ are the absolute residual velocities $u^1$ and $u$ at $B$ and $C$.

By measurement it will be found that

$$E\,a = u_m = 2\cdot875.$$
$$F\,b = u^1 = 4\cdot125.$$
$$G\,c = u = 4\cdot125.$$

The square of each of these values is proportional to the loss *per unit of weight* from unutilized energy, but, to take into account the distribution, must be multiplied with the corresponding ratio of the radius at the point in question to the mean radius $r_2{}'''$, thus :—

Loss from unutilized energy at $A$

proportional to $\qquad u_m{}^2 \times \dfrac{r_2{}'''}{r_2{}'''} = \left(2\cdot875\right)^2 \times\ 1.$

Loss from unutilized energy at $B$

proportional to $\qquad u^{12} \times \dfrac{r_2{}^1}{r_2{}'''} = \left(4\cdot125\right)^2 \times\ 0\cdot57.$

Loss from unutilized energy at $C$

proportional to $\qquad u^2 \times \dfrac{r_2}{r_2{}'''} = \left(4\cdot125\right)^2 \times\ 1\cdot43.$

These values can easily be ascertained for any desired number of points, and the curve $XY$ constructed. The average value of the proportional loss, as determined from the mean ordinate of the curve, is
$$U^2 = 11 \text{ (nearly)},$$
whence the mean residual velocity
$$U = \sqrt{11} = 3 \cdot 32.$$

The relative velocity and direction of inflow is found as follows—

The angle $IDZ$ is made equal to $a = 59°$, and $DI = c^m = 10$; $DH$ is the velocity of rotation $w_t$ at the outer circumference, then $HI = c_1$ the relative velocity of inflow, and angle $HIK = a_1 = 37\frac{1}{2}°$.

With regard to the parallel-flow turbine, it is assumed that the proportion of the width of the buckets to the mean diameter is so small that practically the variations in the absolute residual velocity $u$ at different points are negligible, and the latter may be taken as constant for the whole width. This assumption is extremely favourable to the parallel-flow wheel.

The ratio of the velocity of flow $c_p$ in the parallel-flow turbine to that $c_m$ in the mixed-flow turbine was found to be $1 \cdot 316$; hence $c_p = 10 \times 1 \cdot 316 = 13 \cdot 16$, and the relative velocity of outflow $c_{2p} = 0 \cdot 75 \times 13 \cdot 16 = 9 \cdot 87$.

By construction the residual velocity $u$ is found to be—
$$u = 3 \cdot 75,$$
whence $\qquad u^2 = 14 \cdot 06.$

Notwithstanding, therefore, the relatively great width of the buckets of the mixed-flow wheel, it appears that the loss by unutilized energy is *less* than in the Jonval turbine, although the assumptions in this respect were all in favour of the latter. For a parallel-flow wheel of the given proportions the energy represented by the velocity

of flow $c$ is about *one-half* of that due to the head—$K$
having a value of about 0·7; hence the total available
energy is proportional to $2 \times (13·16)^2 = 346·4$, and the ratio
to this of the loss by unutilized energy is for the parallel-
flow turbine—

$$\frac{14}{346·4} = 0·04, \text{ or } 4 \text{ per cent.,}$$

for the mixed-flow turbine—

$$\frac{11}{346·4} = 0·032, \text{ or } 3·2 \text{ per cent.;}$$

there being consequently a difference in favour of the latter
of 0·8 per cent.

In addition to this trifling advantage, and apart from
those before-mentioned, the mixed-flow turbine gains in
efficiency from another source, namely, that owing to the
velocity of flow being less in it than in the parallel-flow
turbine working with the same head, the various losses by
hydraulic friction and other resistances are reduced propor-
tionally to the square of the velocity of flow.

Taking the same co-efficients as hitherto,
for *the parallel-flow turbine*—
    the loss in the guide passages

$$= \frac{1}{2g} 0·125 (13·16)^2 = \frac{21·6}{2g}$$

the loss in the wheel-buckets

$$= \frac{1}{2g} 0·2 (9·87)^2 = \frac{19·5}{2g}$$

$$\text{Total} \quad . \quad . \quad \frac{41·1}{2g}$$

for *the mixed-flow turbine*—
    the loss in the guide passages

$$= \frac{1}{2g} 0·125 (10)^2 = \frac{12·5}{2g}$$

the loss in the wheel-buckets .

$$= \frac{1}{2g} \; 0 \cdot 2 \; (7 \cdot 5)^2 = \frac{11 \cdot 25}{2g} \; \bullet$$

$$\text{Total} \quad . \quad . \quad \frac{23 \cdot 75}{2g}$$

The difference in favour of the mixed-flow wheel is therefore—
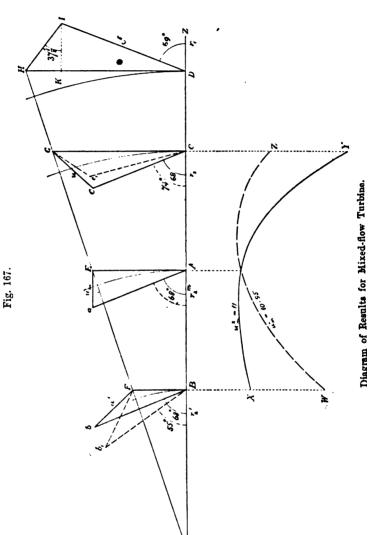
$$\frac{41 \cdot 1 - 23 \cdot 75}{2g} = \frac{17 \cdot 35}{2g} \; ,$$

and this is about **5** *per cent.* of the total available energy.

From this it will be seen that there is theoretically a balance of 5·8 per cent. to the credit of the mixed-flow turbine, apart altogether from any advantages arising from absence of shock and reduced friction of the bearings; assuming that from the latter causes there is a further gain of 1 per cent.—a very moderate estimate—the total advantage would be **6·8** *per cent.* Supposing half of this to be neutralized by the increased loss due to the double deflection of the water previously referred to, there is still a nett gain of **3·4** *per cent.* as compared with the Jonval wheel.

If, instead of a *constant value* of the relative angle of outflow $a_2$, the vane surface at the lower end be *helical*, so that the distribution of the water is *nearly uniform*, the loss by unutilized energy remains almost unchanged, supposing the velocity of flow to be the same. The construction and curve of unutilized energy in this case are shown in the diagram Fig. 167 by the broken lines. The average loss is 10·95, corresponding to a velocity $u = 3·309$.

As a matter of fact, for even distribution of the water, the velocity of flow must be rather less than with a

Fig. 167.

Diagram of Results for Mixed-flow Turbine.

constant angle $a_2$, since the mean ratio $\dfrac{r_1}{r_2}$ to be used in calculating $K_1$ is greater under the former conditions in the proportion of 1·835 to 1·714; with *even distribution*, therefore, the advantage of the mixed-flow over the parallel-flow turbine is even greater than that calculated, and the assumption of equal velocities of flow in both cases errs on the side of caution.

Experiments show a superiority of the best American mixed-flow wheels over the best Jonval turbines of about 4 per cent. (in one case 2 per cent. only), so that the estimate above made agrees very well with facts.

Under otherwise similar conditions, *the greater the ratio of the outer to the average inner diameter, the higher is the efficiency of a mixed-flow turbine.*

The theoretical superiority of the inward-flow wheel was pointed out in comparing the different systems of reaction turbines, but the investigation just completed was necessary to prove that this superiority is not neutralized by the great relative width of the buckets which characterizes so many mixed-flow wheels.

If the mixed-flow turbine be so designed, that instead of the water leaving vertically at the *mean* radius it does so at the *outer* radius $r_2$, it will be found that, with the assumed proportions, the effect on the loss by unutilized energy is comparatively slight, it being about 13 instead of 11, and the speed of the turbine is considerably reduced, while of course the angle $a_1$ must be modified to insure inflow without shock. A reduction of the speed with a very slight reduction in the efficiency is in itself a very desirable result, but this is accompanied by an increase of the velocity of flow which necessitates a smaller area (less width with the same diameter) of the passages and buckets. The increase in the velocity of flow is explained as follows—

The usual formula for determining the velocity is onl[y] strictly accurate on the assumption that the water leave[s] the turbine at right angles to the direction of rotatio[n] at every point. This condition is only fulfilled for *on[e]* diameter, but when that is the *mean* diameter, the resul[t] is practically correct, as the deviations on either sid[e] nearly balance each other: when, on the other hand outflow at right angles to the direction of rotation occur[s] at the outer diameter, it is clear that the deviations fron[m] the assumed conditions being all in one sense must resul[t] in the average velocity of flow being that due to a lowe[r] speed than the speed of the outer circumference. I[n] inward-flow turbines the velocity of flow *increases* as th[e] speed *decreases,* hence it will be greater for the wheel i[n] question than in one designed for vertical outflow at th[e] *mean* diameter.

Increased velocity of flow is accompanied by increase[d] losses by hydraulic friction, &c., and results in a furthe[r] reduction of the efficiency. The best effect is obtained therefore, by arranging the wheel for vertical outflow a[t] the mean diameter, but as the velocity of flow rises onl[y] gradually with diminished speed, a very considerabl[e] departure from this principle may be permitted withou[t] very serious detriment to the performance of the wheel.

This peculiarity is only possessed to a very limite[d] extent by parallel-flow turbines, and not at all by thos[e] with purely inward-flow.

It having been proved that in an inward- and down ward-flow turbine with a—relatively to the diameter— very great width of buckets at the outflow, the loss b[y] unutilized energy is not greater, and may even be les[s] than in a Jonval turbine of good proportions, the efficienc[y] of the former may be estimated by the usual formula fo[r] any given angles $a$ and $a_2$ and ratio $\dfrac{A}{A_2}$.

It will be evident from the preceding investigation that the average loss by unutilized energy with the assumed proportions is to the minimum loss—where the outflow is vertical (or at right angles to the direction of rotation) —as 4 to 3, or in the ratio $1\frac{1}{3}$ : 1. In calculating the efficiency by the well-known formula, that portion of it representing the loss by residual velocity, $\left(\dfrac{A}{A_2}\right)^2 \cos^2 a_2$, must be multiplied by a co-efficient, which in the above case would be $1\frac{1}{3}$, and in others might be greater or less according to the dimensions of the wheel, in order to allow for the excess of actual loss over that due to vertical outflow.
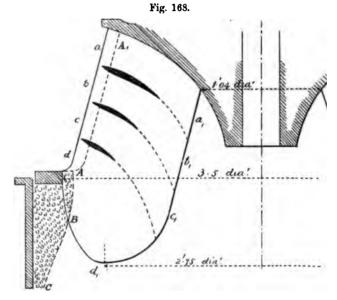
Taking as an example the "Hercules" turbine of which the test results are given on page 405, all the necessary dimensions are given except the relative angle of outflow $a_2$, and if it be assumed that the average value of the latter is 70°, it will probably not be very far from the truth; besides which the co-efficient of velocity $K_1$ is not very sensitive to variations in the relative angle of outflow.

A general description of the "Hercules" turbine has already been given in connection with the trials made with it at Holyoke, but for purposes of calculation it will be necessary to examine its construction somewhat more closely.

Fig. 168 shows a section through the axis of the wheel and some of the principal dimensions kindly furnished to the author by Mr. Clemens Herschel, at the time engineer of the Holyoke Water-power Company.

The flow of the water on leaving the buckets is both inward and downward, but the inward component of the direction of motion preponderates. In entering the buckets the water already has a downward tendency, and

I I

for the greater bulk of it this is comparatively sligh
increased in passing through the motor; a part of t
fluid, however, at the lower end of the buckets lea
axially, or possibly even with a slightly outward rad
tendency.

Fig. 168.



The curved form of the inner edges of the bucket
given with the object of securing a large area of outfl
which with the small diameter would otherwise
impossible.

Mr. Clemens Herschel expresses the opinion that
water "seldom if ever *fills* the bucket orifices." As
previously remarked, the co-efficient of outflow from
guide passages ($K_1 = 0.625$) proves that the turbine
the *reaction* type, and that therefore the water pass
through the buckets is under a pressure greater t

that of the atmosphere; if the bucket orifices are not entirely filled, then a portion of the water itself, not participating in the flow, forms itself into a wall, and the *effective* orifice is that bounded by the body of stationary eddying water. An action of this kind undoubtedly does often take place in the buckets of turbines, and it is extremely probable that it occurs in the "Hercules" wheel.

The water leaves by a short suction or draft-tube, and must take a course in leaving the buckets somewhat like that indicated in Fig. 168; the shaded area represents relatively stationary water in an eddying condition.

If the water *filled* the outflow orifices, a portion of it must leave in a nearly *radial outward* direction, and would immediately impinge nearly at right angles on the inner surface of the suction-tube, its flow being thus abruptly arrested and a great resistance offered to the flow.

The fluid will therefore not take this course, but follow the path of least resistance, and the line $A\ B\ C$ will probably very nearly represent the outer boundary of a section through the axis of the body of water leaving the turbine.

If the edge of the vane at the *outflow* for the greatest diameter be supposed to lie in the plane of the diagram, then the true position of the projection of the *inflow* edge is represented by $A\ A_1$, as the line $a\ b\ c\ d$ shows this edge brought round into the same plane as the outflow edge.

Assuming the course of the water to be as indicated, then only 0·89 of the full outflow area from the buckets, after allowing for contraction, would be occupied, and the ratio $\dfrac{A}{A_2} = 0\cdot673$ instead of $0\cdot509$, as would be the value if the whole area were filled.

From the dimensions given the mean ratio of th
diameters or radii $\frac{r_1}{r_2} = 1\cdot84$; this is arrived at by sup
posing the turbine to be divided into four parts—by th
transverse partitions—through each of which an equ
quantity of water is assumed to flow, and taking th
mean of the ratios $\frac{r_1}{r_2}$ of each division.   The correspondir
outer and inner diameters are measured at $a$ and $a_1$,
and $b_1$, $c$ and $c_1$, $d$ and $d_1$ respectively.

Adding 3 per cent. for shaft friction to the actu
efficiency makes the hydraulic efficiency $\epsilon = 0\cdot888$, or sa
$0\cdot89 = 89$ per cent.

The data are then as follows—

$$a = 75\tfrac{1}{4}°,= a_2 = 70°,\ \frac{A}{A_2} = 0\cdot673,\ \frac{r_1}{r_2}\ \text{(mean)} = 1\cdot84,$$

$\epsilon = 0\cdot89$, $sin\ a = 0\cdot967$, $sin\ a_2 = 0\cdot940$, $r$, mean $= 1\cdot5$

Substituting these values in the formula for $K_1^2$

$$K_1 = \sqrt{\frac{0\cdot89}{2 \times 1\cdot84 \times 0\cdot673 \times 0\cdot967 \times 0\cdot94}} = \mathbf{0\cdot629};$$

the experimental value is **0·625**, so that the agreement
very good.

For the sake of further comparison the theoretic
efficiency may be calculated, allowing **50** *per cent.* mo
loss by unutilized energy than that corresponding
radial outflow in a very narrow wheel.   On this assumptic
the efficiency

$$\epsilon = \cfrac{2\ \frac{r_1}{r_2}\frac{A}{A_2}\ sin\ a\ sin\ a^2}{2\ \frac{r_1}{r_2}\frac{A}{A_2}\ sin\ a\ sin\ a_2 + \zeta_1 + \left(\frac{A}{A_2}\right)^2 (\zeta_2 + 1\cdot5\ cos\ ^2 a_2)};$$

or substituting numerical values

$$\epsilon = 0\cdot881,$$

as against $\epsilon = 0.888$ for the experimental value with 3 per cent. added for shaft friction.

The ratio of the best speed as found by experiment to the velocity due to the head is

$$K_2 = 0.669.$$

For inflow without shock calculated with the observed value of $K_1 = 0.625$—

$$K_1 = 0.582.$$

The value of $K_1$ which should strictly speaking be employed in calculating $K_2$ is probably not quite the same as that given above, since there is a clear space between the guide orifices and the inflow orifices of the buckets, so that the area of the stream on leaving the guides is not the same as at the moment of entering the buckets, the absolute direction of flow being also changed during the passage of the water through the clearance space.

In consequence of the diameter at which the water enters the buckets being considerably less than that at which it leaves the guides, the absolute area of the stream tends to diminish, but this tendency is partly counteracted by the absence of vanes to occupy a portion of the space; at the same time the angle $a$ is very probably less than that at which the water leaves the guides, and this tends to increase the area $A$. It is impossible, owing to the uncertainty as to the direction taken by the water, to know exactly what would be the effective area of the stream, and consequently its absolute velocity at the moment of entering the buckets, even if the clearance between guides and wheel-buckets were given (which in this case it is not), but it is probable that the area is somewhat less than the value previously stated for $A$, and therefore the velocity of flow greater than that from the guide passages. Under these circumstances the velocity $w_1$ of the outer circumference, as calculated, would be

also greater than the value corresponding to $K_2$ 0·582.

The velocity of rotation at the inflow corresponding radial outflow at the mean effective inner diameter $2r$ is 0·7275 $\sqrt{2gh}$, whence $K_2 = 0\cdot7275$.

The mean of the values of $K_2$ resulting from the tv assumptions of inflow without shock and radial outflo

is $$K_2 = \frac{0\cdot582 + 0\cdot7275}{2} = 0\cdot655,$$

which corresponds very satisfactorily with the exper mental value $K_2 = 6\cdot669$.

In this case the correspondence between theory ar practice is exceedingly good.

If the efficiency of a Jonval turbine with the san values of $a$, $a^2$ and $\dfrac{A}{A_2}$ be calculated in a similar mannc it will be found that $\epsilon = 0\cdot818$,

the same co-efficients $\zeta_1$ and $\zeta_2$ being used in both is stances, but no excess of loss by unutilized energy ov that due to vertical outflow allowed for in the paralle flow wheel (which of course tells in *favour* of the latter).

It will be seen that the theoretical advantage of th mixed-flow over the parallel-flow turbine in the case is vestigated is **5·4** per cent., apart from the question any gain arising from absence of shock at the inflov The actual superiority is not so great, as some peculiai ities in the construction and circumstances before allude to in connection with mixed-flow wheels tend to neutraliz the advantages. The actual hydraulic efficiency parallel-flow turbines is often greater than that obtaine above; this is owing to the assumed values of the cc efficients $\zeta_1$ and $\zeta_2$, more especially the latte being too high. Good Jonval wheels have in certai

cases shown efficiencies of 85 per cent., or adding 3 per cent. for shaft friction, hydraulic efficiencies of 88 per cent.

A peculiar feature of the "Hercules" wheel under consideration is, that the efficiency at 0·806 gate was *higher* than at *full* gate.

This is easily accounted for by the construction of the turbine. The buckets are subdivided by partitions parallel with the direction of flow, so as to form practically several narrower wheels connected together and keyed to one shaft. At full gate, when all these divisions are full, the water leaving the uppermost division is, owing to the tapering form of the inside of the wheel and the obstruction of the boss, more abruptly deflected and impeded in its off-flow than the water from the lower divisions. When, therefore, the uppermost portion of the wheel is closed, the fluid meets with less resistance than at full gate, while the top division is filled up with nearly stationary water, which, owing to the presence of the boss, remains shut up between the latter and the sluice-gate regulator in the one direction and the top of the wheel and the flowing stream in the other. In addition to this, the area of the short draft-tube used with the motor, which is about 4 feet diameter, appears to be too small in proportion to the relative area of outflow $A_2$, so that the water is choked in leaving at full gate, while at part gate this does not occur. In calculating the theoretical efficiency the choking action was not taken into account, but when that is done, the efficiency becomes 0·871 instead of 0·881.

The author is of opinion that even better results than those actually achieved would be possible with the type of wheel in question if the inner and outer diameters were both proportionately greater, so that there would be more room for the water to escape, and the necessity might be avoided of giving the inside edge of the buckets

the complicated curved form now required to secure t
stated outflow area $A_2$, at the same time making t
draft-tube of sufficient area to avoid all choking.

There is no reason to suppose that the co-efficients
and $\zeta_2$ would be higher in a turbine of the "Hercule
type than in a Jonval wheel; on the contrary, for t
same angles $a$ and $a_2$, the curvature of the vanes is le
abrupt in an inward-flow than in a parallel-flow whe
while owing to the relatively greater pitch and width
buckets compared with their length, the surface area
contact with a given volume of water is less in the form
than in the latter motor. Both these circumstanc
would tend rather to reduce the co-efficients of resistan
$\zeta_1$ and $\zeta_2$ than otherwise.

Against this may be set the double curvature (whic
however, in the "Hercules" wheel is only slight for tl
greater proportion of the water passing through it), ai
the transverse divisions which offer additional resistan
to the water.

In the "Hercules" turbine the transverse divisior
besides facilitating regulation, enable the guide passag
and buckets to be made wider than would otherwise l
practicable, since they transform a wheel of very gre;
width in proportion to the diameter into several whee
of very moderate width. The drawback of making tl
total width of the buckets relatively great, in order i
secure a small diameter, which is common to mar
American turbines, is illustrated in the "Hercules
wheel by the choking of the outflow. Through th
choking, as has been mathematically shown, the efficienc
is reduced below what it would otherwise be, and this
confirmed by the experiments, which prove the motor i
work better with the gate partially closed than when
is fully open.

A more accurate method of analyzing the performanc

of this motor would be to treat each division as a separate turbine, but in that case a knowledge of the relative outflow areas of the different parts would be necessary, and this information the author does not possess. For each division the ratio $\dfrac{A}{A_2}$ is probably somewhat different, its value being least for the lowest division. Under these circumstances the resultant velocity of flow should be taken as the mean of the four different values obtained.

At the Centennial Exhibition in Philadelphia a large number of turbine trials were carried out, and the motor which on this occasion showed the best results was the " Risdon " Turbine. This motor is illustrated on pages 27 and 28; it is of the inward- and downward-flow type, the whole of the water leaving axially. It is reported to have shown an efficiency of 87 per cent. when tested at the Centennial Exhibition, and as much as 90 *per cent.* when tested elsewhere.

The author is of opinion that even the efficiency of 87 per cent. should be accepted with caution, while 90 per cent. may be rejected as practically impossible with the turbine in question. The ratio of the guide outflow area to the bucket outflow area in this " Risdon " turbine was, without allowing for contraction, as 1·25 to 1; allowing for contraction at the guide orifices, and assuming there was none at the bucket orifices, the ratio $\dfrac{A}{A_2}$ would be nearly 1. As to the proportions between the diameter at the inflow and the diameter at the outflow, no data are published, but judging from the illustrations of this type of wheel in the makers' catalogues, the average ratio $\dfrac{r_1}{r_2}$ would be about $\frac{4}{5}$. With low values of the co-efficients $\zeta_1$ and $\zeta_2$ and the above ratios, a hydraulic efficiency of

about 90 per cent. or a little over is theoretically possib
which, allowing 3 per cent. for shaft friction, would lea
an actual efficiency of about 87 per cent. The questi
is whether, with the abrupt deflection which a portion
the water is compelled to make from a radial to a verti
course, the assumption of such low values for the co-efficie
is justified.

## Comparison of Water Measurements.

As it might very naturally be suggested that t
exceptionally good results obtained with the "Hercule
wheel are due to errors in the measurement of the wa
used by the motor, a comparison of the co-efficient us
at Holyoke with that obtained from Braschmann's form
and by other experimenters appears desirable.

The mean length of the weir at Holyoke during t
trials of the "Hercules" turbine was $L = 20\cdot035$ fe
being the same as that of the canal of approach.

For the best speed at full gate the depth on the w
was $H = 1\cdot2$ feet, and the quantity of water estimated
have passed through the turbine $Q = 88\cdot33$ cubic f
per second. If from these data the co-efficient $C$
calculated in the formula

$$Q = CL\sqrt{2gH^3},$$

it will be found that

$$C = 0\cdot4179.$$

By Braschmann's formula

$$C = \left(0\cdot3838 + 0\cdot0386\,\frac{L}{B} + \frac{0\cdot00174}{H}\right);$$

since $\dfrac{L}{B} = 1$

$$C = 0\cdot3838 + 0\cdot0386 + 0\cdot0014 = 0\cdot4238.$$

This gives a difference of only $1\cdot4$ per cent. betw

the two values, which is well within the limits of probable error.

Fteley and Stearns found a co-efficient $C = 0.4106$ for a weir 19 feet wide without end contractions, with a depth of water on the weir of 1.166 feet; the difference between this and the Holyoke value is 1.75 per cent.

Before dismissing the subject of American turbines, it may be recalled to mind that with an inward-flow wheel the loss from unutilized energy with a *given outflow-angle* $a_2$ is not so great as in a parallel-flow turbine with the *same angle*, and this fact has been taken advantage of in some American motors, by making the outflow angle *less* than would be desirable in axial turbines in order to secure a greater outflow area without increasing the diameter. The smaller the angle $a_2$ can be made, the less sudden is the change of sectional area of the buckets for a given length of the latter. The loss in the buckets is reduced by making the change of shape more gradual, and this may in some instances compensate for the accompanying increase of loss from unutilized energy resulting from the smaller angle $a_2$. For this reason, of two turbines of the same class, but with different outflow angles, that with the smaller angle may give the higher efficiency, in spite of the greater residual velocity of the water leaving it.

The following Tables I. and II. (previously referred to) give respectively the values of the co-efficients of flow and efficiencies corresponding to various proportions of the outflow areas and radii, and to various angles of outflow. The figures contained in Table II. are not intended to be taken as having any absolute value, but merely illustrate the manner in which modifications in the proportions tend to influence the efficiency, on the assumption that the experimental co-efficients of friction, &c. remain unaltered.

# TABLE I.

*Co-efficients of Flow $K_1$ for various proportions of Turbines.*

$$a_2 = 80°.$$

| $\dfrac{A}{A_2}$ | 2 | | | | | 1·75 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\dfrac{r_1}{r_2}$ | 1 | $\tfrac{3}{4}$ | $\tfrac{1}{2}$ | $\tfrac{1}{4}$ | $\tfrac{1}{8}$ | 1 | $\tfrac{3}{4}$ | $\tfrac{1}{2}$ | $\tfrac{1}{4}$ | $\tfrac{1}{8}$ |
| **a** | | | | | | | | | | |
| 75° | 0·528 | 0·494 | 0·454 | 0·415 | 0·385 | 0·573 | 0·535 | 0·490 | 0·448 | 0·414 |
| 70° | 0·533 | 0·499 | 0·459 | 0·420 | 0·389 | 0·578 | 0·540 | 0·496 | 0·453 | 0·419 |
| 65° | 0·540 | 0·506 | 0·466 | 0·426 | 0·395 | 0·586 | 0·548 | 0·503 | 0·460 | 0·426 |
| 60° | 0·549 | 0·515 | 0·474 | 0·434 | 0·403 | 0·595 | 0·557 | 0·512 | 0·468 | 0·434 |
| 55° | 0·559 | 0·525 | 0·489 | 0·444 | 0·412 | 0·607 | 0·569 | 0·522 | 0·479 | 0·445 |
| 50° | 0·572 | 0·538 | 0·496 | 0·456 | 0·424 | 0·622 | 0·583 | 0·537 | 0·492 | 0·457 |

| $\dfrac{A}{A_2}$ | 1·5 | | | | | 1·25 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\dfrac{r_1}{r_2}$ | 1 | $\tfrac{3}{4}$ | $\tfrac{1}{2}$ | $\tfrac{1}{4}$ | $\tfrac{1}{8}$ | 1 | $\tfrac{3}{4}$ | $\tfrac{1}{2}$ | $\tfrac{1}{4}$ | $\tfrac{1}{8}$ |
| **a** | | | | | | | | | | |
| 75° | 0·627 | 0·585 | 0·535 | 0·487 | 0·451 | 0·695 | 0·647 | 0·591 | 0·538 | 0·497 |
| 70° | 0·633 | 0·591 | 0·541 | 0·493 | 0·456 | 0·702 | 0·654 | 0·598 | 0·544 | 0·503 |
| 65° | 0·642 | 0·599 | 0·549 | 0·501 | 0·463 | 0·712 | 0·664 | 0·607 | 0·553 | 0·511 |
| 60° | 0·653 | 0·610 | 0·559 | 0·510 | 0·472 | 0·724 | 0·676 | 0·618 | 0·563 | 0·521 |

$a_2 = 80°$.

| $\frac{A}{A_2}$ | | | 1 | | | | | | 0·75 | | | |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| $\frac{r_1}{r_2}$ | | | | | | | | | | | | |
| $a$ | $\frac{3}{8}$ | $\frac{1}{2}$ | 1 | $\frac{1}{4}$ | $\frac{1}{8}$ | $\frac{5}{8}$ | $\frac{1}{2}$ | 1 | $\frac{1}{4}$ | $\frac{1}{8}$ |
| 75° | 0·785 | 0·730 | 0·665 | 0·605 | 0·558 | 0·912 | 0·847 | 0·771 | 0·700 | 0·646 |
| 70° | 0·793 | 0·738 | 0·673 | 0·612 | 0·565 | 0·921 | 0·856 | 0·780 | 0·709 | 0·654 |
| 65° | 0·805 | 0·749 | 0·684 | 0·622 | 0·574 | 0·934 | 0·869 | 0·792 | 0·720 | 0·665 |
| 60° | 0·819 | 0·762 | 0·696 | 0·634 | 0·586 | 0·950 | 0·885 | 0·807 | 0·734 | 0·678 |
| 55° | 0·836 | 0·779 | 0·713 | 0·649 | 0·600 | 0·971 | 0·904 | 0·827 | 0·753 | 0·695 |
| 50° | 0·857 | 0·800 | 0·733 | 0·668 | 0·618 | 0·996 | 0·929 | 0·849 | 0·774 | 0·716 |

| $\frac{A}{A_2}$ | | | 0·5 | | | | | | 0·25 | | | |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| $\frac{r_1}{r_2}$ | | | | | | | | | | | | |
| $a$ | $\frac{3}{8}$ | $\frac{1}{2}$ | 1 | $\frac{1}{4}$ | $\frac{3}{8}$ | $\frac{5}{8}$ | $\frac{1}{2}$ | 1 | $\frac{1}{4}$ | $\frac{1}{8}$ |
| 75° | 1·107 | 1·029 | 0·939 | 0·854 | 0·788 | 1·480 | 1·386 | 1·274 | 1·164 | 1·082 |
| 70° | 1·118 | 1·041 | 0·950 | 0·864 | 0·798 | 1·495 | 1·400 | 1·288 | 1·179 | 1·096 |
| 65° | 1·134 | 1·056 | 0·966 | 0·879 | 0·811 | 1·514 | 1·419 | 1·307 | 1·198 | 1·112 |
| 60° | 1·154 | 1·075 | 0·983 | 0·894 | 0·827 | 1·537 | 1·443 | 1·330 | 1·219 | 1·133 |
| 55° | 1·178 | 1·099 | 1·005 | 0·917 | 0·847 | 1·565 | 1·483 | 1·358 | 1·247 | 1·159 |
| 50° | 1·208 | 1·128 | 1·033 | 0·943 | 0·872 | 1·600 | 1·507 | 1·392 | 1·280 | 1·191 |

TABLE I.—continued.

$a_2 = 75°$.

Upper group (α = 75°–50°):

| $\dfrac{A}{A^2}$ | 2 | | | | | 1·75 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\dfrac{r_1}{r_2}$ | ⅝ | ¾ | 1 | ¾ | ⅝ | ⅝ | ¾ | 1 | ¾ | ⅝ |
| α | | | | | | | | | | |
| 75° | 0·522 | 0·489 | 0·450 | 0·413 | 0·384 | 0·566 | 0·530 | 0·488 | 0·446 | 0·414 |
| 70° | 0·526 | 0·494 | 0·455 | 0·418 | 0·388 | 0·569 | 0·536 | 0·493 | 0·451 | 0·418 |
| 65° | 0·532 | 0·500 | 0·462 | 0·424 | 0·394 | 0·582 | 0·543 | 0·500 | 0·458 | 0·425 |
| 60° | 0·540 | 0·508 | 0·469 | 0·432 | 0·401 | 0·588 | 0·552 | 0·508 | 0·466 | 0·433 |
| 55° | 0·550 | 0·518 | 0·479 | 0·441 | 0·410 | 0·599 | 0·563 | 0·519 | 0·476 | 0·443 |
| 50° | 0·562 | 0·530 | 0·491 | 0·452 | 0·421 | 0·612 | 0·576 | 0·532 | 0·489 | 0·455 |

Lower group (α = 75°–65°):

| $\dfrac{A}{A^2}$ | 1·5 | | | | | 1·25 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\dfrac{r_1}{r_2}$ | ⅝ | ¾ | 1 | ¾ | ⅝ | ⅝ | ¾ | 1 | ¾ | ⅝ |
| α | | | | | | | | | | |
| 75° | 0·621 | 0·581 | 0·533 | 0·486 | 0·451 | 0·691 | 0·644 | 0·590 | 0·538 | 0·497 |
| 70° | 0·627 | 0·587 | 0·539 | 0·492 | 0·456 | 0·697 | 0·651 | 0·596 | 0·544 | 0·503 |
| 65° | 0·635 | 0·595 | 0·546 | 0·499 | 0·463 | 0·707 | 0·660 | 0·605 | 0·553 | 0·511 |

$$\alpha_2 = 75°.$$

| $\frac{A}{A_2}$ | 1 | | | | | 0·75 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\frac{r_1}{r_2}$ | ½ | ¾ | 1 | 1½ | 2 | ½ | ¾ | 1 | 1½ | 2 |
| $\alpha$ | | | | | | | | | | |
| 75° | 0·782 | 0·728 | 0·665 | 0·606 | 0·560 | 0·910 | 0·847 | 0·773 | 0·703 | 0·649 |
| 70° | 0·790 | 0·736 | 0·673 | 0·613 | 0·566 | 0·919 | 0·856 | 0·781 | 0·711 | 0·657 |
| 65° | 0·801 | 0·747 | 0·683 | 0·623 | 0·576 | 0·933 | 0·869 | 0·794 | 0·722 | 0·668 |
| 60° | 0·815 | 0·760 | 0·696 | 0·635 | 0·587 | 0·949 | 0·884 | 0·809 | 0·737 | 0·681 |
| 55° | 0·831 | 0·776 | 0·712 | 0·650 | 0·601 | 0·968 | 0·904 | 0·827 | 0·754 | 0·698 |
| 50° | 0·852 | 0·796 | 0·731 | 0·668 | 0·619 | 0·992 | 0·927 | 0·850 | 0·775 | 0·718 |

| $\frac{A}{A_2}$ | 0·5 | | | | | 0·25 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\frac{r_2}{r_1}$ | ½ | ¾ | 1 | 1½ | 2 | ½ | ¾ | 1 | 1½ | 2 |
| $\alpha$ | | | | | | | | | | |
| 75° | 1·109 | 1·033 | 0·943 | 0·858 | 0·793 | 1·486 | 1·394 | 1·282 | 1·175 | 1·090 |
| 70° | 1·119 | 1·043 | 0·953 | 0·868 | 0·802 | 1·499 | 1·407 | 1·295 | 1·187 | 1·102 |
| 65° | 1·135 | 1·059 | 0·968 | 0·882 | 0·815 | 1·519 | 1·426 | 1·314 | 1·205 | 1·119 |
| 60° | 1·156 | 1·080 | 0·988 | 0·899 | 0·831 | 1·542 | 1·449 | 1·337 | 1·227 | 1·140 |
| 55° | 1·179 | 1·101 | 1·009 | 0·920 | 0·852 | 1·569 | 1·476 | 1·364 | 1·253 | 1·166 |
| 50° | 1·208 | 1·130 | 1·035 | 0·946 | 0·876 | 1·604 | 1·510 | 1·397 | 1·285 | 1·198 |

## TABLE I.—continued.

$a_2 = 70°$

| $\dfrac{A}{A_2}$ | 2 | | | | | 1·75 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\dfrac{r_1}{r_2}$ | ¾ | ½ | 1 | ¾ | ⅜ | ⅜ | ½ | 1 | ¾ | ⅜ |
| $a$ | | | | | | | | | | |
| 75° | 0·512 | 0·482 | 0·446 | 0·410 | 0·382 | 0·558 | 0·524 | 0·484 | 0·444 | 0·413 |
| 70° | 0·517 | 0·487 | 0·451 | 0·415 | 0·386 | 0·562 | 0·529 | 0·489 | 0·449 | 0·417 |
| 65° | 0·522 | 0·492 | 0·456 | 0·420 | 0·392 | 0·569 | 0·536 | 0·495 | 0·455 | 0·423 |
| 60° | 0·530 | 0·500 | 0·464 | 0·428 | 0·399 | 0·577 | 0·544 | 0·503 | 0·463 | 0·431 |
| 55° | 0·539 | 0·509 | 0·473 | 0·437 | 0·408 | 0·588 | 0·554 | 0·514 | 0·473 | 0·441 |
| 50° | 0·549 | 0·520 | 0·484 | 0·448 | 0·418 | 0·600 | 0·567 | 0·526 | 0·485 | 0·453 |

| $\dfrac{A}{A_2}$ | 1·5 | | | | | 1·25 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\dfrac{r_1}{r_2}$ | ¾ | ½ | 1 | ¾ | ⅜ | ⅜ | ½ | 1 | ¾ | ⅜ |
| $a$ | | | | | | | | | | |
| 75° | 0·614 | 0·576 | 0·530 | 0·485 | 0·451 | 0·685 | 0·641 | 0·588 | 0·538 | 0·498 |
| 70° | 0·620 | 0·581 | 0·535 | 0·491 | 0·456 | 0·691 | 0·647 | 0·594 | 0·544 | 0·504 |
| 65° | 0·627 | 0·589 | 0·543 | 0·498 | 0·463 | 0·700 | 0·656 | 0·603 | 0·552 | 0·512 |
| 60° | 0·637 | 0·598 | 0·552 | 0·507 | 0·471 | 0·711 | 0·667 | 0·614 | 0·562 | 0·522 |
| 55° | 0·648 | 0·610 | 0·564 | 0·518 | 0·482 | 0·725 | 0·680 | 0·627 | 0·575 | 0·534 |

α₂ = 70°

| A/A₂ | | 1 | | | | | 0·75 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| r₁/r₂ | ⅜ | ½ | 1 | ¾ | ⅝ | ⅝ | ¾ | 1 | ½ | ⅜ |
| α | | | | | | | | | | |
| 75° | 0·778 | 0·726 | 0·665 | 0·607 | 0·562 | 0·909 | 0·847 | 0·775 | 0·706 | 0·653 |
| 70° | 0·785 | 0·734 | 0·672 | 0·614 | 0·568 | 0·918 | 0·856 | 0·784 | 0·714 | 0·661 |
| 65° | 0·796 | 0·744 | 0·683 | 0·625 | 0·578 | 0·931 | 0·869 | 0·796 | 0·726 | 0·671 |
| 60° | 0·809 | 0·757 | 0·695 | 0·636 | 0·589 | 0·946 | 0·884 | 0·810 | 0·739 | 0·685 |
| 55° | 0·825 | 0·773 | 0·710 | 0·650 | 0·603 | 0·965 | 0·903 | 0·830 | 0·756 | 0·701 |
| 50° | 0·844 | 0·792 | 0·729 | 0·670 | 0·624 | 0·988 | 0·926 | 0·850 | 0·777 | 0·721 |

| A/A₂ | | 0·5 | | | | | 0·25 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| r₁/r₂ | ⅜ | ½ | 1 | ¾ | ⅝ | ⅝ | ¾ | 1 | ½ | ⅜ |
| α | | | | | | | | | | |
| 75° | 1·112 | 1·037 | 0·948 | 0·864 | 0·799 | 1·493 | 1·403 | 1·292 | 1·184 | 1·100 |
| 70° | 1·123 | 1·048 | 0·959 | 0·874 | 0·809 | 1·509 | 1·417 | 1·306 | 1·198 | 1·113 |
| 65° | 1·138 | 1·063 | 0·973 | 0·888 | 0·822 | 1·527 | 1·435 | 1·324 | 1·216 | 1·129 |
| 60° | 1·157 | 1·082 | 0·991 | 0·904 | 0·837 | 1·549 | 1·457 | 1·346 | 1·237 | 1·149 |
| 55° | 1·181 | 1·104 | 1·013 | 0·926 | 0·858 | 1·578 | 1·486 | 1·374 | 1·264 | 1·175 |
| 50° | 1·209 | 1·131 | 1·040 | 0·951 | 0·882 | 1·612 | 1·520 | 1·407 | 1·296 | 1·208 |

K K

## TABLE II.

*Calculated Efficiencies with $\alpha_2 = 70°$ for various proportions of Turbines.*

| $\frac{A}{A_2}$ | 2 | | | 1·75 | | | 1·5 | | | 1·25 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\frac{r_1}{r_2}$ | $\frac{2}{3}$ | 1 | $\frac{3}{2}$ | $\frac{2}{3}$ | 1 | $\frac{4}{3}$ | $\frac{2}{3}$ | 1 | $\frac{3}{2}$ | $\frac{3}{4}$ | 1 | $\frac{3}{2}$ |
| $\alpha$ | | | | | | | | | | | | |
| 75° | 0·634 | 0·722 | 0·796 | 0·659 | 0·743 | 0·813 | 0·684 | 0·765 | 0·829 | 0·709 | 0·785 | 0·846 |
| 70° | 0·628 | 0·717 | 0·792 | 0·653 | 0·738 | 0·809 | 0·678 | 0·760 | 0·826 | 0·703 | 0·781 | 0·842 |
| 65° | 0·620 | 0·710 | 0·786 | 0·644 | 0·731 | 0·803 | 0·670 | 0·753 | 0·820 | 0·695 | 0·774 | 0·837 |
| 60° | 0·609 | 0·700 | 0·778 | 0·634 | 0·722 | 0·796 | 0·660 | 0·744 | 0·813 | 0·686 | 0·766 | 0·831 |
| 55° | 0·595 | 0·688 | 0·751 | 0·621 | 0·711 | 0·786 | 0·648 | 0·733 | 0·805 | 0·674 | 0·756 | 0·823 |
| 50° | 0·579 | 0·674 | 0·756 | 0·605 | 0·697 | 0·775 | 0·632 | 0·720 | 0·794 | 0·659 | 0·743 | 0·813 |

| $\frac{A}{A_2}$ | 1 | | | 0·75 | | | 0·5 | | | 0·25 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\frac{r_1}{r_2}$ | $\frac{2}{3}$ | 1 | $\frac{3}{2}$ | $\frac{3}{4}$ | 1 | $\frac{3}{2}$ | $\frac{3}{4}$ | 1 | $\frac{3}{2}$ | $\frac{3}{4}$ | 1 | $\frac{3}{2}$ |
| $\alpha$ | | | | | | | | | | | | |
| 75° | 0·732 | 0·804 | 0·860 | 0·749 | 0·818 | 0·871 | 0·747 | 0·816 | 0·869 | 0·677 | 0·758 | 0·824 |
| 70° | 0·727 | 0·800 | 0·857 | 0·744 | 0·813 | 0·867 | 0·742 | 0·812 | 0·866 | 0·670 | 0·753 | 0·820 |
| 65° | 0·719 | 0·794 | 0·852 | 0·737 | 0·808 | 0·863 | 0·735 | 0·806 | 0·862 | 0·662 | 0·746 | 0·815 |
| 60° | 0·710 | 0·786 | 0·847 | 0·728 | 0·801 | 0·858 | 0·727 | 0·799 | 0·856 | 0·652 | 0·737 | 0·808 |

The curves shown in Fig. 169 illustrate clearly the
ɛct of the ratio of inflow to outflow radius on the
ɔrgy available in the kinetic form—as velocity—at the
.de outflow, and on the various losses.

It will be seen that the loss from residual velocity
:reases as the ratio $\frac{r_1}{r_2}$ increases.

Neglecting the loss in the guide passages, the ordinates
the curve of kinetic energy measured from the lower
rizontal line upwards give the proportion of energy
ɹilable as pressure at the guide outflow $h - \frac{c_2}{2g}$.

# CHAPTER XIII.

## HYDRAULIC PRESSURE ENGINES.

Definition.—Mode of action.—Motion of piston.—Influence of reciprocating masses.—Pressure on piston.—Available energy.—Useful work.—Losses.—Efficiency.—Motors of Armstrong, Rigg, Schmid, Wyss and Studer, Haag, Meyer, and Hastie.—General remarks.

A HYDRAULIC pressure engine is a motor in which water is made to do work by means of its hydrostatic pressure only acting on a piston or plunger reciprocating in a cylinder, or, in some cases, on a revolving piston similar to those employed in rotary steam-engines.

Reciprocating motors of this class have been made both single and double acting, and for pressure up to one ton per square inch and even greater.

In the case of turbines the pressure of a column of water is in the first instance employed in giving motion to the fluid itself, and after this motion has been produced the energy due to it is utilized in doing work. In hydraulic pressure engines the pressure of the water, instead of first being expended on accelerating the motion of the latter, is applied direct to the performance of work, by overcoming through the medium of a piston or plunger an approximately equal and contrary force or resistance. A portion of the available energy due to the pressure is, it is true, unavoidably expended in giving the fluid the velocity necessary to follow the piston and escape from the motor,

but this portion should be as small as practicable, and an ideal pressure motor would be one in which the energy represented by the velocity is a vanishing quantity compared with the work done on the piston; this will be approximately the case when the pressure is very high and the speed of the engine low.
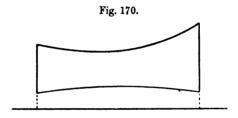
For pressure engines, as for other hydraulic motors, the available energy is expended in (1) doing useful work, (2) overcoming hydraulic resistances, and (3) producing the residual velocity of the fluid.

If the motion of the piston or plunger were uniform throughout the whole of its stroke, the pressure exerted on it would be constant, provided the pressure in the main did not vary; as a matter of fact, however, in reciprocating engines the speed of the piston varies very greatly at different points of the stroke, being *nil* at the commencement, a maximum near the middle, and again *nil* at the end. The water in the cylinder and in the pipes connected with it, as far back as the air vessel or its equivalent which is generally used, has to follow this varying motion of the piston, and consequently alternate acceleration and retardation of the whole of this mass of fluid, as well as the reciprocating masses of the engine, must occur. At the beginning of the stroke, when the piston is for a moment at rest, the whole available pressure will be transmitted through it to the crank-pin, but as the speed increases towards the middle of the stroke, a part of this pressure will be required to cause the acceleration of the moving masses, and the force acting on the crank-pin will be diminished by a corresponding amount. During the latter part of the stroke the energy thus absorbed will be again restored as the masses are brought to rest.

The pressure exerted on the piston is, in consequence of the action described, not constant, but varies according

to the amount required to produce the velocity of the water necessary to follow the piston. The indicator diagram would have a form similar to that shown in Fig. 170.

The pressure, as recorded by an indicator, is not affected by the absorption of energy through the reciprocating masses of the engine, which only has the effect of altering the distribution of effort on the crank-pin.

Fig. 170.



During the exhaust, in addition to the power required to overcome the atmospheric pressure, a certain amount of work has to be done in expelling the water from the cylinder, all of which represents loss of energy, and has the immediate effect of increasing the back pressure. The greater the piston speed the greater, with a given exhaust orifice, must be the back pressure, and the latter increases as the outflow area is reduced; it is therefore obviously desirable to keep the speed of the motor low relatively to the velocity due to the available pressure or head, and at the same time to make the area of the exhaust passages as great as possible.

An exact mathematical investigation of the effect of the acceleration and retardation of the masses of water and metal on the distribution of effort on the piston or crank-pin leads to very complicated expressions and has very little practical use ; it may therefore be dispensed with.

The well-known formula
$$E = W + L,$$
· where (as before) $E$ denotes the available energy, $W$ the useful work done, and $L$ the losses, applies to water-pressure as to other motors; it remains to be seen of what components these quantities are made up.

## Available  Energy.

In the case of small water-pressure motors driven from the water-supply in the town mains, the available energy at the disposal of the motor is the product of the pressure in the pipes, measured as *head*, multiplied with the weight of water supplied.  The pressure must be measured at a point immediately behind the orifice through which the water enters the motor.

The amount of the available energy for any given case is by no means invariable, but depends upon the position of the valve or cock through which the water leaves the main.  If $F$ denote the sectional area of the pipe, $f$ the (variable) valve area, $a$ the co-efficient of contraction, then the velocity of outflow

(1)
$$c = \sqrt{\frac{2gh}{1 + \left(\dfrac{F}{af} - 1\right)^2}}$$

where $a$ also varies under different conditions.  The quantity of water passing through the valve-area $f$ is $Q = f\,c$, so that $Q$ is dependent on $f$ in a double sense, and is not merely proportioned to the opening of the valve.  If $p$ denote the pressure near the valve-orifice, then the available energy

(2)
$$E = c\,s\,f\,p = Q\,s\,p,$$

where $s$ is the weight of the unit of volume of the fluid.

The quantity $p$ is, however, dependent to a high degree on $f$ (the valve-outlet area), and attains its minimum value when the valve is fully open, and its maximum (the full pressure in the main) when the valve is closed. The volume, on the other hand, increases with $f$, and hence it follows that $E$ must reach its maximum with some intermediate position of the valve.
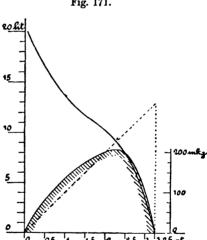
Fig. 171.



To follow the variations of $E$ mathematically would be a somewhat tedious and unnecessary process, but experiments made on this subject by Schaltenbrand, the results of which were published in the *Zeitschrift des Vereins Deutscher Ingenieure* in 1881, p. 655, give valuable information on this point. The experiments were made with an 8 mm. valve in the Berlin mains, to which a pressure-gauge was connected.

When the valve was closed, the gauge indicated a

pressure of 3·25 atmospheres, and with fully open valve only 0·05 atmosphere.

The diagram (Fig. 171) shows graphically the results of this investigation. The pressures in atmospheres plotted as abscissæ and the quantities of water flowing out as ordinates give the curve shown by the full plain line. The product of the pressure and corresponding quantity of water is proportional to the available energy in each case, and is also plotted as a curve represented by the full line with shaded margin. The efficiency is shown by the dotted line. The maximum value of the available energy occurs at a pressure of 2·3 atmospheres, and with a quantity of water equal ·to 8·9 litres, and amounts to 205 metre-kilograms per minute, or 0·045 horse-power.

The amount of energy available for every cubic foot of water is evidently proportional to the pressure behind the valve-orifice.

The mean velocity of flow $c$ through the main valve with a given motor is obviously directly proportional to the mean speed of the latter, so that with a given head in the main, there is some speed of the motor at which a maximum quantity of energy *is supplied to it* for conversion into work. It does not follow that the maximum work done by the motor is also coincident with this, since, as previously stated, the efficiency is greatest when the speed is least.

If $h$ denote the total head of water available in the main, then for the relation between velocity of flow $c$ and pressure $p$, the hydrodynamic equation applies—

$$(A) \qquad h = \frac{c^2}{2g} + p + \text{Losses} = (1 + \zeta)\frac{c^2}{2g} + p,$$

where $\zeta$ is co-efficient including all losses.

By (2)
$$E = c f s p,$$
and since from $(A)$

$$c = \sqrt{\frac{(h - p) \, 2g}{1 + \zeta}}$$

(2a)
$$E = f s p \sqrt{\frac{h - p}{1 + \zeta} \, 2g},$$

showing very clearly that the *total* available energy is not simply directly proportional to the pressure, although this is the case with regard to the energy available per unit of weight—

$$E_u = p.$$

Expressed in terms of $c$ (instead of $p$ as above)

(2b)
$$E = f s c \left\{ h - \frac{c^2}{2g} (1 + \zeta) \right\}$$

## Useful Work.

Let $A$ denote the effective area of the piston,

$S$    the stroke,

$P$    the mean effective pressure on the piston, per square inch,

$p_m$    the mean working pressure on the piston, per square inch,

$p_o$    the mean back pressure above the atmosphere,

$a$    the atmospheric pressure,

$n$    the number of strokes per second of the piston,

then $P = p_m - p_o$, assuming the area of the piston to be the same on both sides, and the useful work per stroke $= P \, A \, S = (p_m - p_o) \, A \, S.$

The useful work *per second*

$$W = n \, P \, A \, S = n \, (p_m - p_o) \, A \, S,$$

or since the mean piston velocity per second

$$v = n \, S,$$

$$W = v \, (p_m - p_o) \, A = v \, P \, A.$$

The pressure $p$ in the main, measured in the air vessel or its equivalent, would, if the velocity were uniform, equal the working pressure $p_m$, neglecting the losses due to friction and hydraulic resistances of various kinds. In practice, $p$ and $p_m$ are only equal at the commencement of a stroke, when the piston or plunger has the velocity *nil;* as the velocity increases, the pressure $p$ (which for the sake of simplicity is assumed to be constant, as would be approximately the case with a very large air vessel), instead of being wholly expended in doing work on the piston, is partially employed in accelerating the mass of water between the air vessel and the piston, as well as the masses of the reciprocating portions of the engine. This has the effect of reducing the pressure on the piston below that in the air vessel, apart from hydraulic resistances which tend in the same direction, and causes the depression of the indicator diagram, shown in Fig. 170, previously referred to. As the piston is brought to rest during the latter part of the stroke, the pressure rises, and the kinetic energy due to the velocity of the moving fluid is reconverted into pressure, so that at the termination of the stroke, when the piston velocity is again *nil,* not only does the full pressure $p$ in the air vessel make itself felt, but also an excess of pressure over this due to the restoration of the energy previously absorbed in acceleration. Hence the ordinate of the indicator .diagram corresponding to the end of the stroke is higher than that at the beginning.

For practical purposes it is sufficient to assume a mean constant pressure $p_m$ throughout each stroke, which is less than the mean pressure $p$ in the air vessel, by the amount due to hydraulic resistances in the pipes, passages and cylinder.

With a small air vessel, as generally used with hydraulic reciprocating motors, the pressure $p$ is not constant, since at the commencement and termination of every stroke (where only a single motor is in use) the body of fluid between the air vessel and piston is brought to rest while water continues to flow into the air vessel from the main, thus producing compression of the enclosed air and a corresponding increase of pressure. The actual conditions occurring are, it will be seen, of a very complicated character, owing mainly to the periodically varying piston speed.

### *Losses of Energy.*

The hydraulic losses of energy taking place in water-pressure motors are similar to those occurring in the case of turbines, and are due to the following causes: fluid friction in pipes and passages, flow through orifices with sharp or imperfectly rounded edges, sudden changes of stream section, sharp bends in pipes and passages, and unutilized energy in outflowing water.

Some of these losses occur during the working stroke, others during the exhaust stroke.

Generally speaking, the water enters the cylinder through ports which are considerably less in area than the cylinder, so that in passing into the latter a sudden enlargement of the stream section takes place. If, as before, $v$ be the piston speed and $v_1$ the velocity of flow through the ports, the loss for the unit of weight will be

$$L_1 = \frac{1}{2g} (v_1 - v)^2,$$

or, if $R_1$ denote the ratio of the area of the cylinder to

that of the ports, $L_1 = \dfrac{(R_1 - 1)^2 v^2}{2g}$,

since $\qquad\qquad v_1 = R_1 v.$

In passing through the ports a loss is sustained repre-

sented by $\qquad L_2 = \dfrac{\zeta_1 v_1^2}{2g} = \dfrac{1}{2g} \zeta_1 R_1^2 v^2.$

Previous to entering the cylinder a certain quantity of energy is absorbed by fluid friction in the pipe leading from the air vessel to the motor. A similar loss occurs in the cylinder both during the working and exhaust stroke.

Owing to the relatively small length of the cylinder, the loss in it by hydraulic friction may be neglected; that occurring in the supply pipe is expressed by the
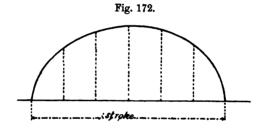
well-known formula $\qquad L_o = \dfrac{l}{d_o} \zeta_r \dfrac{v_o^2}{2g},$

where $l$ is the length of the pipe, $d_o$ its diameter, $v_o$ the velocity of the water in it, and $\zeta_r$ a co-efficient. If the exhaust takes place through a pipe of any considerable length, the loss by friction in this must also be taken into account. Denoting by $R_o$ the ratio of the cylinder area to that of the pipe with the diameter $d_o$,

$$L_o = \dfrac{l}{d_o} \zeta_r R_o^2 \dfrac{v^2}{2g}.$$

The loss from unutilized energy has next to be considered. At every point of the exhaust stroke the velocity of the water expelled from the motor is different, varying with the piston speed, and the loss of energy, for

a given quantity of water expelled is of course proportional to the square of the piston speed. If the squares of the piston velocities be plotted as ordinates, and the corresponding portions of the stroke traversed as abscissæ, a curve will result similar to that shown in Fig. 172. The mean ordinate of this curve—the height of a rectangle of the length of the stroke and of the area comprised between the curve and the axis of abscissæ— represents the *average loss* per unit of weight of water expelled, and the square root of this value the equivalent velocity of flow, which is in excess of the mean piston

Fig. 172.



velocity. In a motor with an oscillating cylinder, and the distance of the centre of oscillation from the crank centre equal to three times the throw of the crank, the average loss by unutilized energy is to that due to the mean piston velocity as about 6·5 to 4, and the ratio of the equivalent velocity to the mean piston velocity as about 2·54 to 2, or say $1\frac{1}{4}$ times the latter.

In calculating the various hydraulic losses, this value should be taken for $v$ instead of the actual mean piston velocity—it may be denoted by $C\,v$. The loss from unutilized energy may then be expressed by

$$L_3 = R_3{}^2 \frac{C^2 v^2}{2g},$$

where $R_3$ is the ratio of the cylinder area to that of the exhaust ports.

There will finally be a further loss during the passage of the water through the exhaust ports and passages, depending on the length, shape and surface of these, and whether the edges are well rounded or sharp.

All these losses may be summed up together in the form

$$L = \frac{v^2}{2g} \left\{ R_o{}^2 \frac{l}{d_o} \zeta_r + (R_1 - 1)^2 + R_1{}^2 \zeta_1 + R_3{}^2 C^2 + \ldots \right\};$$

let the quantities within the brackets be for brevity

denoted by $\zeta$, then $\qquad L = \zeta \frac{v_2}{2g}.$

The mean effective pressure can now be expressed by

$$P = \left( \frac{p}{s} - \zeta \frac{v^2}{2g} \right) s.$$

Loss by leakage has not been included in the losses thus far considered, but it should in a well-constructed motor be very trifling.

### Efficiency.

The efficiency of a hydraulic pressure engine

$$\epsilon = \frac{W}{E} = \frac{Q P}{Q p} = \frac{P}{p},$$

where both $P$ and $p$ are measured in pounds per square inch.

Substituting for $P$ its value as previously found

$$\epsilon = \frac{p - \zeta \frac{v^2}{2g} s}{p}.$$

This equation shows very plainly how with a given velocity $v$ the efficiency must increase with the pressure, or with given pressure increase as the velocity decreases.
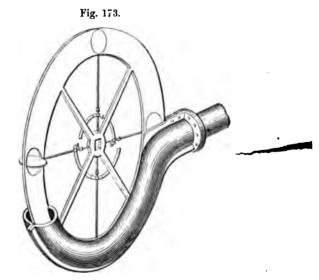
## *Sir William Armstrong's Motor.*

To Sir William Armstrong appears to be due the credit of having designed and constructed the first hydraulic pressure engine. His original idea in connection with the subject was to adopt the rotary type of engine, and having arrived at a design which he considered fulfilled the desired conditions, he sent it to the *Mechanic's Magazine*, in which journal it was fully described and illustrated under date December 29th, 1838. Fig. 173 shows this design, which, in the inventor's own words,[1] "may be briefly described as consisting of a wheel with a flat rim, containing four equidistant pistons folding into circular apertures, and intersecting longitudinally a curved tube open at the lower end, and communicating at the upper end with the supply pipe. The pistons open out as they enter the tube and fold up on leaving it, and each piston takes the pressure of the column before the preceding one loses it. The opening and closing of the pistons in the order required is effected by external cams and slides giving motion to the pistons through the axles on which they turn."

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[1] "Minutes of Proceedings of the Institution of Civil Engineers," vol. i. p. 65.

Subsequently a working model of this motor was constructed and tried at Newcastle and Gateshead, in connection with the town water-pipes.

Its efficiency is stated to have been 95 per cent. of the theoretic power of the fall as represented by the pressure. The pressure used was equivalent to a head of 131 feet, and at a speed of 30 revolutions per minute the motor developed about 5 horse-power. The rotary

Fig. 173.



Sir William Armstrong's Rotary Motor.

type of engine seems never to have been actually applied in practice, and the first of Sir William Armstrong's hydraulic pressure engines commercially used was constructed on the reciprocating principle. "Each engine was composed of two cylinders placed at an angle of 90° to each other and working upon the same crank-pin." "Balanced cylindrical valves were used, and the passages were made very large to keep down the velocity

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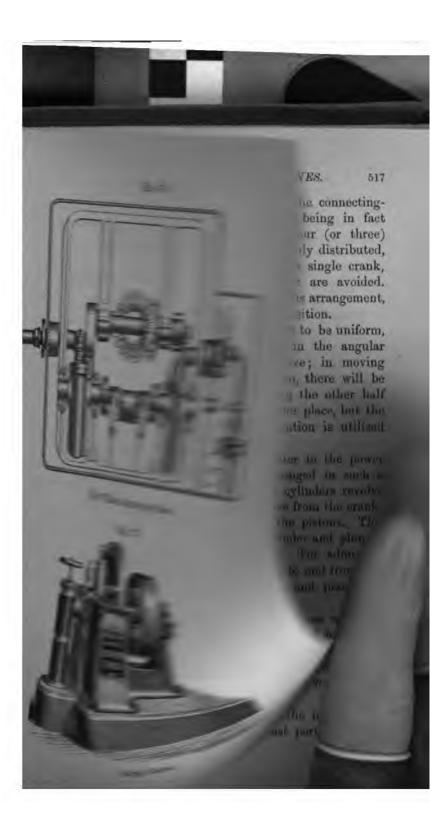
of the water. Relief valves were also applied to prevent shock at the end of the stroke." The engines in question replaced overshot water-wheels at Allenhead, and were used for crushing and washing lead ore and for raising material from the shafts. The head of water available was about 200 feet.

Subsequently this type of motor was adapted to working with accumulators under very high pressure. The pattern now generally used by Sir William Armstrong has two oscillating cylinders with differential pistons working on over-end cranks placed at right angles to each other. "The upper side of the piston presents only one-half the effective area of the lower side, in consequence of the displacement of the ram, and the pressure upon that half area is constant, a passage being always open to the supply pipe. The other side communicates alternately with the pressure and the exhaust, so that the engine acts by difference of pressure in one direction and by an unopposed pressure in the opposite direction." "The action is governed by a two-port slide valve of a cylindrical form, placed either within the trunnion or in a prolongation of it, and worked direct by the oscillation of the cylinder, and a relief valve is applied to prevent concussion from shut-in water." This form of motor is illustrated in Fig. 174.[1]
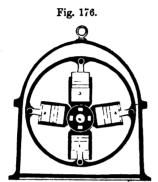
## A. Rigg's Hydraulic Motor.

In this motor the inventor has sought to combine the advantages of a rotary with those of a reciprocating engine while avoiding their drawbacks. A perspective view of it is shown in Fig. 175. Its construction

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[1] For this and the preceding illustration the author is indebted to the courtesy of the Institution of Civil Engineers.

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will be best understood by reference to the diagram, Fig. 176;[1] from this it will be seen that the engine consists of four cylinders—in hydraulic motors there are generally only *three,* but the essential features are the same —which revolve freely and independently about a common central stud. The pistons of these cylinders, which are single acting, are connected through the piston-rods to corresponding crank - pins, equidistant from each other, attached to a disc keyed to the main engine-shaft. The latter is not concentric with the stud above-mentioned, but the axes of both are parallel; if, therefore, the disc revolves, so must also the cylinders, but about another centre, and it is clear that to allow this movement every piston must reciprocate relatively to its cylinder, although both have absolutely only a simple rotary motion. In principle the result is the same as though the engines worked in the usual manner on a crank of a radius equal to the eccentricity of the two centres (that of the shaft and that of the stud), the stud taking the place of the crank-pin, but as the cylinders and pistons are respectively balanced among themselves, the effects resulting from the unbalanced dynamic action of reciprocating masses in the ordinary type of engine are avoided.

Fig. 176.

The stroke of each piston is equal to twice the eccentricity of the two axes, and the power developed may be calculated in the usual way.

   [1] Reproduced, with the kind permission of Mr. A. Rigg and the editor, from *The Engineer.*

The turning effort on the disc to which the connecting-rod ends are attached is very constant, being in fact exactly equivalent to that exerted by four (or three) single-acting engines with the cranks evenly distributed, only in this case all act in one plane on a single crank, so that alternating stresses on the shaft are avoided. There are of course no dead centres with this arrangement, so that the engine can be started in any position.

Assuming the rotary motion of the shaft to be uniform, there must necessarily be a variation in the angular velocity with which the cylinders revolve; in moving from the position 3 to 1 in the diagram, there will be an acceleration, while from 1 to 3, during the other half of the revolution, a retardation must take place, but the power absorbed in effecting the acceleration is utilized in the subsequent retardation.

To proportion the expenditure of water to the power required, the hydraulic motors are arranged in such a manner that the centre about which the cylinders revolve can be moved so as to alter its distance from the crank-shaft centre, and thereby the stroke of the pistons. This movement is effected by a hydraulic cylinder and plunger acting on a cross-head carrying the stud. The admission and exhaust of the water under pressure to and from this is controlled by a centrifugal governor, and practically absorbs no power.

Each cylinder has at its lower end a boss with flat faces, through which the central stud referred to passes, and each boss is so arranged that its faces fit accurately against those of the neighbouring bosses, and contain ports for the passage of the water on its way to the cylinders.

The outside boss, at the end at which the inflow takes place, contains the admission and exhaust ports to all the

cylinders; as the latter revolve, these ports are opened and closed in succession by a stationary valve, nearly balanced, which works against the outer face of the boss.

These motors run at a high speed. One of them, driving a dynamo in connection with glow-lamps, works regularly at 250 revolutions a minute, and can be made to revolve at a much higher speed if required.

It may not at first sight be obvious in what way the effects produced by rapidly reciprocating masses in ordinary engines are avoided in the case of Mr. Rigg's motor, since a relative reciprocation and reversal of the motion of the plungers actually occurs. Consideration will however make it clear, that the variation in the angular velocity with which the cylinders revolve takes the place of the acceleration and retardation of the reciprocating masses in an ordinary motor; the smaller the eccentricity of the central stud relatively to the crank centre, the less is the amount of relative reciprocation and variation in the angular velocity.

## A. Schmid's Hydraulic Motor.

This was one of the first motors of the class under consideration designed to meet the wants of small manufacturers and for domestic purposes, and is in many respects the best. Its construction is shown in Figs. 177, 178, and 179.[1]

The cast-iron cylinder is closed at the end next the crank by a cover furnished with a long and massive stuffing-box, which serves as a guide for the piston-rod.

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[1] For these and many of the subsequent illustrations of hydraulic pressure engines the author is indebted to Knoke's work, *Kraft-maschinen des Kleingewerbes.*

Fig. 177.
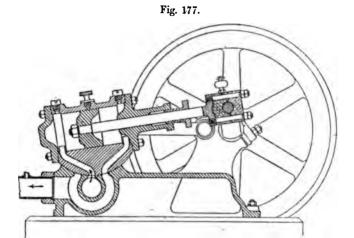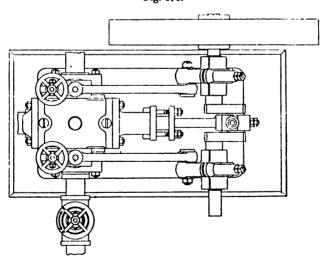
Fig. 178.
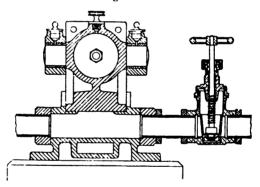
Schmid's Hydraulic Motor.

This is necessary, owing to the engine being of the oscillating type.

The cylinder rests on a cylindrical surface, about the axis of which the oscillation takes place, and this surface forms at the same time the valve-face and works in a corresponding hollow in the bed-plate, which contains the admission and exhaust passages, and in reality takes the place of a slide-valve, only in this case the usual order is reversed, and the valve is stationary while the cylinder moves. The valve has three ports, of which

Fig. 179.

the middle one is in communication with the admission pipe, and as the cylinder oscillates communicates through the passages in the latter alternately with opposite ends of the cylinder. Through the two remaining ports the exhaust takes place, both being connected with one off-flow pipe. The construction will be clear from the accompanying illustration, Fig. 179.

On either side of the cylinder are necks, the axis of which coincides with that about which oscillation takes place. These necks have their bearings in two levers pivoted at one end to the frame near the crank-axle, and

by means of a hand-wheel and screw acting on the other
end of each lever the cylinder can be pressed down upon
its bearing surface. In order to lubricate or inspect
the valve-face the cylinder can be lifted quite off the
bed by unscrewing the hand-wheel.

The piston is solid. In small machines it is accurately
turned and ground in; in larger, a leather collar is used
to make it water-tight.

A small copper air vessel is used in connection with
the motor, and for larger sizes a small air-pump to
replace loss by leakage and absorption of the air by the
water.

The capacity of the air vessel is from 2 to $2\frac{1}{2}$ times
the volume swept through by the piston.

Experiments made by Professors Zeuner and Kronauer
show that an efficiency as high as 90 per cent. can be
obtained with Schmid's motors; the maker himself
guarantees 80 per cent.

### *Hydraulic Motor of Wyss and Studer.*

This engine, illustrated in Figs. 180, 181, 182, and 183,
is also of the oscillating type, and differs from Schmid's
chiefly in the fact that the valve surface is flat instead of
curved.

The cylinder rocks on two necks, which have their
bearings in the main frame. On the under side of the
cylinder a chamber is formed, in the vertical sides of
which are the ports for the distribution of the water.
The fluid enters simultaneously from both sides into this
chamber, and thus any one-sided pressure is avoided.
A vertical partition (the prolongation of which would pass
through the axis of oscillation) divides the chamber into
two parts, each of which communicates through one

Fig. 180.
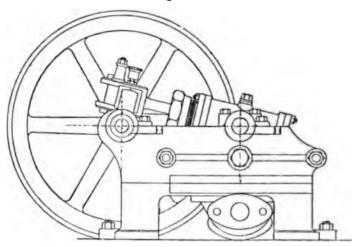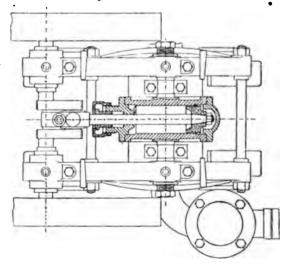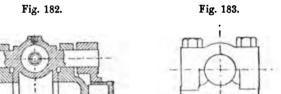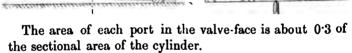


Fig. 181.



Motor of Wyss and Studer.

opening with one end of the cylinder, and through two lateral ports with the admission and exhaust passages alternately. The vertical surfaces, on the outside of the cylinder chamber containing the ports, work against corresponding faces on the sides of two valve-chests, attached to the main frame, through which the water enters and leaves the cylinder. Fig. 183 shows a section through one of these chests, which has three divisions. Through the middle the water is admitted, and by the two side passages exhausted.

Fig. 182.

Fig. 183.



The area of each port in the valve-face is about 0·3 of the sectional area of the cylinder.

The valve-chests above referred to can be pressed by means of screws on both sides against the bearing surfaces on the cylinder chamber, so as to make a water-tight joint. Below the main frame is a casting containing the admission and exhaust passages to which the inlet and off-flow pipes are connected. This casting is so arranged that it can be turned round through 180°, so that, according to circumstances, the connections can be made on either side of the motor. The joint between the frame casting and each valve-chest is made tight by

means of two conical gun-metal tubes, one within the other, which pass easily through the frame-plate, and are forced by the pressure of the water against the sides of the openings through which they project. The valve-chests have bearings at the upper ends through which the cylinder necks pass, to insure their being kept in the proper relative position.

## Haag's Hydraulic Motor.

In this hydraulic engine—of which Figs. 184 and 185 show the construction — the necks about which the cylinder oscillates contain the ports and passages for admission and exhaust, which takes place on both sides. The water enters and flows off through the main frame, as in Schmid's motor, but the necks do not rest directly on the valve surface of the frame, but upon intermediate bushes, which serve as the lower brasses of the bearings and can be renewed when worn out. The cylinder and frame are of cast-iron, the piston of gun-metal; the latter is made water-tight by a leather collar.

The connecting-rod head is guided in a continuation of the cylinder casting so as to relieve the stuffing-box of all lateral pressure, and thus serves the purpose of a cross-head.

It has been claimed as an advantage of this motor over that of Schmid, that the surfaces for friction are smaller, but as the friction depends chiefly on the total pressure and very little on the area of the bearing surface, it is desirable to make the latter as large as practicable in order to reduce wear.

The following are the results of experiments made at Erfurt in 1878 with one of Haag's hydraulic motors, having a diameter of 50 millimètres (2 inches) and 75

Fig. 184.
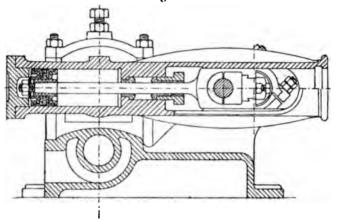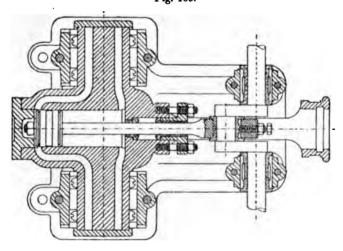


Fig. 185.



Haag's Hydraulic Engine.

millimètres (3 inches) stroke, as given in Knoke's work on *Kraftmaschinen des Kleingewerbes.*

| Number of revolutions per minute. | Horse-power as measured on the brake. | Pressure in equivalent head, feet. | Water consumed in cubic feet per minute. | Available horse-power. | Efficiency. |
|------|------|------|------|------|------|
| 351 | 0·406 | 98·4 | 3·920 | 0·728 | 0·56 |
| 372 | 0·451 | 114·8 | 4·132 | 0·900 | 0·50 |
| 335 | 0·406 | 123·0 | — | — | — |

## *Philip Meyer's Engine.*
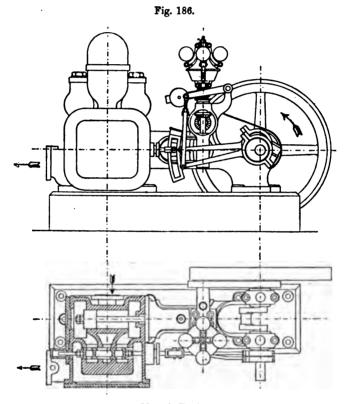
The hydraulic engine of Philip Meyer is of a special type, deserving notice chiefly on account of the fact that the admission of water can be cut off before the full stroke is completed.

The cylinder of the motor, which in general construction resembles an ordinary stationary steam-engine, has at either end a chamber filled with air, the contents of which at the end of a stroke must be compressed to the initial pressure. The water as it enters the cylinder therefore encounters an elastic cushion which prevents the occurrence of shocks.

Fig. 186 illustrates the construction of the engine. The cylinder, with the air chambers at each end, is cast in one piece with the valve chest, and has only one cover, at the back end; at the front end it is bolted to a strong forked frame containing the crank-shaft bearings and the cross-head guides.

The slide valve has parallel surfaces at top and bottom, and is balanced and kept tight by an adjustable plate on the back. This plate has recesses corresponding to the ports in the cylinder, which have the effect of making the

area of admission for a given opening of the valve twice as great as it would otherwise be, and at the same time serve to relieve more completely the pressure on the valve.

Fig. 186.



Meyer's Engine.

This arrangement, although it has not been found to answer with steam, gives very satisfactory results with water where the latter carries no sand or grit.

The valve is worked by an ordinary link motion, with open eccentrics (Stephenson's).

The governor, which is of a powerful type, acts on the link and regulates the cut-off, the peculiar feature of this motor which has yet to be explained.

The air vessel is placed on the top of the valve chest, and the water enters the latter from below.

In order to replace any air lost by leakage or absorption, the following arrangement is employed: Over each end of the air chambers, and in connection with the latter, are small expansion air vessels; at the side of each of these vessels is a suction or snifting valve, which, when the pressure in the cylinder sinks below that of the atmosphere, allows air to enter. At the top of the same air vessels are check valves communicating through a pipe with the main air vessel on the valve chest, which are kept down on their seats by the pressure. When the pressure of the air in the cylinder exceeds the pressure in the main, the air escapes through the check valves, but, instead of being lost in the atmosphere, passes into the main air vessel to compensate for loss.

Coming now to the action of the motor, let it be assumed that before the slide valve opens the air chamber at one end of the cylinder is filled with air at the pressure of the water in the main; water is then admitted, and does its work throughout the forward stroke. At the commencement of the return stroke, when the exhaust takes place, the pressure sinks very rapidly to that of the atmosphere. At a certain point before the end of the return stroke is reached, the exhaust passage is closed, and the contents of the cylinder are compressed during the remainder of the stroke, until the volume is reduced again to that of the air chamber. The work required for this compression, when water is admitted throughout the whole stroke, is entirely

lost. The matter is however different when the cut-off takes place before the stroke is completed; in that case, up to the point of cut-off, the water does its work at —approximately—constant pressure, and during the rest of the stroke the compressed air expands, giving out again —when the proportions are correct—the energy required for compression. During the latter period the water in the cylinder merely acts as an extension, so to speak, of the piston. Under these conditions no loss is involved in the use of the air cushion, but it is necessary that for a given maximum pressure and air-chamber volume the latter should bear a definite ratio to the volume behind the piston at the moment the cut-off takes place. A deviation from this ratio entails loss. It will be seen that Meyer's motor is in reality a combined air and water engine, in which the air is compressed and expanded.

### *Theory of Meyer's Engine.*

As in the case of this motor not merely the action of water under pressure, but the expansion of air, is concerned, it is necessary to undertake a separate theoretical investigation.

Let $p_1$ be the initial pressure, $v_1$ the volume of each air chamber, $V_1$ the volume swept through by the piston up to the point of cut-off, $V$ the total volume swept through, and $V_2$ that at which the exhaust closes (not including $v_1$); further, $p_0$ the pressure of the atmosphere.

As a result of the contact with a large body of water, the expansion or compression of the air in the cylinder may be assumed to take place at constant temperature, according to the law expressed by

$$p\, v = \text{constant},$$

M M

where $p$ and $v$ are respectively the corresponding pressure and volume at any given moment.

At the time the exhaust closes all water has been expelled from the cylinder, and there remains only air of atmospheric pressure $p_0$ and the volume $v_1 + V_2$; this is compressed to the volume $v_1$ of the air chamber and pressure $p_1$ of the main; hence there follows

$$p_1 v_1 = p_0 (v_1 + V_2)$$

and

$$v_1 = -\frac{p_0}{p_1 - p_0} - V_2;$$

from this it is clear that for a given pressure $p_1$ the exhaust must always close at the same volume $V_2$ in order to obtain the necessary amount of compression.
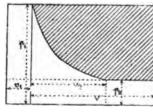
The work required for this compression is

$$W = (p_1 - p_0)\, v_1\, log.\ hyp.\ \frac{V_2 + v_1}{v_1},$$

$$W = (p_1 - p_0)\, v_1\, log.\ hyp.\ \frac{p_1}{p_0}$$

and when the admission takes place throughout the whole stroke, Fig. 187, this work is lost; in order to minimize this loss $v_1$ should be as small as possible. Under these conditions it will be seen that to avoid shocks efficiency has to be sacrificed to some extent. The efficiency $\epsilon$ is expressed by

Fig. 187.



$$\epsilon = \frac{(p_1 - p_0)\, V - W}{(p_1 - p_0)\, V} = 1 - \frac{v_1}{V}\ log.\ hyp.\ \frac{V_2 + v_1}{v_1}.$$

When the cut-off takes place at the volume $V_1$, Fig. 188, before the end of the stroke, the following relations exist between the pressures and volumes :—

$$p_1 v_1 = p_o (v_1 + V - V_1),$$

and
$$p_o (V_2 + v_1) = p_1 v_1,$$

whence $\qquad V - V_1 = V_2;$ consequently

$$v_1 = \frac{V - V_1}{p_1 - p_o} p_o = \frac{p_o}{p_1 - p_o} (1 - E) V,$$

where $E$ is the cut-off ratio $\frac{V_1}{V}$. It will be seen from

this that $v_1$ the volume of the air chamber is dependent on the expansion ratio, and if this is changed with a constant value of $v_1$, the action of the engine ceases to be quite correct.



Fig. 188.

In the case just considered, when $v_1$ is correctly proportioned with regard to the expansion ratio, no loss is incurred by the use of air, the work done in compression being restored during expansion, on the assumption that both compression and expansion are isothermal.
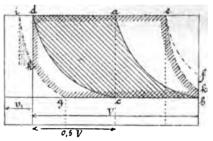
When the ratio of expansion is changed, while $v_1$ remains unaltered, the process occurring is as follows : Supposing 0·5 to be the correct point of cut-off for a motor working with a pressure of say 60 lbs. per square inch absolute, and that this is suddenly changed to 0·8, the air, instead of expanding from $v_1$ to $v_1 + 0·5 \; V$, can only expand to the volume $v_1 + 0·2 \; V$, and consequently

the corresponding pressure is above that of the atmosphere at which compression commenced. When the exhaust is opened this excess of pressure is blown off, and compression commences again at a volume of $v_1 + 0{\cdot}2\ V$, *less* than that of the previous stroke owing to the change in the cut-off. When the volume of the charge has been reduced to $v_1$ the pressure will only be increased in the ratio of $\dfrac{v_1 + 0{\cdot}2\ V}{v_1}$ (instead of as before $\dfrac{v_1 + 0{\cdot}5\ V}{v_1}$), and will be *less* than 60 lbs., so that on

Fig. 189.



the admission of water at the latter pressure from the main the air has to be compressed by the water before equilibrium is established, and the work required for this purpose is *lost*.

The diagram, Fig. 189, illustrates the process; *a b c d* is the original indicator diagram with a cut-off at $0{\cdot}5\ V$; *e* is the new point of cut-off, and *e f* the expansion curve for the latter, with which the terminal pressure is in excess of that of the atmosphere by *b f*.

At *g* compression commences under the new conditions; *g h* is the compression curve up to the end of the back stroke, and *h i* represents the further compression by the

inflowing water. The area shaded with dotted lines is equivalent to the work done by the water in compressing; *c i* is proportional to the total quantity of water used, while *e d* represents that usefully employed, the difference *d i* being wasted. *d e k b g h* is the effective diagram for the new point of cut-off when regular working has been established.

When the cut-off takes place at an earlier point than the correct one, the pressure at the end of compression on the return stroke rises above that in the main, and to meet this the check valves already described as communicating with the main air vessel are provided.

Fig. 190.
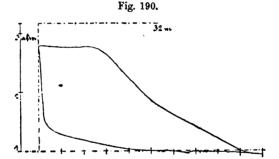


Fig. 190 shows an indicator diagram from Meyer's engine.

## *Hastie's Motor, with Variable Stroke.*

About the year 1879 Mr. John Hastie of Greenock brought out a water engine designed to remove the chief objection to hydraulic pressure motors, viz. their incapacity for economic regulation. In Hastie's machine the stroke is variable, and adapts itself automatically to the power required. Figs. 191, 192, 193, 194, 195, and 196 show

Fig. 191.



Fig. 192.

Ins. 12    6    0    1    2    3    4 Feet

Scale 1/16th
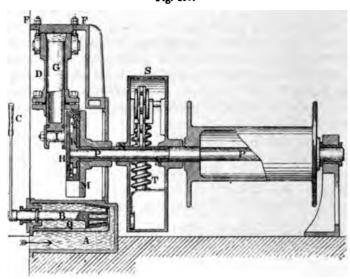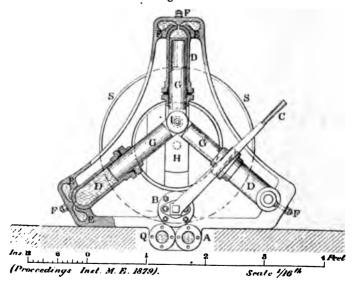
the construction as applied to a hoist. There are three oscillating cylinders *D*, to which water is admitted through passages at *E*, acting alternately as admission and exhaust ports; the oscillating ends of the cylinders take the place of valves in distributing the water. The supply pipe *A* communicates with the cock *B*, worked by the handle *C*, which controls the action of the hoist, and can be used
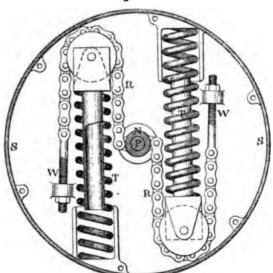
Fig. 193.



as a reversing valve at the extreme positions and a brake in the central position. In the latter case both ports are placed in connection with the exhaust passage *Q*, which is made with a bend so as to contain at least as much water as will fill the three cylinders.

All the plungers of the three cylinders are connected to the same crank-pin *I*, which, however, is not rigidly

secured to the crank disc *M*, but is attached to a block



Fig. 194.
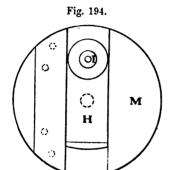


Fig. 195.
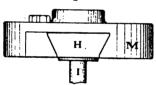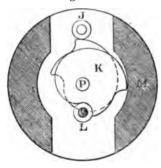


Fig. 196.

*H*, sliding in a radial groove in the latter. At the back of this block, revolving on pins projecting from it at either end, are two small rollers *J* and *L*, which run on the circumference of a peculiarly shaped cam *K*. The cam is keyed to a spindle *P* passing through a concentric hollow shaft *N* carrying the crank disc. Supposing the spindle to be held fast while the hollow shaft and crank disc revolve, the block *H* will be displaced radially by the cam *K*, and with it the crank-pin, thus altering the stroke of the pistons. A hollow cylindrical casing *S* or drum is keyed to the spindle *P* and runs loose on *N*; within this are two rollers carried in forks at the ends of hollow rods. These rods with the rollers are pressed outwards towards the circumference by helical springs *T*, while over each roller passes a chain *R*, one end of which is attached to a snug
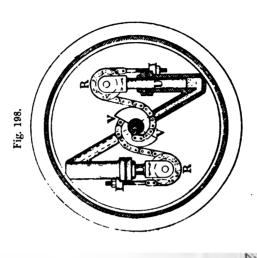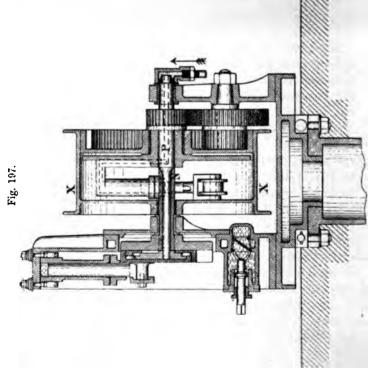
*W* projecting from the side of the drum, the other to the hollow shaft *N*.

When water is admitted to the cylinder under pressure the crank disc begins to revolve, and with it the hollow shaft *N*, while the spindle *P* carrying the cam *K* is held fast by the resistance of the load applied through the hoisting chain to the circumference of the chain barrel. The result of this is that the chains *R* are wound up on *N*; at the same time, owing to the motion of the crank disc relatively to the cam, the block *H* with the crank-pin is pushed outwards and the stroke increased. The winding up of the chains compresses the springs, and this compression and the simultaneous increase of the stroke go on until the resistance of the springs balances that of the pulley, and then the latter is driven round and a state of equilibrium established, which lasts as long as no change occurs in the load or pressure. If the load is increased, a motion of the crank disc relatively to the cam again takes place, until the turning moment on the crank equals that of the load on the pulley by the alteration of the crank radius or stroke.

In engines constructed to work with very high pressures the springs are not employed, but the arrangement shown in Figs. 197 and 198 is used instead. In this case the springs are replaced by two water rams *U*, which are in connection with the supply pipe through the centre of the shaft *P*, and are consequently under the same pressure as that used to work the engine. The chains *R* act in the same way as with springs, but instead of being wound directly on the hollow shaft *N*, they are wound on cams *V*; in this way greater power is required to force back the rams in proportion as the chains *R* act at an increasing distance from the centre of the shaft.

Fig. 198.


Fig. 197.

The following are the results of experiments made by Mr. Kinniple, chief engineer to the Greenock Harbour Trustees, on a small hoist for the Greenock Infirmary ; the lift was 22 feet and the pressure in the cylinder 80 lbs. per square inch :—

Weight lifted $\left\{ \begin{matrix} \text{Chain} \\ \text{alone} \end{matrix} \right\}$ 427 633 745 857 969 1081 1193

Water used, gallons. $\left. \right\}$ $7\frac{1}{2}$   10   14   16   17   20   21   22

From this it will be seen that the water consumed is approximately proportional to the work done.

## *General Remarks.*

Hydraulic pressure engines are suited for utilizing comparatively small quantities of water at higher pressures than are generally advantageous with turbines. Whereas the speed of a turbine is determined by the velocity of flow, which in its turn depends on the head, irrespective of the quantity of water supplied, in a hydraulic pressure engine with a given supply and head the speed may, within reasonable limits, be made high or low as desirable, by giving the piston a smaller or greater diameter and inversely corresponding stroke. Efficiency is promoted by a low piston velocity, but of course with high pressures the speed may, for a given efficiency, be greater than with lower pressures. Probably no other class of motor, except an electric motor, can be so efficient as a hydraulic pressure engine, but the latter suffers under the disadvantage, as generally constructed, that any regulation of the power developed is accompanied by a serious reduction in the efficiency. Such regulation can only be effected, where a constant speed is required, through
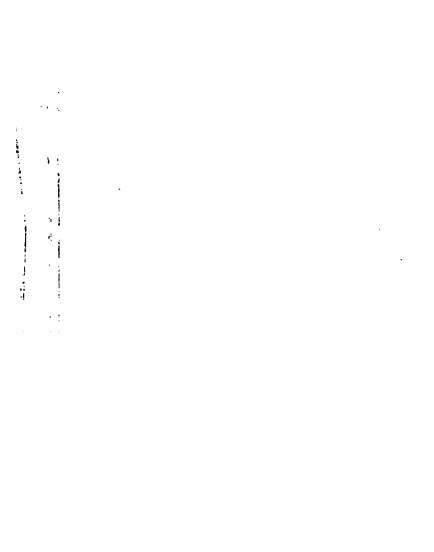
a reduction of the pressure on the piston by throttling the supply of water. This means purposely wasting the available energy with the object of reducing the pressure. If, for example, the resistance to the piston decreases to one-half of its maximum value, and no change in the speed is allowed, the stop-valve orifice is reduced so that the velocity of flow through it is increased, and the sudden change from this velocity to that of the piston results in a greater loss of energy than occurs with the valve orifice fully open. The effect shows itself in a diminution of pressure, which must be adjusted to the resistance to be overcome, that is, in the assumed instance, reduced to one-half the pressure employed at full power. As the loss of energy is proportional to the reduction of pressure the disadvantages of this method of regulation are very evident.

The hydraulic motors of Rigg and Hastie, previously described, have been designed to overcome the defect in question by varying the *stroke* of the engines according to the power required. In this way the pressure on the piston is maintained practically constant, while the quantity of water used is varied, and no loss of efficiency is necessarily entailed by the variation.

In some places on the continent of Europe small hydraulic pressure engines are much used in connection with the water from the town mains. In mines hydraulic engines of a larger kind are often employed, on account of the comparative ease and safety with which the water power can be transmitted through pipes to different distant parts of the workings.

Water under high pressure is in many respects a good medium for the transmission of power to considerable distances, but its application to that purpose does not come within the scope of this work.

There are, of course, many varieties of hydraulic pressure engines besides those described, but the latter, it is believed, include the leading types with the exception of the hydraulic ram, which is used exclusively for pumping, and does not come within the category of general motors.

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